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Propagation and Interaction of Ultrashort Light Pulses in Nonlinear Media: Numerical Solution of Maxwell Equations

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Finite-difference time-domain method for numerical solution of Maxwell equations has been applied to the problems of propagation and interaction of ultrashort light pulses in the media with linear dispersion and kerr-like nonlinearity. The evolution of temporal and spectral structures of femtosecond laser pulses has been analyzed in the modes of self- and cross-phase modulation, formation of shock waves, and soliton-like temporal pulses. Interaction of two ultrashort laser pulses in nonlinear media has been studied for the case of collinear propagation. The peculiarities of transformation of spatial-temporal structure of probe pulse have been analyzed numerically in the regime of its reflection from the pump pulse.

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1. Introduction

The development of research in the field of optics of ultrashort light pulses in the last decades is due to the prospects of their application in a variety of optical phenomena [1–3]. Extremely short duration of light pulses allows us to study fast processes under the pump-probe scheme. By the nature of changes in the probe light pulse one can estimate the changes of the properties of the medium induced by pump pulse. Ultrashort light pulses can also be used in fiber-optic communication lines to transmit data at an extremely high rate. Besides, high intensity of ultrashort laser pulses can lead to initiation of nonlinear effects, e.g. generation of frequency harmonics [4].

A necessary step in the study of the interaction of laser radiation with matter is a theoretical analysis and modeling of spatial and temporal structure of ultrashort pulses under they propagation in linear and nonlinear media, taking into account the processes of diffraction-limited spatial beams and dispersion of short pulses. At present, special attention is paid to the methods of modeling the propagation of ultrashort pulses, based on the direct numerical integration of Maxwell equations [5]. This approach allows us to trace the dynamics of the interaction of electromagnetic radiation with various kinds of complex optical systems [6–8], without resorting to any approximations and simplifications. Adequate numerical simulation makes it possible to largely replace labor-intensive

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and costly experimental work in the early stages of research.

The aim of this work is the development of theoretical and numerical model to describe the evolution of the space-time structure of ultrashort light pulses under they propagation and interaction in nonlinear media.

The paper is structured as follows. Section 2 is devoted to the development of theoretical and numerical models for the study of the laws of propagation of high-power ultrashort laser pulses in a medium with linear dispersion of optical properties and the Kerr nonlinearity. The developed approach is based on the numerical solution of Maxwell equations by finite-difference approximation in spatial and temporal domain. Section 3 presents the results of modeling of propagation and collinear interaction of ultrashort pulses. The obtained results are discussed in terms of transformation of spatial, temporal and spectral characteristics of laser pulses.

2. Theoretical model

In a theoretical consideration of propagation of ultrashort laser pulses and beams in media with dispersion and nonlinearity a wave equation is usually used in the form [1]:

$$\frac{\partial E}{\partial z} = \frac{i}{2k_0}\nabla_{\perp}^2 E + ik_0\frac{n_2}{2}|E|^2E - i\frac{k_2}{2}\frac{\partial^2 E}{\partial t^2}, \quad (1)$$

a solution of which, as a rule, numerical, allows to model the spatio-temporal evolution of ultrashort light pulses and beams. Despite the high prevalence in the literature of this type of equation, it should be noted that in its derivation priori laid an approximate description of the dispersion properties of the medium $k(\omega) = n(\omega)\,\omega/c$ in the form of the expansion in the vicinity of the main frequency of the pulse ω_0, that is meant also quasi-monochromatic pulse:

$$k(\omega) = k_0 + k_1(\omega-\omega_0) + \frac{1}{2}k_2(\omega-\omega_0)^2 + \frac{1}{6}k_3(\omega-\omega_0)^3 + \ldots \quad (2)$$

where $k_m = \left[\frac{d^m k}{d\omega^m}\right]_{\omega=\omega_0}$ $(m = 0, 1, 2, 3, \ldots)$ are valid for quasi-monochromatic approximations. As a rule, the terms of the third and higher orders are negligible for pulses with low bandwidth in this equation as $\Delta\omega \ll \omega_0$ and should be considered only k_1 and k_2:

$$k_1 = \frac{1}{c}\left(n + \omega\frac{dn}{d\omega}\right) = \frac{1}{v_g}, \quad (3)$$

$$k_2 = \frac{1}{c}\left(2\frac{dn}{d\omega} + \omega\frac{d^2 n}{d\omega^2}\right) = -\frac{1}{v_g^2}\frac{dv_g}{d\omega} \quad (4)$$

where v_g is the group velocity at the frequency ω_0. From the expression (3) it follows that k_1 determines the velocity of propagation of the pulse. The expression (4) for k_2 determines the change in the group velocity of the pulse as a function of frequency. Therefore, parameter k_2 is the group velocity dispersion.

Application of this method for picosecond pulses is apparently the most desirable, however, to describe the evolution of femtosecond pulses alternative approaches are developed [9–11]. In particular, the promising is the use of FDTD-method of direct numerical integration of Maxwell equations.

The finite-difference time-domain method (FDTD) is a powerful approach to numerical analysis of Maxwell equations [12]. The FDTD method is applicable for a wide range of complex dielectric structures, constrained only by the size of the computational space required for the simulation. Due to its accuracy, the FDTD method is widely used for simulation of light propagation linear and nonlinear media [13–15], in optical waveguides [16, 17], scattering media or photonic crystals [6–8]. FDTD is a grid method for solving Maxwell equations. It uses the explicit scheme of second order accuracy in time step Δt and spatial grid Δx, Δy, Δz, the sampling proposed by Yee [12].

Solving the problem of propagation of the light pulses in a medium with non-linear dependence of the optical properties of the intensity of acting radiation is based on the use of Maxwell equations for the field vectors in the form of:

$$\nabla \times \vec{E} = -\frac{\mu}{c}\frac{\partial \vec{H}}{\partial t}, \tag{5}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}. \tag{6}$$

For cubic nonlinearity $\chi^{(3)}$ of Kerr-type the relationship between the vectors of the electric displacement $\vec{D}$ and the electric field $\vec{E}$ taking into account finite response time of nonlinearity $\tau_{\rm rel}$ is defined by the equations:

$$\vec{D}\left(\vec{r},t\right) = \left[\varepsilon_{\rm lin} + 4\pi\chi\left(t,\vec{E}\right)\right]\vec{E}\left(\vec{r},t\right), \tag{7}$$

$$\tau_{\rm rel}\frac{\partial \chi}{\partial t} + \chi = \chi^{(3)}\vec{E}^2. \tag{8}$$

In one-dimensional case we will consider the propagation of a plane electromagnetic wave taking into account nonlinear effects. Then the equations (1–2) take the dimensionless form:

$$\frac{\partial \tilde{H}_{\tilde{z}}}{\partial \tilde{t}} = -\frac{\partial \tilde{E}_{\tilde{y}}}{\partial \tilde{x}}, \tag{9}$$

$$\frac{\partial \tilde{D}_{\tilde{y}}}{\partial \tilde{t}} = -\frac{\partial \tilde{H}_{\tilde{z}}}{\partial \tilde{x}} \tag{10}$$

where $\tilde{H}_{\tilde{z}} = H_{\tilde{z}}/E_0$, $\tilde{E}_{\tilde{y}} = E_{\tilde{y}}/E_0$, $\tilde{D}_{\tilde{y}} = D_{\tilde{y}}/E_0$, E_0 is the maximum value of the amplitude of the light pulse in the entrance of nonlinear medium; $\tilde{x} = x/\lambda_0$, $\tilde{t} = t/T$ where λ_0 is the wavelength in vacuum, $T = 1/\nu$ is the period (ν is the frequency) of electromagnetic oscillations.

According to the method used in the finite-difference approximation differential equations (5–6) are replaced by the following finite-difference equations in space and time [18]:

$$\tilde{H}_{\tilde{z}}^{l+1/2}\left(i+1/2\right) = \tilde{H}_{\tilde{z}}^{l-1/2}\left(i+1/2\right) - \frac{\Delta\tilde{t}}{\Delta\tilde{x}}\left[\tilde{E}_{\tilde{y}}^{l}\left(i+1\right) - \tilde{E}_{\tilde{y}}^{l}\left(i\right)\right], \tag{11}$$

$$\tilde{D}_{\tilde{y}}^{l+1}\left(i\right) = \tilde{D}_{\tilde{y}}^{l}\left(i\right) - \frac{\Delta\tilde{t}}{\Delta\tilde{x}}\left[\tilde{H}_{\tilde{z}}^{l+1/2}\left(i+1/2\right) - \tilde{H}_{\tilde{z}}^{l+1/2}\left(i-1/2\right)\right]. \tag{12}$$

In these equations, $\Delta\tilde{x}$ is the step over spatial grid along the coordinate $\tilde{x}$, $\Delta\tilde{t}$ is the time step; the unknown functions $F^l\left(i\right)$ are connected with the mesh nodes in the following way: $F^l\left(i\right) = F\left(i\Delta\tilde{x}, l\Delta\tilde{t}\right) = F\left(\tilde{x},\tilde{t}\right)$.

Kinetic equation for determining the nonlinear susceptibility of the medium (4) is approximated as:

$$\chi_{\text{nl}}^{l+1}(i) = \chi_{\text{nl}}^{l}(i) - \frac{\Delta\tilde{t}T}{\tau_{\text{rel}}}\chi_{\text{nl}}^{l}(i) + \frac{\Delta\tilde{t}T}{\tau_{\text{rel}}}\left[\chi^{(3)}E_0^2\right]\left(\tilde{E}_{\tilde{y}}^{l}(i)\right)^2 \qquad (13)$$

and to find the amplitude of the electric field in the next time step we use the following approximation of the equation (3):

$$\tilde{E}_{\tilde{y}}^{l+1}(i) = \frac{\tilde{D}_{\tilde{y}}^{l+1}(i)}{\varepsilon_{\text{lin}}(i) + 4\pi\chi_{\text{nl}}^{l+1}(i)}. \qquad (14)$$

Thus, the numerical solution of the system (7–10) allows us to describe the propagation of the light pulse of arbitrary shape and duration in the nonlinear medium. In the work, the implementation of the FDTD method is complemented by the use of so-called absorbing boundary conditions [18], thus avoiding non-physical wave reflection from the computational domain boundaries. Also it is complied a necessary stability condition in accordance with Courant criterion: $\Delta t/\Delta x \leq v$ where v is the speed of light in the medium.

Let us notice, FDTD-method does not provide a table setting of dependence of the dielectric permittivity on the frequency. One of the approaches of the dispersion modeling within the FDTD-method is the auxiliary differential equation method [19].

For calculations we use the model of Drude – Lorentz dispersion of the dielectric permittivity:

$$\varepsilon_{\text{lin}}(\omega) = 1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{e0}^2 - i\omega\gamma_e}. \qquad (15)$$

To simulate the dispersion properties of the medium displacement vector can be written as:

$$\vec{D} = \varepsilon_{\text{lin}}(\omega)\,\vec{E}. \qquad (16)$$

In one-dimensional case, equation (12), taking into account the frequency dependence of the dielectric permittivity (11), can be rewritten as follows:

$$\tilde{D}_{\tilde{y}} = \left(1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{e0}^2 - j\gamma_e\omega}\right)\tilde{E}_{\tilde{y}}. \qquad (17)$$

Equation (13) can be transformed via the inverse Fourier transform as:

$$\frac{\partial^2\tilde{D}_{\tilde{y}}}{\partial t^2} + \gamma_e\frac{\partial\tilde{D}_{\tilde{y}}}{\partial t} + \omega_{e0}^2\tilde{D}_{\tilde{y}} = = \frac{\partial^2\tilde{E}_{\tilde{y}}}{\partial t^2} + \gamma_e\frac{\partial\tilde{E}_{\tilde{y}}}{\partial t} + \left(\omega_{e0}^2 + \omega_{ep}^2\right)\tilde{E}_{\tilde{y}}. \qquad (18)$$

Let us proceed to dimensionless form by substituting $t = \tilde{t}T$ in (14):

$$\frac{\partial^2\tilde{D}_{\tilde{y}}}{\partial\tilde{t}^2} + \gamma_e T\frac{\partial\tilde{D}_{\tilde{y}}}{\partial\tilde{t}} + \omega_{e0}^2T^2\tilde{D}_{\tilde{y}} = = \frac{\partial^2\tilde{E}_{\tilde{y}}}{\partial\tilde{t}^2} + \gamma_e T\frac{\partial\tilde{E}_{\tilde{y}}}{\partial\tilde{t}} + T^2\left(\omega_{e0}^2 + \omega_{ep}^2\right)\tilde{E}_{\tilde{y}}. \qquad (19)$$

For the convenience of further calculations, we introduce the following coefficients: $a_1 = \gamma_e T$, $a_2 = \omega_{e0}^2T^2$, $a_3 = T^2\left(\omega_{e0}^2 + \omega_{ep}^2\right)$. After the final transformation equation (15) can be written as follows:

$$E_y^{l+1} = D_y^{l+1} + D_y^l\left(\frac{a_2\left(\triangle\,\tilde{t}\right)^2 - 2}{1 + \frac{a_1\triangle\tilde{t}}{2}}\right) + D_y^{l-1}\left(\frac{1 - \frac{a_1\triangle\tilde{t}}{2}}{1 + \frac{a_1\triangle\tilde{t}}{2}}\right) + E_y^l\left(\frac{2 - a_3\left(\triangle\,\tilde{t}\right)^2}{1 + \frac{a_1\triangle\tilde{t}}{2}}\right) - E_y^{l+1}\left(\frac{1 - \frac{a_1\triangle\tilde{t}}{2}}{1 + \frac{a_1\triangle\tilde{t}}{2}}\right). \qquad (20)$$

Thus, the modeling of the light pulse evolution in media with dispersion of the dielectric permittivity is reduced to the numerical solution of equations (7–8) and (16).

3. Results and discussions

First, let us construct the frequency dependence of the real part of the dielectric permittivity defined by the Drude – Lorentz formula, and compare it with the dependence given by the empirical formula of Sellmeier [20] for quartz glass:

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} \quad (21)$$

where the coefficients are: $B_1 = 0.6961663$, $B_2 = 0.4079426$, $B_3 = 0.8974794$, $C_1 = 4.67914826 \cdot 10^{-3}\ \mu\text{m}^2$, $C_2 = 1.35120631 \cdot 10^{-2}\ \mu\text{m}^2$, $C_3 = 97.9340025\ \mu\text{m}^2$.

The following settings were selected to calculate the spectral dependence of the dielectric permittivity from the Drude – Lorentz formula: $\lambda_{e0} = 0.7\ \mu$m, $\lambda_{ep} = 0.75\ \mu$m, $\gamma_e = 0.001$ fs. As it shown in Fig. 1a of the dispersion dependence of the refractive index in the wavelength range of $1 \le \lambda \le 2\ \mu$m the real part of the dielectric permittivity (curve 1) has the form characteristic for the case of the medium with normal dispersion. As seen in the region $1 \le \lambda \le 2\ \mu$m dispersion of the refractive index significantly higher than that for a real material calculated by the Sellmeier formula (curve 2). Such a choice of these parameters enables to demonstrate more clearly the role of dispersion in the evolution of the space-time structure of the laser pulse. The imaginary part of permittivity in this spectral range is positive but close to zero $\text{Im}(\varepsilon) \approx 0$, which should lead to a slight damping of the amplitude of the optical radiation propagating in the material. In our case, the dispersion relation calculated from Dure-Lorentz formula (15), $k_2 > 0$ (Fig. 1b) in considered spectral range $1 \le \lambda \le 2\ \mu$m. Thus, the evolution of ultrashort light pulses will be investigated in the positive group velocity dispersion regime.

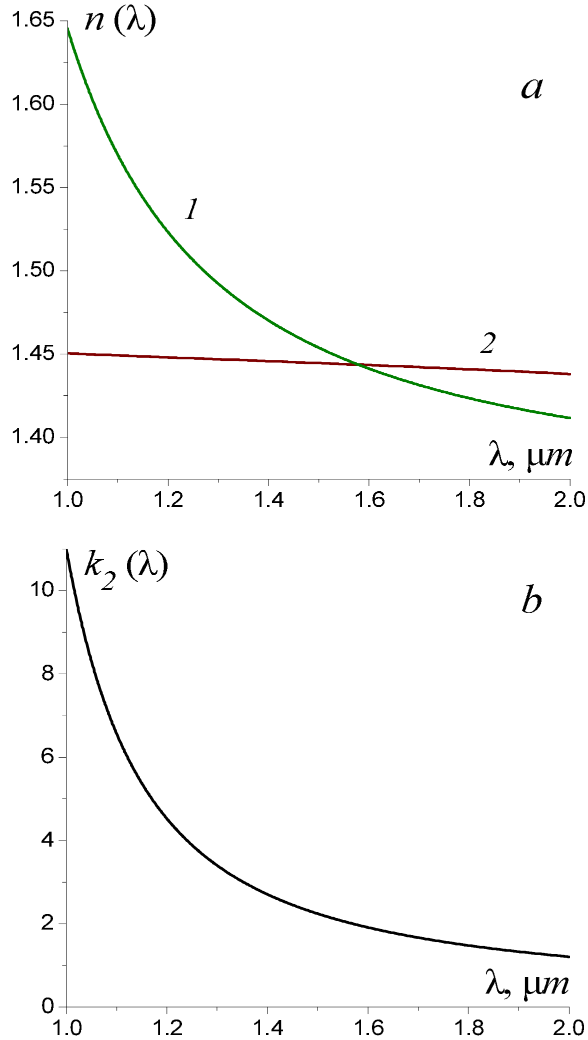

FIG. 1. Dispersion dependencies: (a) - $n(\lambda)$, (b) - $k_2(\lambda)$. 1 – Drude – Lorentz formula; 2 – Sellmeier formula.

Numerical modeling of the problems for short light pulses propagation in the media with linear dispersion of the refractive index and kerr-like nonlinearity has been performed for the following cases: propagation of light pulses in linear medium with dispersion; propagation of intense light pulses in nonlinear regimes, joint action of linear dispersion and nonlinearity. In the numerical modeling, it was assumed that the source of the electromagnetic field is in the form of a quasi-monochromatic wave with pulse profile of Gaussian-like shape: $E_y(t) = E_0 \exp\left[-\frac{(t-t_0)^2}{2\tau_p^2}\right] \sin\left(2\pi\frac{c}{\lambda_0}t\right)$ (where τ_p is a pulse duration, and λ_0 is a wavelength in vacuum). The pulse is started from the left border of the computation domain $x = 0$. The time evolution of spatial distribution of electric field has been calculated, and the results are presented in Figs. 2–6.

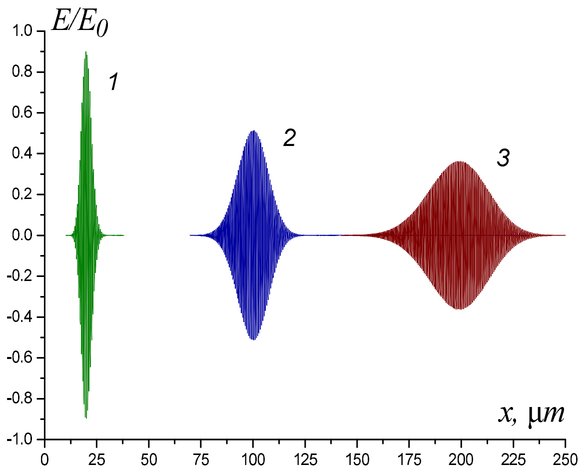

FIG. 2. Evolution of ultrashort light pulse in linear medium with dispersion. $\tau_p = 25$ fs, $\lambda_0 = 1$ μm. (in color)

First, let us consider the effect of dispersion on the temporal structure of a ultrashort pulse with duration $\tau_p = 25$ fs and wavelength $\lambda_0 = 1$ μm. Numerical calculation results are presented in Fig. 2. One can see, there is a broadening of the dispersion characteristic of the light pulse as a result of differences in velocity of propagation of various spectral components. Low-frequency components propagates faster than high-frequency components, and as a result the pulse obtains a positive frequency modulation, which corresponds to the normal dispersion $dn/d\lambda > 0$. The delay of the spectral components is determined by the difference between its frequency and main frequency. Since the spectral width is inversely proportional to the duration of pulses, the pulses of shorter duration disperse more quickly than pulses of longer duration. Also, the nature and broadening rate of the pulses depend on its the shape and on the presence of frequency modulation.

Next, consider the problem of propagation of ultrashort laser pulses in a Kerr medium with instantaneous response nonlinearity ($\tau_{\rm rel} = 0$). The nonlinearity of the medium is given in units $\chi^{(3)}E_0^2$. Fig. 3 shows the evolution of temporal structure of pulse and its spectrum at different depths of penetration of the nonlinear medium. It is seen that the pulse shape, despite the passage of a considerable distance varies slightly, but the characteristic broadening of the spectrum occurs sufficiently symmetrical about the main frequency $\omega_0 = 2\pi c/\lambda_0$. The broadening is caused by a phase shift which is linearly dependent on the intensity and the distance traveled: $\varphi_{\rm nl} = -n_2E^2(t)\,z\omega_0/c$. This phase shift leads to a broadening of the frequency spectrum of the pulse, the value of which can be estimated as: $\Delta\omega_{\rm nl} \approx \varphi_{\rm nl}(0)/\tau_p$, that is inversely proportional to the pulse duration. Note also that the frequency modulation is positive, that is, long-wavelength components arranged in front of the pulse and the short-located at the trailing edge. For small values of the nonlinearity parameter results are in good agreement with the typical solutions of the wave equation (1), however, we used an approach that provides much more information about the spatial and temporal structure of the ultrashort light pulse, at the time, as a solution to the slowly varying amplitude wave equation allows us to investigate a change in the envelope of the wave packet only.

When the initial pulse amplitude increases, in units $\chi^{(3)}E_0^2$, there is a significant expansion of the frequency spectrum of the pulse during

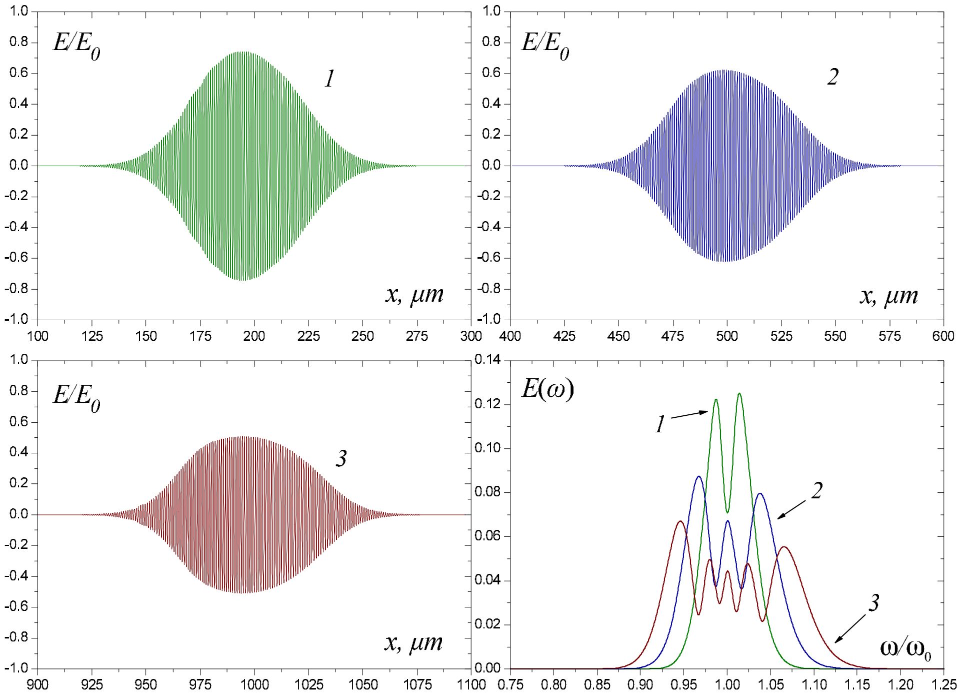

FIG. 3. Evolution of ultrashort light pulse and its spectrum in nonlinear medium $\tau_p = 100$ fs, $\lambda_0 = 1$ μm, $\chi^{(3)}E_0^2 = 0.01$. (in color)

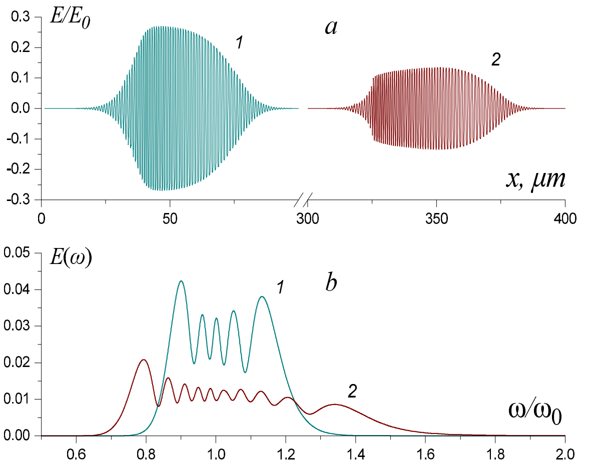

FIG. 4. Evolution of ultrashort light pulse and its spectrum in nonlinear medium $\tau_p = 50$ fs, $\lambda_0 = 1$ μm, $\chi^{(3)}E_0^2 = 0.05$. (in color)

its propagation in the nonlinear medium. Also, with the propagation in a nonlinear medium steepening of the pulse tail develops, due to the fact that the center of the Gaussian pulse with high intensity has a great additive to the refractive index (positive Kerr nonlinearity $n_2 > 0$) and, consequently, a smaller group velocity. As numerical calculations show, for a sufficiently deep penetration of the pulse in the nonlinear medium a shock wave of the optical pulse envelope may develop (Fig. 4).

The combination of group velocity dispersion and nonlinear refraction creates the preconditions for the implementation of a variety of phenomena such as the formation of optical solitons, spatial-temporal self-focusing or defocusing. Various combinations of the coefficient of nonlinearity n_2 and group velocity dispersion can be quite varied alter the spatial and temporal characteristics of the light pulses [1]. By analogy with light beams, short pulses (wave packets) may experience self compression at $k_2 n_2 < 0$ or decompression at $k_2 n_2 > 0$. This is due to the fact that a pulse having a Gaussian

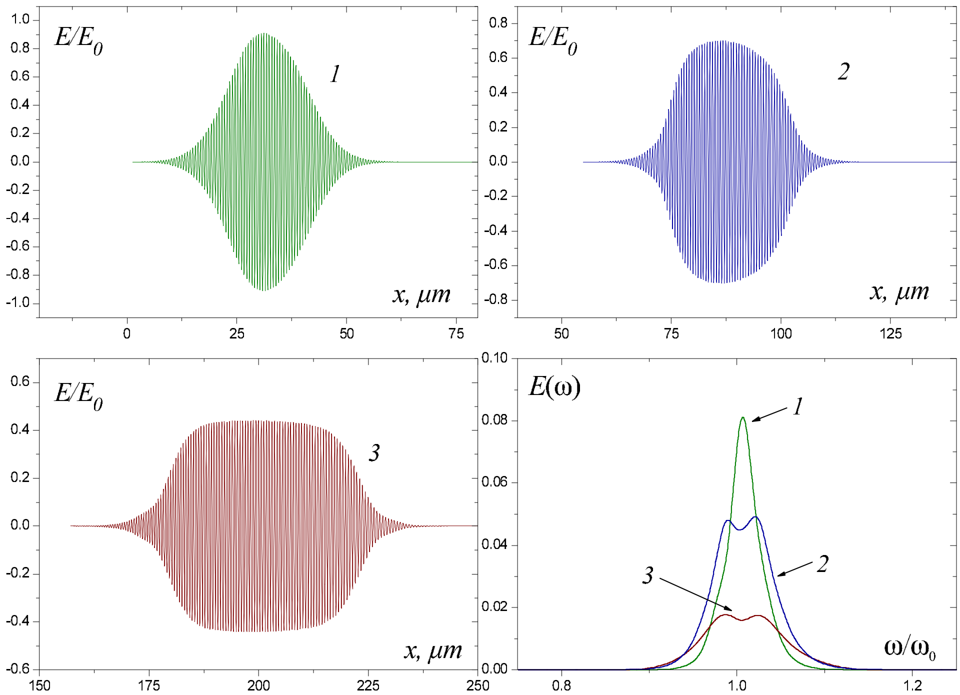

FIG. 5. Evolution of ultrashort light pulse and its spectrum in nonlinear medium with dispersion $\tau_p = 100$ fs, $\lambda_0 = 1$ μm, $\chi^{(3)}E_0^2 = 0.01$. (in color)

temporal profile acquires negative frequency modulation in medium with $n_2 > 0$. Such an impulse is compressed itself with anomalous group velocity dispersion, i.e. at $k_2 n_2 < 0$. With a positive sign of the product of the nonlinearity coefficients and group velocity dispersion decompression develops. Thus, the ratio of the linear effects of dispersion and nonlinearity determines the resulting change in the temporal and spectral structure of an ultrashort light pulse.

Let us consider the evolution of an ultrashort light pulse in a medium with dispersion and nonlinearity (Fig. 5). As shown by numerical calculations, even a small quantity of non-linearity leads to a greater broadening of a light pulse in comparison with the case of linear dispersion. The nature of this broadening is determined by the dominant process. Thus, for short pulse duration the group velocity dispersion

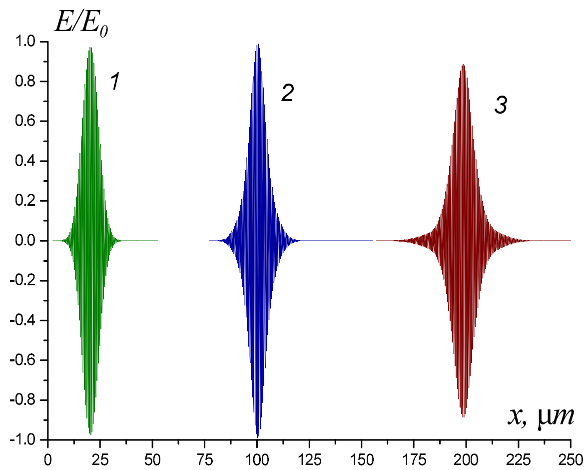

FIG. 6. Formation of soliton-like pulse. $\tau_p = 50$ fs, $\lambda_0 = 1$ μm, $\chi^{(3)}E_0^2 = -0.0002$. (in color)

dominates and the effect of nonlinearity simply increases the broadening rate, but its nature is no different. The situation is changing significantly for long pulse duration. In this case, in addition to increasing the broadening rate change occurs and the pulse waveform. Its envelope approaches the rectangular shape. Increasing the duration is resulting in that the effect of dispersion on the initial broadening is small and its evolution is determined by self-action effects. This leads, firstly, to the steepening of the pulse shape. Secondly, due to self-phase modulation new frequency components are generated and shifted to longer wavelengths in the forefront and to shorter wavelengths at the back front of the pulse. These circumstances lead to an increase the role of dispersion, since it depends on the frequency modulation and pulse shape. Therefore steepening will begin to flatten, leading to a rectangular pulse shape. Broadening rate is increased because in the normal dispersion mode the blue components are gathering in the tail (due to self-phase modulation) of pulse, and are moving more slowly than the front-line red. Also, the nature of the broadening of the pulse spectrum is changing. From Fig. 5c we see that the spectral broadening occurs practically without oscillations characteristic of a "pure" nonlinearity. Thus, the dispersion effect smooths these oscillations.

In accordance with the selected dispersion dependence of the spread of an ultrashort light pulse is considered for the case when $k_2 > 0$. If we select a negative nonlinearity ($n_2 < 0$), one can get a soliton regime of propagation of ultrashort light pulses. The results of numerical modeling of this mode are shown in Fig. 6 for the case of a pulse width and negative nonlinearity. One can see that the pulse in the medium at first is slightly compressed itself and then propagates over significant distances, practically maintaining the form.

In conclusion, we consider the problem of collinear interaction of ultrashort light pulses in a medium with linear dispersion and Kerr-type nonlinearity. As it was shown in the work [21], at a certain ratio parameters of light pulses and the

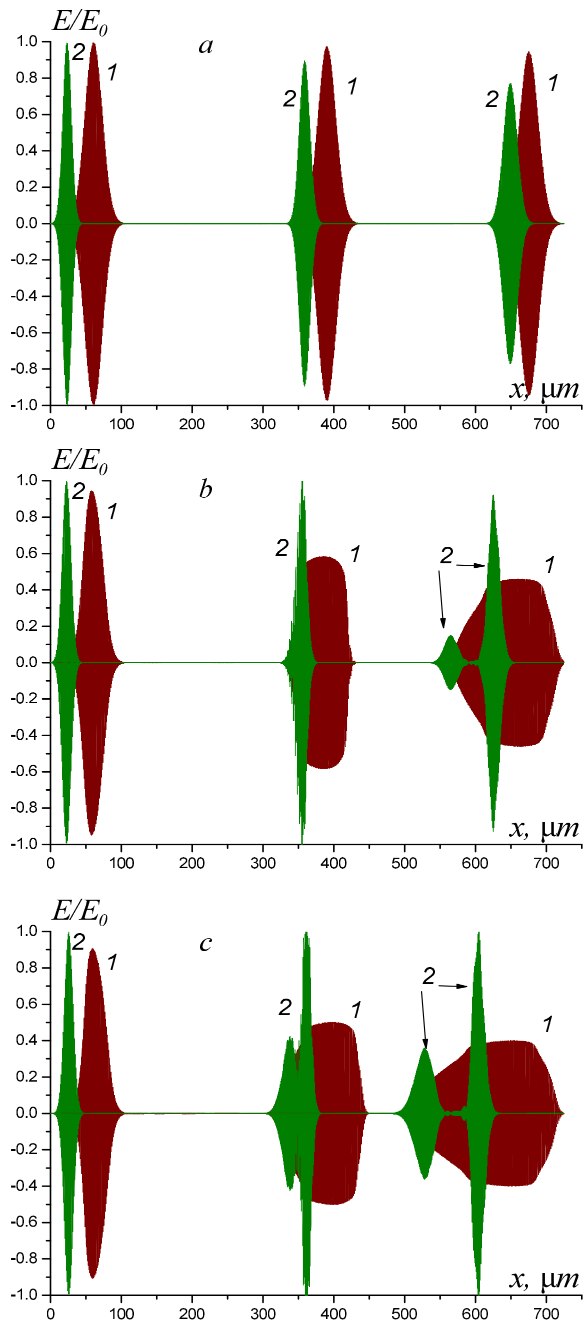

FIG. 7. Reflection of the signal pulse (2) from the pump pulse (1). τ_{pp} = 100 fs, τ_{ps} = 50 fs, λ_{0p} = 1.45 μm, λ_{0s} = 1.5 μm, $\chi^{(3)}E_{0p}^2$ = $0(a), 0.002(b), 0.004(c)$ (in color)

nonlinear medium in which they are propagated, the mode of reflection of a weak signal pulse from a powerful pump pulse can be realized. This mode is an analogue of the well-known mode of total internal reflection of light beams from induced inhomogeneity of the refractive index in the nonlinear defocusing medium [22].

The following parameters were chosen for the implementation of the light pulses reflected mode: the pump pulse wavelength is $\lambda_{0p} = 1.45$ μm, the pump pulse duration is $\tau_{pp} = 100$ fs, the signal pulse wavelength is $\lambda_{0s} = 1.5$ μm, the probe pulse duration is $\tau_{ps} = 50$ fs, the initial time delay between the pulses was varied in the range of $50-100$ fs, and intensity of the pump pulse ranged from $\chi^{(3)}E_{0p}^2 = 0.001 \div 0.01$; optical properties of the medium dispersion were modeled as before, using the Drude – Lorentz.

Numerical simulation results are shown in Figure 7 for different values of nonlinearity of the medium (in the units of $\chi^{(3)}E_{0p}^2$). In the linear mode (Fig. 7) due to the velocity difference between the light pulses (velocity of the probe pulse is greater than the velocity of the pump pulse, as $n(\lambda_{0s}) < n(\lambda_{0p})$) the probe pulse catches up with the pump pulse and then passes through it without interacting. Increasing the pump pulse amplitude increases the refractive index of the medium in the space through which the pulse is propagating because $n_2 > 0$. Consequently, the refractive index is increased for the signal pulse and, as a consequence, decreases its velocity of propagation in the space occupied by the pumping pulse. The collision of the probe pulse with induced inhomogeneity of the refractive index leads to its partial reflection. Probe pulse is divided into two components; the reflected component is slowing, when the transmitted through the optical heterogeneity component passes through the pump pulse (Fig. 7b). A further increase of the pump pulse amplitude leads to an increase in the coefficient of reflection of the probe pulse from induced inhomogeneity of the refractive index (Fig. 7c). Thus, by changing the amplitude of the pump pulse, it is possible to control the dynamic interaction between the two ultrashort light pulses propagating collinearly in a nonlinear medium.

4. Conclusions

The FDTD-method for numerical solution of Maxwell equations, which allows at the same time clearly and in detail present the process of propagation of electromagnetic waves in various media, has been applied to solve problems of the propagation and interaction of ultrashort light pulses in nonlinear materials. To simulate the dispersion properties of optical materials, the auxiliary differential equation method was used. The combined use of these methods has allowed the development of an effective modeling algorithm to describe transformation of the space-time structure of a light field in a Kerr nonlinear medium.

Performed numerical experiments allow demonstrating the effects of self-phase modulation of high-power ultrashort light pulses in nonlinear media, the mode of formation of the shock wave of the envelope, the formation of optical solitons of the envelope. Comparison of the evolution of the envelope for temporal shape of ultrashort light pulses and their spectra with the results of solution of the nonlinear wave equation leads to the conclusion that used in this paper Drude – Lorentz model satisfactorily describes the effect of dispersion in the presence of a non-linear response of the medium. The developed method of numerical solution of the Maxwell equations can not only correctly describe the basic features of change in temporal and spectral shape of an ultrashort light pulse, but also to get detailed information on its internal structure without any simplifying assumptions.

It is also shown that by changing the

amplitude of a pump pulse, it is possible to control dynamic interaction between two ultrashort light pulses propagating collinearly in a nonlinear medium.

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