RETRIEVAL OF SPECTRA FROM INTERFEROGRAMS WITH A SMALL OPTICAL PATH DIFFERENCE USING A GENETIC ALGORITHM

I. M. Gulis* and A. G. Kupreyeu

UDC 681.785.57+543.42

We propose a method for retrieval of an optical line spectrum limited to a narrow spectral range using a short interferogram, based on the use of a multi-stage genetic algorithm. We have run simulations for the example of spectra with lines having wavenumbers lying in the 100 cm^{-1} range, and the maximum optical path difference for the interferograms was $1200\text{ }\mu\text{m}$. We show that the spectral resolution can be improved by a factor of 3–4 compared with Fourier transform estimation.

Keywords: spectral analysis, interferogram, genetic algorithm.

Introduction. Interference spectral instruments have high luminosity, low signal-to-noise ratio as a result of multiplexing, and a broad working spectral range. Retrieval of the spectrum in conventional Fourier transform spectroscopy is based on the Wiener–Khinchin theorem, relating the power spectral density for a stationary random process to its autocorrelation function (interference pattern or interferogram) by means of a Fourier transform [1]. In this case, the theoretical spectral resolution is inversely proportional to the maximum optical path difference, and therefore it is low in compact instruments.

It is difficult to substantially improve the resolution of compact instruments strictly with hardware. Fast continuous scanning, used in popular laboratory Fourier transform spectrometers, requires a high-precision drive and constant operating parameters of the instrument for high-quality retrieval of the spectrum, which is complicated to achieve in a small instrument [2]. Experimental systems using scanning of the optical path difference in a more compact size (for example, using a Wollaston prism [3] or in a standing-wave interferometer [4]) do not provide a substantial increase (by a factor of 3–5) in the spectral resolution and generally form a poorer quality interferogram. Recording a stationary interferogram based on a multi-element photodetector makes it possible to dispense with the scanning mechanism and thus to reduce the size of the instrument [5–7]. However, due to the small number of pixels in the detector, an increase in the length of the interferogram causes narrowing of the spectral range for retrieval. Parallel recording of sections of the interferogram on a two-dimensional photodetector and "splicing" them into one interferogram with a large effective length requires, in addition to complicated post-processing, an increase in the dimensions of the optical part of the instrument while maintaining light beam homogeneity [8]. Recording the interferograms from different spectral intervals on different regions of the same two-dimensional array decreases the influence of noise at individual frequencies on the rest of the spectrum, but the spectral resolution for each of the intervals remains poor since the interferogram is short [9].

Alternatives to the Fourier transform method for spectral estimation in some cases (more often for quasi-line spectra) let us improve the spectral resolution without increasing the maximum path difference. For example, knowing the mathematical model for the signal *a priori* in autoregression methods [10, 11] and eigenvector methods [12] lets us estimate the characteristics of the interferogram beyond the recorded segment. Disadvantages of these methods include the nonlinearity of the spectral estimation, the problem of selecting the parameters for correct retrieval, and the decrease in accuracy as the signal-to-noise ratio decreases [11, 13]. Another approach is based on describing the intensity distribution at the exit from the spectral instrument using an integral equation. In the simplest case, it is reduced to a system of equations

*To whom correspondence should be addressed.

that can be solved in the presence of noise by regularization [14] or singular value decomposition [15]. Expansion of the kernel in eigenfunctions gives a more stable solution with better resolution and reduced noise level [16], but to do this we need an analytical description of the instrumental function for the interferometer, and it is not applicable to all types of interferometers. A nonparametric algorithm, determining the spectral estimation by the iterative method of regularization in the Cauchy–Gauss model, gives a spectral resolution exceeding the resolution for both the conventional and some alternative methods, but may retrieve spurious lines [11].

We consider the possibility of retrieving spectral information using a "short" interferogram (for a small optical path difference), based on the following considerations. We can assume that using an interferogram for a spectrum lying in a narrow range has advantages over the limitation to only the Nyquist frequency typical for most spectral estimation methods. First of all, according to the generalized Kotelnikov theorem, the number of readings required for the retrieval decreases, which is important when using pixel photodetectors. Secondly, the influence of noise on the individual spectral segments is reduced over the entire interferogram [9]. Due to the smaller size of the instrumental function matrix, the error in spectral estimation by solution of an integral equation, reduced to an ill-posed system of equations [16], decreases and consequently the spectral resolution increases [14]. Lowering the dimension of the spectra/solutions space lets us use a genetic algorithm (GA) to search for the solution.

The GA approach is a type of heuristic (non-rigorous) method for solving optimization problems. During operation of the GA, the set of variants for the solution to the studied problem is transformed by means of "crossover," "mutation," and "selection" procedures and drifts toward promising regions of the solution space, defined by the extrema of the target function. Selection stimulates choosing solutions with better (closer to the extremum) values of the target function; crossover promotes convergence to the optimal pattern found; mutations permit a search in solution space. Genetic algorithms are popular because of their simplicity, the possibility for using them together with other optimization methods, and their suitability for parallelization; they are suitable for solving any problems which can be formulated in optimization terms [18]. Since no general-purpose optimization algorithm exists that has the best performance in all classes of problems [19], the advantage of a genetic algorithm over other heuristic algorithms can be demonstrated in specific types of problems. In optics and spectroscopy, genetic algorithms have been successfully applied for analysis and processing of spectra [20], approximation of the phase shift in interferograms [21], retrieval of multispectral images [22], and spectral calibration [23].

The use of genetic algorithms is effective for solving an ill-posed problem (in this case, finding the spectral distribution corresponding to the recorded segment of the interferogram) if an external mechanism is available for screening out wrong solutions (local extrema of the target function) to which sometimes the calculation converges [24]. However, successful examples are known for the use of genetic algorithms without additional evaluation of results for solving ill-posed problems with a high noise level [25].

In this paper, we consider the prospects for a GA-based method for obtaining a stable spectral estimation of a section of the interferogram for a light beam with a line spectrum (in principle, the method lets us work with arbitrary types of spectra) in a narrow spectral range. A narrow range can be identified, for example, by using a dispersive element crossed with a static interferometer and detection on a two-dimensional array detector as in [9].

Description of the Method. The proposed method operates on an interferogram I_{orig} for radiation whose spectrum must be retrieved, and a set of calibration interferograms M_{cal} for nearly monochromatic sources, recorded on the same instrument (in the current version, the interval between wavenumbers for the calibration lines is 4 cm^{-1}). We are looking for a spectral estimation S : a matrix composed of columns of wavenumbers ν_i and intensities A_i for m lines:

$$S = [\nu_i | A_i], \nu_i \in [\nu_{\min}, \nu_{\max}], A_i \in [0, 1], i = \overline{1, m}, \quad (1)$$

for which the difference between its interferogram I_{found} and the original I_{orig} is minimum for some target function:

$$\arg \min_S \| I_{\text{found}} - I_{\text{orig}} \|, \quad I_{\text{found}} = M^* A, \quad (2)$$

where M^* is a matrix whose rows are the interferograms for lines of unit intensity with wavenumbers ν_i . Each interferogram M_j^* is obtained by interpolation (extending with respect to frequency and averaging over the intensity) for two calibration interferograms from the set M_{cal} for the wavenumbers nearest to ν_j . In order to improve the quality of the retrieval and speed up the calculations in the cycle, the interferograms M_{cal} are interpolated beforehand using cubic splines for up to 10 times more readings. Our method includes finding and refining the spectrum/solution (run cycle) and generalizing the results of several such cycles (Fig. 1).

Fig. 1. Flowchart for spectral estimation algorithm: creation of the generation (a), iteration of the genetic algorithm (b), and creation of the final result (c).

An elementary iteration in the GA (Fig. 1a) is performed on a set of N line spectra using m spectral lines in each (a line is defined by frequency and intensity). In the generation calculation step, the target function is calculated for each spectrum (in the text below, "better" means a spectrum with a smaller target function). The best results with respect to the accuracy of convergence of the result are displayed by two types of target functions: the norm of the variation

$$f_{\text{norm}}(I) = \|\Delta I - M[\Delta I]\|_2, \Delta I = I_{\text{found}} - I_{\text{orig}}, \quad (3)$$

and the norm of the final differences between the interferograms

$$f_{\text{diff}}(I) = (|\Delta I_{\text{found}} - \Delta I_{\text{orig}}|)^{1/2}, \Delta I_{\text{found}} = I_{\text{found } k} - I_{\text{found } k+1}, \Delta I_{\text{orig}} = I_{\text{orig } k} - I_{\text{orig } k+1}, k = \overline{0, n-1}, \quad (4)$$

where $M[\Delta I]$ is the expectation value for the difference between the interferograms; n is the number of readings in the interferogram. The target function (3) estimates the pointwise similarity of the interferograms, (4) estimates similarity from the differentials, favoring those variants of the spectra with similar intensities. In order to reduce the probability of falling on a local minimum, besides increasing the role of mutations, the indicated target functions are also alternated in different iterations of the run cycle (in the current version, one iteration with f_{norm} after every two iterations with f_{diff}). In order to eliminate the appearance of lines with close frequencies, we use a multiplicative factor, increasing the value of the target function for each pair of lines spaced $< 0.5 \text{ cm}^{-1}$ apart (the *a priori* resolution limit).

After sorting and ranking the spectra according to the value of the target function, the next generation is created. It consists of N_{elite} "elite" spectra, N_{xover} crossover child spectra, and N_{mut} mutation children, while the size of the generation remains unchanged: $N = N_{\text{elite}} + N_{\text{xover}} + N_{\text{mut}}$. Some of the best elite spectra (five in the current version) pass to the next generation unchanged; the rest of the generation is created from the crossover and mutation results in equal proportions. To do this, using uniform selection we select $2N_{\text{xover}} + N_{\text{mut}}$ "parent" spectra (the same parent can participate more than once). Uniform selection makes sure that the "parent" is sampled with probability proportional to the reciprocal root of the number of the spectrum in the general fitness rank for the given generation, after which the parent spectra are additionally randomly mixed. $2N_{\text{xover}}$ parent spectra participate in the heuristic crossover, in which the lines of the child spectrum acquire characteristics intermediate between the characteristics of the lines for the two randomly selected parent spectra, but closer to the better one:

$$x_{\text{child}} = x_{\text{worsePar}} + r_{\text{xover}}(x_{\text{betterPar}} - x_{\text{worsePar}}), \quad (5)$$

where x is the intensity or wavenumber of the line; the subscript "child" marks the characteristics of the child spectrum, the subscripts betterPar and worsePar mark the better parent spectrum and the worse parent spectrum; the empirical coefficient $r_{\text{xover}} = 1.2$. The mutation children are created by applying a normally distributed variation $N(0, 1^2)$ in wavenumber ν and intensity A to the lines of the parent spectra. The amplitude of the variation decreases linearly with an increase in the number of the generation, ensuring stability of the convergence, but does not drop down to zero in order to preserve the variability of the results:

$$\begin{cases} v_{\text{child}} = v_{\text{par}} + N(0, 1^2) r_{\text{scaleFreq}} (1 - r_{\text{shrinkFreq}} g / N_{\text{gen}}), \\ A_{\text{child}} = A_{\text{par}} + N(0, 1^2) r_{\text{scaleInt}} (1 - r_{\text{shrinkInt}} g / N_{\text{gen}}), \end{cases} \quad (6)$$

where g is the number of the generation; N_{gen} is the total number of generations, the subscript "par" indicates the parameters of the line in the parent spectrum, the empirical coefficients $r_{\text{shrinkInt}} = 0.5$, $r_{\text{scaleInt}} = 0.1$, $r_{\text{shrinkFreq}} = 0.5$, $r_{\text{scaleFreq}} = 0.01$. The experiments showed that simultaneously changing the wavenumber and intensity of the line is rarely favorable (i.e., refines the solution or brings it out from the region of a local minimum), and so its probability is established as 0.1. In the remaining cases, one or the other is varied with equal probability.

The set of spectra obtained becomes the parent for the next generation, and the calculation is repeated. In the current version, after 40 generations the GA iteration is terminated (Fig. 1b). After one iteration, generally the interferogram is far from the original, since the method converges slowly. Increasing the size of the population by a factor of 2–5 does not have a substantial effect, and so in order to refine the result we perform several iterations, and the initial population for the next iteration is created from the result for the previous generation and the refinement procedure (Fig. 1c). The refinement procedure involves searching by trial and error for the position of lines in best spectrum obtained for which the target function is minimum (the entire working range is inspected in 0.4 cm^{-1} steps). According to estimates, the procedure reduces the value of the target function after 13 GA iterations by more than an order of magnitude compared with the version not including it.

Due to the stochastic nature of the genetic algorithm, the results of individual run cycles are different, and so we consider two approaches to obtain a more stable final result. The first approach is weighted averaging, in which the worst are discarded from the file of run cycle results, while the rest are summed with weights $1/\text{score}^3$ (score means the value of the target function for the given spectrum), increasing the contribution to the final sum from the best results. In the second approach, using the nonparametric KDE estimation (kernel density estimation), we determine the centers of clusters of wavenumber groupings for the retrieved lines from the entire set of spectra with better correlation distance between the original and the retrieved interferograms. Only $A_i \geq 0.02$ rel. units is used for the noise probability cutoff; the weights of the spectra are also taken into account in order to give preference to lines from the better spectra. The centers obtained are considered in terms of the wavenumbers for the lines in the result; their intensity is determined by the least-squares method with non-negative constraints (NNLS, non-negative least squares) using the matrix M^* (NNLS was calculated by a function from the Octave mathematics software package). It seems that the KDE+NNLS procedure can more effectively estimate the frequencies of the spectral lines.

The proposed algorithm for the calculation demonstrated the best convergence and stability with respect to the outcome of experiments with several variants of the genetic algorithm and run cycles (the parameters of the genetic algorithm, the target function, the method for calculating the generation and refining the result were varied). For this problem, formal proof of the convergence to the optimal solution is difficult (it can be obtained only for the very simplest types of "individuals"), and so validation of the method involves a number of criteria [26], including determination of the range of applicability and comparison with known spectral estimation methods.

Results and Discussion. The interferograms for dual-beam interference of a light beam with a line spectrum, containing lines of intensity $A_k = 0.01$ –1 relative units and wavenumber $v_k = 2902$ – 3002 cm^{-1} , was simulated with sampling along the x coordinate by superimposing normally distributed noise with variance $\sigma = 0.001$ and mean $\mu = 0$:

$$I(x) = (1 + N(x; \mu, \sigma^2)) \sum_{k=1}^m A_k (1 + \cos 2\pi v_k x) . \quad (7)$$

In order to take into account the pixel nature of the detector, as the reading of the interferogram at point x we took the sum of the readings for five points in the vicinity of x , corresponding to a single pixel. In the first series of experiments, carried out using eight tests with a random initial population of size 110 spectra for each type of problem, with length of the recorded segment equal to $1200 \mu\text{m}$ and 6001 readings (pixels), the number of lines in the spectrum was $m = 8$. Due to the stochastic nature of the algorithm, small variations in the lines (for example, splitting into a few closely spaced lines) can give similar interferograms and so estimation of broadened spectral patterns (the original and the retrieved patterns) is of interest. Below we present the results for a broadening of 2 cm^{-1} , since such resolution is several times better than the conventional resolution and effectively evens out the indicated variations.

Note that the proposed method is preferentially used near the Nyquist frequency (for the indicated segment length and number of readings, this may be in particular the visible range). In this case, for a substantial (many-fold) lengthening of the section of the interferogram projected onto the detector (in order to increase the spectral resolution), less than two pixels

fall within one of its fringes. However, even in other cases, the method may be useful because of the simple measurement procedure and the little need for correction of the interferogram. In order to evaluate the suitability of the method in principle, the basic calculations were performed for the IR region of the spectrum, since in this region the number of interference fringes in the recorded section is substantially smaller than in the visible region, and interpolation of the interferograms is easier.

When retrieving a spectrum containing a single line at 2943.7 cm^{-1} with broadening up to 2 cm^{-1} , the deviation of the maximum in the final distribution is $\leq 0.1\text{ cm}^{-1}$ with respect to wavenumber and $\leq 3\%$ with respect to intensity, and spurious lines are not retrieved (for broadening up to 0.1 cm^{-1} , they were retrieved in four tests). In retrieval of the spectral distribution for a doublet of lines with identical intensity, spaced 2 cm^{-1} apart ($2967.4/2969.4\text{ cm}^{-1}$), with broadening 2 cm^{-1} a better result is shown with KDE+NNLS treatment: wavenumber deviation $\leq 0.3\text{ cm}^{-1}$, intensity deviation $\leq 8\%$ of the maximum. Instead of two lines, we retrieved one line in one test, while when using weighted averaging treatment, this happened in seven out of eight tests. A doublet of lines with identical intensity, spaced 3 cm^{-1} apart ($2966.4/2969.4\text{ cm}^{-1}$), for broadening 2 cm^{-1} was retrieved by both methods with comparable results. The wavenumber deviation was less for weighted averaging ($\leq 0.2\text{ cm}^{-1}$ compared with 0.3 for KDE+NNLS); the intensity deviation was less for KDE+NNLS ($\leq 15\%$ compared with 17% for weighted averaging). In one test, KDE+NNLS retrieved a spurious line with intensity $\leq 5\%$ of the maximum, 3.5 cm^{-1} away from the original line.

In randomly generated spectra with 5, 6, and 7 lines (distance between lines $\leq 2.1\text{ cm}^{-1}$), the quality of the retrieval depends substantially on the relative position of the lines and their intensity (we can arbitrarily divide them into small lines with intensity $<30\%$ of the maximum in the spectrum, and large lines for all the rest). In retrieval of a 5-line spectrum with broadening up to 2 cm^{-1} , both methods showed similar results in estimation of the line position (wave number deviation ≤ 0.2 and $\leq 0.1\text{ cm}^{-1}$ for the large lines for weighted averaging). For KDE+NNLS, the intensity deviation is three times smaller (3% and 2% , compared with 10% and 3% for small and large lines respectively). Similar results are obtained for a 6-line spectrum with broadening up to 2 cm^{-1} , but in this case for the small lines the wavenumber deviation increases ($\leq 0.5\text{ cm}^{-1}$ using KDE+NNLS, and $\leq 0.7\text{ cm}^{-1}$ using weighted averaging) and the intensity deviation increases (20% using KDE+NNLS and 35% using weighted averaging). For the large lines, the intensity deviation is $\leq 3\text{--}4\%$. For a 7-line spectrum with broadening 2 cm^{-1} , a better result is obtained when using weighted averaging: for large and small lines, the wavenumber deviation is 0.3 and 1.9 cm^{-1} and the intensity deviation is $\leq 16\%$ and $>50\%$ (compared with 2 and 2.6 cm^{-1} , $\leq 45\%$ and $>50\%$ for KDE+NNLS). Furthermore, for weighted averaging, only in one test do we observe a spurious line with intensity 3% of the maximum (for KDE+NNLS, in five tests the intensity reaches 25% of the maximum for the original).

Retrieval results averaged over eight tests for 5-line and 7-line spectra, broadened up to 2 cm^{-1} , and also spectral estimation using an inverse Fourier transform, estimation with regularization of the minimum norm (MNS) [27] (regularization parameter 0.1) and ESPRIT [28] (rank of the covariance matrix 50 , rank of the model 900) are presented in Figs. 2 and 3. We chose specifically these techniques for comparison because MNS is actually an alternative regularization method for solving problem (2), and ESPRIT is one of the most effective and noise-stable parametric methods for spectral estimation. On the whole, the quality of the retrieval by our proposed method is higher than for these analogs. With an increase in the number of lines, the accuracy of the retrieval by all the methods decreases. The weighted averaging result becomes more accurate and more stable than the KDE+NNLS result, which can be explained by the appearance of a large number of spurious clusters of lines and complication of the problem for NNLS. We can conclude that the quality of retrieval of the line is primarily affected by the total number of lines in the segment, the presence of adjacent lines and the relative intensity of the line compared with the others (when its intensity is $<30\%$ of the maximum in the spectrum and there are intense lines nearby, the accuracy is less). The variability of the individual results is characterized by a standard deviation which is <0.01 for a 5-line spectrum (Fig. 2a), <0.03 for a 6-line spectrum, and <0.05 for a 7-line spectrum (Fig. 3a), where it is minimum near the maxima of the retrieved lines and increased on their wings, especially for low-intensity lines.

For the possibility of spectral retrieval in the visible range ($20,001\text{--}20,101\text{ cm}^{-1}$), we carried out retrieval of a 5-line spectrum, similar to that presented in Fig. 2, all other conditions being equal. Weighted averaging (Fig. 4a) shows some worsening of the retrieval accuracy, which is mainly explained by the increase in the interpolation error for an interferogram with a large number of fringes. The standard deviation in this case is practically the same as in the calculation for the IR region.

An important feature of our method is its applicability to different types of interferograms, such as those obtained as a result of multiple-beam interference. We carried out spectral estimation of the interferogram obtained for multiple-beam interference on a rotating plane-parallel plate (analog of a Fabry–Perot interferometer) of thickness $200\text{ }\mu\text{m}$ with refractive index 1.5 and reflection coefficients for the faces equal to 0.3 for a 5-line spectrum similar to Fig. 2a, in the range

Fig. 2. Results for spectral estimation of an interferogram for a five-line spectrum using weighted averaging and broadening up to 2 cm^{-1} (a), inverse Fourier transform (b), MNS (c), and ESPRIT (d). Dashed line: original. Solid line: estimation.

Fig. 3. Results for spectral estimation of an interferogram for a seven-line spectrum using weighted averaging and broadening up to 2 cm^{-1} (a), inverse Fourier transform (b), MNS (c), and ESPRIT (d). The arrow indicates the position of the 2931.0 cm^{-1} line with intensity 0.24 rel. units, which is not visible after broadening on the background of more intense adjacent lines. Dashed line: original. Solid line: estimation.

Fig. 4. Results for spectral estimation of the interferogram for a five-line spectrum using weighted averaging in the visible range (a) and results obtained from a multiple-beam interferogram (b) with broadening up to 2 cm^{-1} . Dashed line: original. Solid line: estimation.

$2902\text{--}3002\text{ cm}^{-1}$ for 6001 readings (Fig. 4b). The retrieval result has significant similarity to the original, but the estimation accuracy is lower than in the case of a dual-beam interferogram. This is probably explained by the increase in the interpolation error due to the complex form of the interferogram, since in preliminary tests with more frequent calibration interferograms (every 2 cm^{-1}), the retrieval error is lower (for example, the spurious band at 2965.9 cm^{-1} does not appear).

In the best of the results obtained, the relative interferogram retrieval error exceeds the noise level by only a factor of 3–5, but in some complex spectra the retrieval result is unstable in this case. This suggests that for the parameters used, further refinement of the result is difficult and for such cases the method needs to be modified. Preliminary experiments showed that the result depends considerably on the number of interferogram sampling points, the length of the recorded segment, and the accuracy of the interpolation. The quality of the retrieval is affected by the number of lines m in the retrieval spectrum: the closer it is to the number in the original spectrum, the better the result. For some spectra, the retrieval accuracy can be increased by repeatedly running the algorithm, where the initial population for each run is created based on the previous result. It is theoretically possible to use *a priori* information about the spectrum in the calculation, such as the position of individual spectral lines. It is also important to develop a special interpolation for interferograms, for which fewer calibration measurements are needed while the interferogram retrieval accuracy is increased.

Conclusions. Study of the feasibility of using our proposed method for spectral estimation of a short interferogram in a narrow spectral range is promising because of the better quality of retrieval of the spectrum compared with some alternative methods, the simplicity of the measurement cycle (calibrated interferograms in a real spectrometer can be recorded immediately after fabrication, calibrating as needed), no need for post-processing or phase correction of the interferograms, and support for different types of interferograms. The most important disadvantage is the calculation time (on a dedicated server with an Intel Xeon E5/2.5 GHz single-core processor, it takes about 14–16 hours to obtain the final solution using a program written in C++ for the indicated parameters of the method). The calculation can presumably be speeded up by parallelization of the steps in the calculation and by using more powerful computers.

REFERENCES

1. R. J. Bell, *Introductory Fourier Transform Spectroscopy*, Academic Press, New York (1972).
2. V. A. Vagin, M. A. Gershun, G. N. Zhizhin, and K. I. Tarasov, *Wide-Aperture Spectral Instruments* [in Russian], Nauka, Moscow (1988).
3. A. R. Harvey and D. W. Fletcher-Holmes, *Opt. Express*, **12**, No. 22, 5368–5374 (2004).
4. V. Jovanov, J. Ivanchev, and D. Knipp, *Opt. Express*, **18**, No. 2, 426–438 (2010).
5. M. J. O'Dwyer and M. J. Padgett, *Strain*, **39** 107–110 (2003).
6. F. S. Chau, Y. Du, and G. Zhou, *J. Micromechan. Microeng.*, **18**, No. 2, 025023 (2008).
7. T. Okamoto, S. Kawata, and S. Minami, *Appl. Opt.*, **23**, No. 2, 269–273 (1984).
8. D. Komisarek, K. Reichard, D. Merdes, D. Lysak, P. Lam, S. Wu, and S. Yin, *Appl. Opt.*, **43**, No. 20, 3983–3988 (2004).

9. J. V. Sweedler, R. D. Jalkian, G. R. Sims, and M. B. Denton, *Appl. Spectrosc.*, **44**, No. 1, 14–20 (1990).
10. K. Minami, S. Kawata, and S. Minami, *Appl. Opt.*, **24**, No. 2, 162–167 (1985).
11. S. Prunet, C. Durieu, and B. Journet, *Opt. Eng.*, **42**, No. 4, 985–993 (2003).
12. X. Jian, C. Zhang, B. Zhao, and B. Zhu, *Opt. Commun.*, **281**, No. 9, 2424–2428 (2008).
13. Gang Han, Xue-Bin Liu, Bing-Liang Hu, and Cai-Ling Wang, in: *International Conference on Electronics, Communications and Control (ICECC)*, 9–11 September 2011, Ningbo, IEEE (2011), pp. 2112–2115.
14. T. Yang, C. Li, Z. Wang, and H. Ho, *Optik*, **124**, No. 13, 1377–1385 (2013).
15. Y. Wang, Z. Wen, Z. Nashed, and Q. Sun, *Appl. Opt.*, **45**, No. 13, 3111–3126 (2006).
16. K. Okada, Y. Ishibashi, and A. Kitade, *Appl. Opt.*, **28**, No. 8, 1546–1555 (1989).
17. T. Bäck and H. Schwefel, *Evol. Comput.*, **1**, No. 1, 1–23 (1993).
18. S. N. Sivanandam and S. N. Deepa, *Introduction to Genetic Algorithms*, Springer-Verlag, Berlin (2008), pp. 10–12.
19. D. H. Wolpert and W. G. Macready, *IEEE Trans. Evolution. Comput.*, **1**, No. 1, 67–82 (1997).
20. R. Leardi, *J. Chemometr.*, **14**, Nos. 5–6, 643–655 (2000).
21. F. J. Cuevas, J. H. Sossa-Azuela, and M. Servin, *Opt. Commun.*, **203**, No. 3, 213–223 (2002).
22. S. Wang and A. P. Dhawan, *Comput. Med. Imaging and Graphics*, **32**, No. 6, 429–441 (2008).
23. M. Arakawa, Y. Yamashita, and K. Funatsu, *J. Chemometr.*, **25**, No. 1, 10–19 (2011).
24. N. S. Mera, L. Elliott, and D. B. Ingham, *Invers. Probl. Engin.*, **11**, No. 2, 105–121 (2003).
25. N. Mera, L. Elliott, and D. Ingham, *Comput. Mechan.*, **33**, No. 4, 254–262 (2004).
26. C. Reeves and J. E. Rowe, *Genetic Algorithms: Principles and Perspectives. A Guide to GA Theory*, Springer, Boston (2002), pp. 277–278.
27. C. L. Byrne, *Signal Processing: A Mathematical Approach*, A. K. Peters, Wellesley (2005), pp. 111–112.
28. P. Stoica and R. L. Moses, *Introduction to Spectral Analysis*, Prentice Hall, Upper Saddle River (1997), pp. 163–164.