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Linear and Nonlinear Light Beam Propagation in Liquid-Crystal Waveguide Arrays

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Methods of spatial switching of laser beams in a system of coupled optical liquid-crystal waveguides with an electrically controlled depth of refractive index modulation are experimentally realized. Tuning of the parameters of the waveguide system was carried out using the electrooptic effect and the nonlinear-optical response of the liquid-crystal medium. The proposed theoretical model allows us to describe the processes of low-frequency and optical modulation of the dielectric properties of a liquid crystal, calculate the profiles of optical waveguides induced in the system, and interpret the basic laws governing the propagation of light beams in a system of coupled waveguides.

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1. Introduction

Optical liquid crystal (LC) systems with spatial modulation of the refractive index represent a promising technological platform for creation of modern photonic devices with enhanced functional characteristics. In the last decade, the features of the manifestation of linear and nonlinear effects in discrete optical systems have been actively investigated [1–7]. The design of discrete systems based on functional materials with special optical properties allows to significantly improve the performance characteristics of the photonic devices manufactured. In particular, nematic liquid crystals (NLC) are successfully used to create discrete systems with tunable optical parameters.

An abnormally high optical anisotropy of NLC, controlled under the action of low voltages (of the order of units of volts)[8, 9], makes it possible to realize discrete LC structures with a tunable refractive index modulation depth. Also, NLCs proved its importance for nonlinear optics: the study of the interaction of NLC with light led to the discovery of giant optical nonlinearity, 8-9 orders of magnitude higher than the Kerr nonlinearity of ordinary liquids [10]. Under the influence of polarized light, at comparatively low levels of laser radiation intensity (~ 1 kW/cm^2), optical reorientation of the direction of the NLC director (light-induced Fredericks transition) is performed [11–13], which allows the development of a new class of photonic devices.

The pronounced electro-optical response of the NLC makes it possible to transform a planar NLC layer into a one-dimensional array of waveguides. Thus, in the presence of an external spatially modulated electric field within the NLC layer, a discrete structure with a controlled refractive index contrast depth is formed. For radiation polarized as an extraordinary wave, the guiding properties are realized when it propagates in the system of electrically induced NLC waveguides [14].

In this paper we present a method for creating a system of coupled optical NLC waveguides with an electrically controlled depth of modulation (contrast) of the refractive index. Numerical simulation methods have been used to calculate linear and nonlinear propagation of a Gaussian light beam with TM polarization

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in the developed waveguide NLC system. The spatial control of radiation in a system of coupled NLC waveguides based on the electro-optical and nonlinear-optical response has been experimentally realized.

2. Liquid-crystal cell with waveguide array

To create a spatial modulation of the refractive index within the NLC layer, a planar cell of the sandwich type was used, which contained an opaque, comb-shaped electrically conductive layer on the lower substrate. The switching of a NLC cell from a planar waveguide to a one-dimensional system of identical weakly coupled optical waveguides was carried out by means of an external low-frequency electric voltage. Fig. 1a, b shows the schematic diagram of a NLC cell and illustrates the principle of the formation of a system of electrically induced optical waveguides (period of the structure $\Lambda = 40\ \mu$m).

As the core of the planar waveguide, a layer of a NLC material with a positive dielectric anisotropy ($\varepsilon_{\|} > \varepsilon_{\perp}$) was used, located between two plane-parallel glass plates (substrates). The thickness of the NLC layer was determined by the gap between the substrates and was $d = 100\ \mu$m. The refractive indices of the NLC material used in the work for laser radiation with wavelength $\lambda = 532$ nm were: $n_e = 1.70$ for an extraordinary wave and $n_o = 1.52$ for an ordinary wave. The initial planar orientation of the NLC director (Fig. 1a) along the z axis (i.e., along the propagation direction of the light wave) on the surface of the upper and lower substrates was realized using the photostimulated rubbing alignment technology [15].

To observe propagation of light beams in the NLC layer in the yz plane, glass was used as the upper substrate, uniformly coated with a transparent electrically conductive layer of indium-tin oxide (ITO) 50 nm thick. Deposition of a comb-shaped chromium electrically conductive layer on the lower substrate of the LC cell was carried out by laser lithography. The ratio of the width of the electrically conductive chromium bands to the gap between them is equal to unity. The geometry of the selected texture of the electroconductive layer allows the formation of a spatially modulated voltage across the thickness of the NLC layer (i.e., in the $x-$axis direction). Under the action of an external voltage, the director of the LC molecules in the xz plane is reoriented, that leads to formation of a modulation of the refractive index in the NLC layer along the $x-$ and $y-$ directions. Thus, when an external electric field is connected to a cell, the waveguide distribution of the refractive index for the TM polarization mode ($\overrightarrow{E} \uparrow\uparrow x$) is formed as a result of the Fredericks transition within the NLC layer, as it is shown in Fig. 1b. The electrically induced system of optical waveguides consists of identical parallel channels that are weakly coupled along the y-direction. The coupling coefficient of NLC waveguides is determined by the magnitude of the control voltage [1]. The optical parameters of the represented system of coupled waveguides are rearranged on the basis of the electrooptical response and/or the nonlinear optical response of the NLC medium.

The formation in the NLC layer of the periodic lattice of the refractive index, which is a waveguide system, was verified experimentally by using a polarization microscope that studies objects in transmitted light by placing the cell in crossed polarizers. In Fig. 1c, polarization micrograph of a NLC cell with an electrically induced system of optical waveguides is presented.

3. Theoretical model and numerical results

Theoretical models of propagation of laser beams in LC structures, including waveguides and spatially periodic structures with modulation of the refractive index of material, developed to date, can be divided into three main groups: 1) the phenomenological model of the discrete

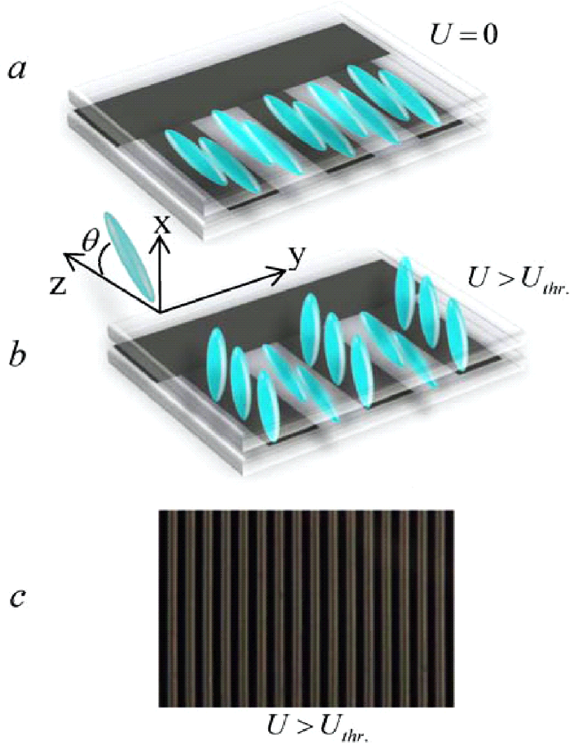

FIG. 1. (Color online) Schematic diagram of a NLC cell (view from the end) (a); the principle of the formation of a system of NLC waveguides (b); a micrograph of a NLC lattice (top view) in polarized light (crossed polarizers) (c).

nonlinear Schrödinger equation for describing propagation of laser radiation in a system of coupled waveguides [16]; 2) the model based on the solution of nonlinear wave equation in the paraxial approximation (nonlinear Schrödinger equation) for the complex light field amplitude, that takes into account the possible nonlinear dependence of the dielectric constant on the light field intensity and/or periodic spatial modulation of the medium parameters [17]; 3) a direct numerical solution of the Maxwell equations for the electric and magnetic field strengths with a given spatial distribution of the dielectric function of the medium and its dependence on the intensity of the light field and external electric or magnetic fields [18].

In this paper we use an approach based on the analysis and solution of the nonlinear wave equation in the paraxial approximation for the complex amplitude of the light wave field (the nonlinear Schrödinger equation), which in general form is written as follows [17]:

$$2ik\frac{\partial \vec{E}}{\partial z} + \Delta_{\perp}\vec{E} = \frac{\omega_0^2}{c^2}\Delta\varepsilon\left(\left|\vec{E}\right|^2, \vec{r}\right)\vec{E}. \qquad (1)$$

Here, the function $\Delta\varepsilon\left(\left|\vec{E}\right|^2, \vec{r}\right)$ takes into account the possible nonlinear dependence of the permittivity on the intensity of the light field and/or the periodic spatial modulation of the parameters of the medium. Allowance for the anisotropy effects characteristic of liquid-crystalline media makes it necessary to consider a system of wave equations of the form (1) for different polarization modes.

In further analysis, we assume that a focused laser beam having an initial Gaussian intensity profile is directed to the NLC cell from the end $z = 0$. In accordance with the experimental conditions, vertical polarization of the radiation (along the x axis) is assumed, the modulation of the dielectric permittivity of the medium is given in the form $\Delta\varepsilon = \Delta\varepsilon\,(x, y) + \varepsilon_{\rm nl}\,(I)$, where the first term is related to the spatially inhomogeneous reorientation of NLC molecules in a low-frequency external electric field $\vec{E}_0$, and the second term describes the orientational action of the electric field of a light wave $\vec{E}$. Under these conditions, the normalized wave equation for the x-component of the electric field E can be written in the form:

$$\frac{\partial E_x}{\partial \tilde{z}} = i\tilde{\Delta}_{\perp}E_x + i\frac{kL_D\Delta\varepsilon}{\varepsilon_{\rm l}}E_x \qquad (2)$$

where $\varepsilon_{\rm l} \equiv \varepsilon_{\perp}$ is the linear dielectric constant, k is the wavenumber, the spatial variables x and y are normalized to the half-width of the Gaussian light beam r_0, and the variable z is normalized to the diffraction length: $z = 2L_D\tilde{z}$, $L_D = kr_0^2 = \frac{2\pi r_0^2 n_0}{\lambda_0}$.

To find the stationary distribution of the orientation of NLC molecules in a low-frequency and/or optical electric field, we find the minimum for the bulk density of free energy $F = F_D + F_E$, where the first term F_D describes the energy of elementary deformations of the bulk NLC, and the second term F_E represents the interaction energy of NLC molecules and electric fields. In the one-constant approximation, in which the elastic Franck constants for each of the deformation types (splay, twist, and bend) are assumed to be the same $K_1 = K_2 = K_3 = K$, the formula for F_D has the form: $F_D = \frac{1}{2}K\left[\left(\vec{\nabla}\cdot\vec{n}\right)^2 + \left(\vec{\nabla}\times\vec{n}\right)^2\right]$, and the interaction energy of NLC molecules and the external electric field is expressed by the formula $F_E = -\frac{\Delta\varepsilon}{2}\left(\vec{n}\cdot\vec{E}\right)^2$. The minimum of free energy is found from the expression [19]:

$$\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial\theta_x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial\theta_y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial\theta_z}\right) - \frac{\partial F}{\partial\theta} = 0 \qquad (3)$$

where θ_x, θ_y, θ_z are the derivatives of the angle of rotation of the director of the LC layer θ along the corresponding coordinate axes.

In accordance with the NLC cell diagram shown in Figure 1, the initial orientation of LC molecules is planar along the z axis. Since the external electric field is modulated in the x, y plane and is a constant along the z axis, we obtain the explicit dependence of the orientation angle of the LC director and the field itself on only two coordinates $\theta\left(x, y\right)$ and $\vec{E}_0\left(x, y\right)$. In the proposed coordinate system, the director of the nematic LC-layer $\vec{n}$ has only $x-$ and $z-$components ($n_x = \sin\theta$, $n_y = 0$, $n_z = \cos\theta$), and the vector of the external electric field has the components E_{0x} and E_{0y}. Using these assumptions, one can obtain an equation for the tilt angle of the NLC director with respect to the initial planar orientation:

$$K\left[\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right] + \frac{\Delta\varepsilon E_{0x}^2}{2}\sin 2\theta = 0. \qquad (4)$$

The intensity of the low-frequency electric field between the electrodes $\vec{E}_0$ can be calculated from the Maxwell equation $\vec{\nabla}\cdot\vec{D} = 0$, taking into account the expression $\vec{D} = \varepsilon_{\mathrm{LF}\perp}\vec{E}_0 + \left(\varepsilon_{\mathrm{LF}\parallel} - \varepsilon_{\mathrm{LF}\perp}\right)\left(\vec{n}\cdot\vec{E}_0\right)\vec{n}$:

$$\frac{\partial}{\partial x}\left[\left(\varepsilon_{\mathrm{LF}\perp} + \left(\varepsilon_{\mathrm{LF}\parallel} - \varepsilon_{\mathrm{LF}\perp}\right)\sin^2\theta\right)E_{0x}\right] + \varepsilon_{\mathrm{LF}\perp}\frac{\partial E_{0y}}{\partial y} = 0 \qquad (5)$$

Taking into account that $\vec{E}_0 = -\nabla\varphi_0$, we get:

$$\frac{\partial}{\partial x}\left[\left(\varepsilon_{\mathrm{LF}\perp} + \left(\varepsilon_{\mathrm{LF}\parallel} - \varepsilon_{\mathrm{LF}\perp}\right)\sin^2\theta\right)\frac{\partial\varphi_0}{\partial x}\right] + \varepsilon_{\mathrm{LF}\perp}\frac{\partial^2\varphi_0}{\partial y^2} = 0. \qquad (6)$$

Thus, the distribution of the NLC director $\vec{n}$, which is completely determined by the tilt angle θ, can be found from the solution of equations (4), (6). In this case, the spatial distribution of the refractive index for a light wave propagating along the z axis and polarized along the x axis is determined by the formula: $n_{\mathrm{eff}}\left(\theta\right) = \frac{n_\perp n_\parallel}{\sqrt{n_\parallel^2\cos^2\theta + n_\perp^2\sin^2\theta}}$. As follows from this expression, as the tilt angle of the NLC director changes, the refractive index for the polarized wave vertically (along the x axis) changes from the value $n_\perp \equiv n_o$ with the initial planar orientation of LC molecules to $n_\parallel \equiv n_e$ with the angle of reorientation $\theta = \frac{\pi}{2}$. Thus, conditions for a waveguide propagation are realized for a vertically polarized wave.

Numerical modeling of the process of the light beam propagation has been carried out for the following cases: 1) linear propagation of a narrow low-power laser beam in an LC layer with an electrically induced array of waveguides; 2) nonlinear self-focusing of a wide laser beam in a system of weakly modulated waveguides; 3) propagation of a wide laser beam in a waveguide array with simultaneous electrically and optically induced modulation of the refractive index of the material. The latter variant corresponds to the conditions of the experimental study. The simulation results are shown in Figure 2.

Figure 2b shows the intensity profiles of the light beam with the initial half-width $r_0 = 4$ μm, and the wavelength $\lambda_0 = 1$ μm propagating in the waveguide array with the period $\Lambda = 4\ \mu$m and depth of modulation $\Delta\varepsilon_{\max} = 0.3$. As can be seen from the calculation, under these conditions at a depth of penetration of the order of $100 - 400\ \mu$m there is a clearly pronounced transfer of the energy of the light beam from the central waveguide to the neighboring ones, which is a typical picture for observing the process of discrete diffraction.

Experimental study of the propagation of laser beams was carried out under conditions of focusing radiation with a wavelength $\lambda_0 = 0.532$ nm on the input of the NLC layer into a spot with diameter 40 μm and for a period of the waveguide structure $\Lambda = 40\ \mu$m, so numerical simulation was performed for these parameters to study the effect of the modulation depth of the waveguide structure and the nonlinear optical response of NLC.

In the case of weak ($\Delta\varepsilon_{\max} \ll 1$) spatial modulation of the permittivity of the NLC layer, the main contribution to the change in the spatial structure of the light beam is determined by the effect of self-focusing (Figure 2, c), which under the indicated conditions manifests itself at penetration depths $z \sim 2000\ \mu$m. For small angles of reorientation of LC molecules ($\theta \ll \frac{\pi}{2}$), the nonlinear term of the permittivity can be described in the framework of Kerr-nonlinearity model [20]: $\varepsilon_{\text{nl}} \approx 2n_0\Delta n = 2n_0 n_2 I$. The saturation of the reorientation effect ($\theta \sim \frac{\pi}{2}$) can be taken into account in the framework of the saturation nonlinearity model, in which $\Delta n = \left(n_{\|} - n_{\perp}\right)\frac{n_2 I}{1+n_2 I}$.

The combined effect of electrically induced spatial modulation of the dielectric permittivity of the NLC layer and optically induced nonlinearity in the propagation of a laser beam with an initial radius $r_0 = 40\ \mu$m in a waveguide array with a period $\Lambda = 40\ \mu$m leads to formation of a complex multimode radiation structure (Figure 2d), a characteristic feature of which is the manifestation of the discrete diffraction at penetration depths $z \sim 400 - 500\ \mu$m.

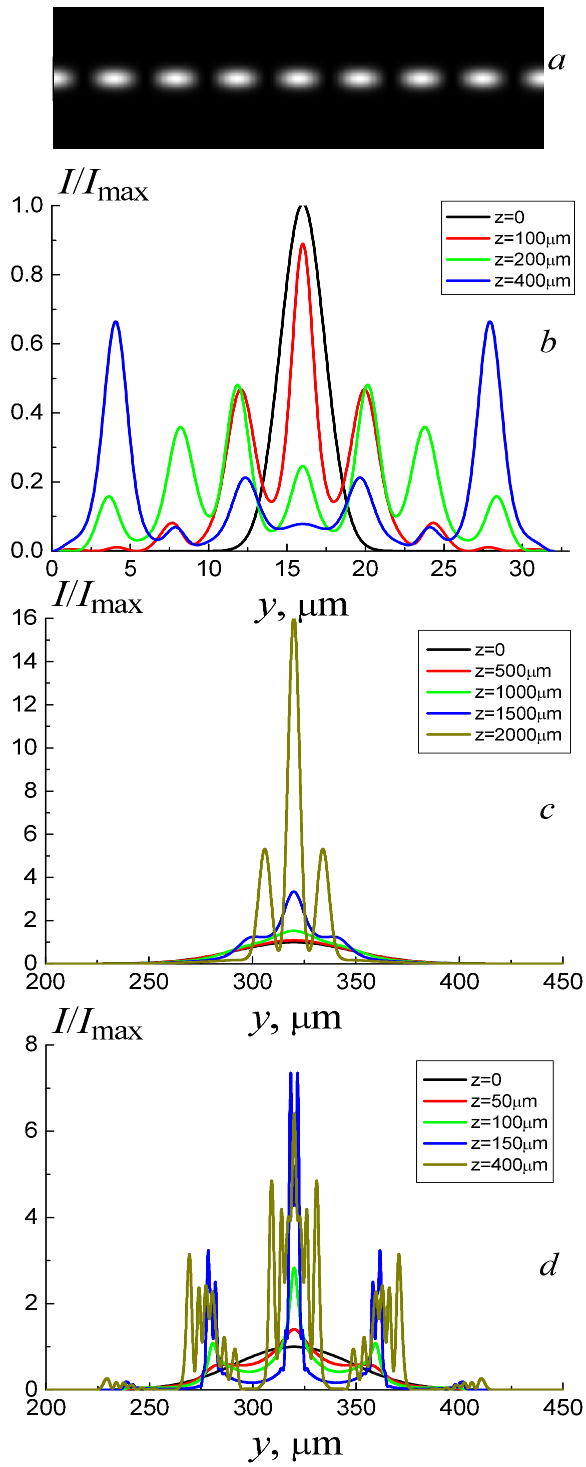

FIG. 2. (Color online) The structure of the waveguide array (a). (b–d) Intensity profiles of the light beam $I(y)$ inside NLC layer.

4. Experimental results

The experiments were performed using the second harmonic of the Nd: YAG laser (λ = 532 nm). To analyze the spatial distribution of the light field in a discrete NLC structure, we used the standard method of fixing the scattering pattern of laser radiation on inhomogeneities in the orientation of the director of a liquid crystal [12]. The registration of the scattering pattern of the light beam in the yz plane was carried out using a high-resolution photosensitive camera coupled with an objective lens. With the help of the multifunctional measuring complex "UNIPRO" , an alternating voltage of 0–10 V amplitude was applied to the electrodes of the NLC cell with a repetition rate of the pulses $\nu = 1$ kHz.

To study the nonlinear propagation of a light beam in a system of coupled optical NLC waveguides, linearly polarized laser radiation ($\lambda = 532$ nm, $\vec{E} \| x$, $P = 5$ mW) was focused by objective lens to a spot with a diameter 40 μm and directed to the end of the NLC cell in the region of the comb-shaped electrode. To reduce the threshold for the nonlinear orientational effect, we used the method of applying a voltage to the LC cell near the threshold of the Fredericks transition (U_{thr} = 1.1 V) [20]. Figure 3a, shows the experimental scattering patterns of a light beam in a system of coupled NLC waveguides under different control voltages, and the corresponding dependencies of the intensity distribution profiles for the propagation length $z = 2$ mm are shown in Figure 3b.

At a voltage U = 1.0 V, in the region of the textured electrode, the pretilt angle of the director of the NLC molecules in the xz plane is created, which leads to a decrease in the threshold value of the intensity of the light beam providing the photoinduced Fredericks transition [21]. Since the maximum optical power is reached at the center of the beam cross section, within a given region the light field self-channeling due to the nonlinear orientation effect and acquires the form of an optical spatial soliton (nematicon) with the energy concentrated within one NLC channel. At a voltage on the cell U = 2.0 V, an external electric field plays the predominant role

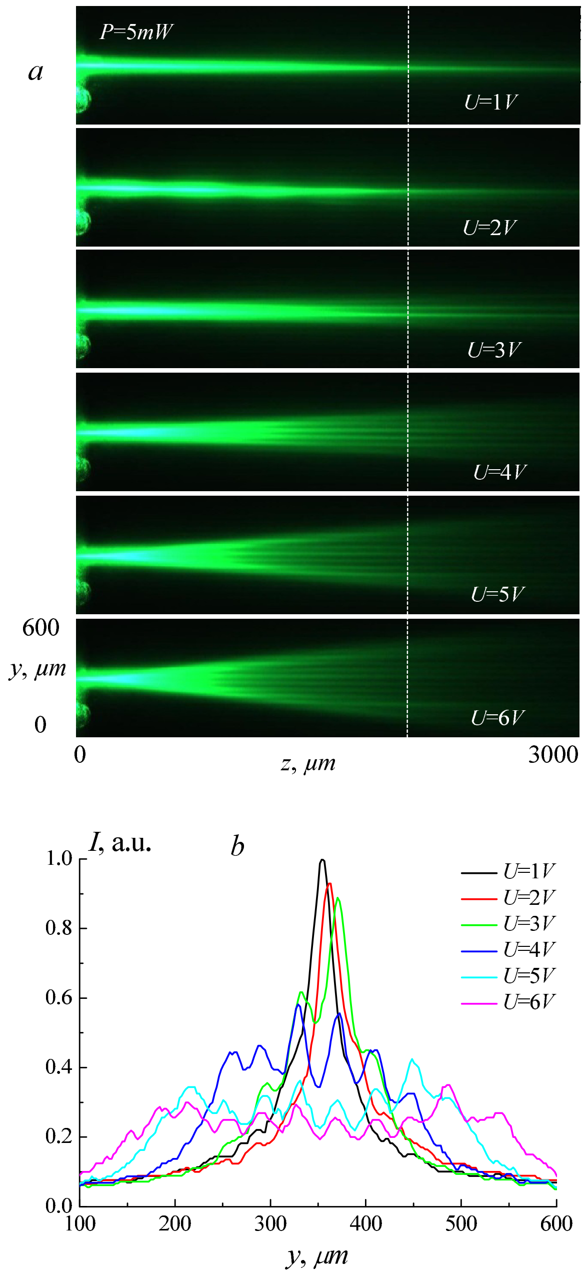

FIG. 3. (Color online) Scattering patterns of laser radiation in a system of electrically induced NLC waveguides under different control voltages (a). Dependencies of the distribution profiles of the intensity $I(y)$ of the light beam at a depth $z = 2$ mm (dashed line) (b).

in reorientation of the NLC director, as a result of which the effective refractive index in the core region of the NLC waveguide increases.

With further increase of the voltage on the cell ($U \gg 2.0$ V) saturation of the reorientation process of NLC molecules occurs in the region of the comb-shaped electrode, which results in smoothing the modulation of the refractive index within the discrete system. Indeed, at excessively high voltages, the refractive index increases not only in the core region, but also in the region of the cladding of NLC waveguides, which is due to the nonlocal response of the NLC medium, as well as the output of the electric field lines beyond the electrodes. The electrically induced change in the coupling coefficient of NLC waveguides, responsible for the process of transfer (redistribution) of light energy between adjacent channels, is accompanied by the manifestation of discrete diffraction of the light beam in a system of coupled NLC waveguides. In accordance with Figure 3, as a result of discrete diffraction, a redistribution of the energy of light radiation occurs between adjacent NLC waveguides. Thus, at a voltage of $U = 6.0V$, the light energy from the region of one NLC waveguide is redistributed between ten adjacent channels of the discrete system. As a result, an external voltage can be used to redistribute the energy of the light field between a given number of waveguide channels within a discrete NLC system.

5. Conclusion

The method of forming a discrete NLC structure with electrically controllable depth of refractive index modulation for spatial control of light fields is proposed. The features of linear and nonlinear propagation of light beams in a system of coupled optical NLC waveguides are studied experimentally. It is shown that, depending on the power of the light beam and the magnitude of the control voltage, the mode of waveguide propagation of light is realized, as well as the regime of the discrete diffraction of the light beam in a system of coupled NLC waveguides.

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