walls and filling is modeled as a linear function of difference between the angular speed of the body and the vortex vector of the filling.

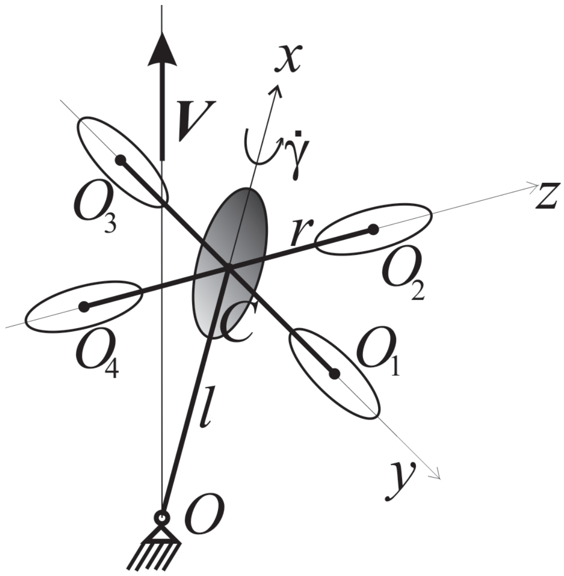

Fig. 1. Vane with cavity in a flow

The nonlinear dynamic system describing motion of the mechanical system is composed of 3 first-order and 3 second-order equations for 9 variables. The dynamic system was numerically integrated for chosen set of parameters and wide range of values of coefficient of interior friction. Two types of body (oblate and oblong) as well as two types of cavity were considered. Calculations show that the mechanical system in consideration has several different modes of motion: steady rotation, damped oscillations, precessions, and so on. It is shown that the mechanical system has variable dissipation.

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STABLE SELF-SUSTAINED COUNTER-OSCILLATIONS OF AERODYNAMIC PENDULA

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The mechanical system consists of two aerodynamic pendula that are intended for rotation around a fixed axis z in the opposite directions

(Fig.1). It is assumed that a viscous friction force exists between shafts of the pendula. The axis z is orthogonal to a wind flow, V is the speed of the flow. Let φ_1 and φ_2 be the angles of rotation of the first and the second pendulum respectively. These angles are measured counterclockwise and clockwise, correspondingly, if observe from the tip of the axis z of rotation. Quasi-steady model of aerodynamic action is used (similar to [1–3]).

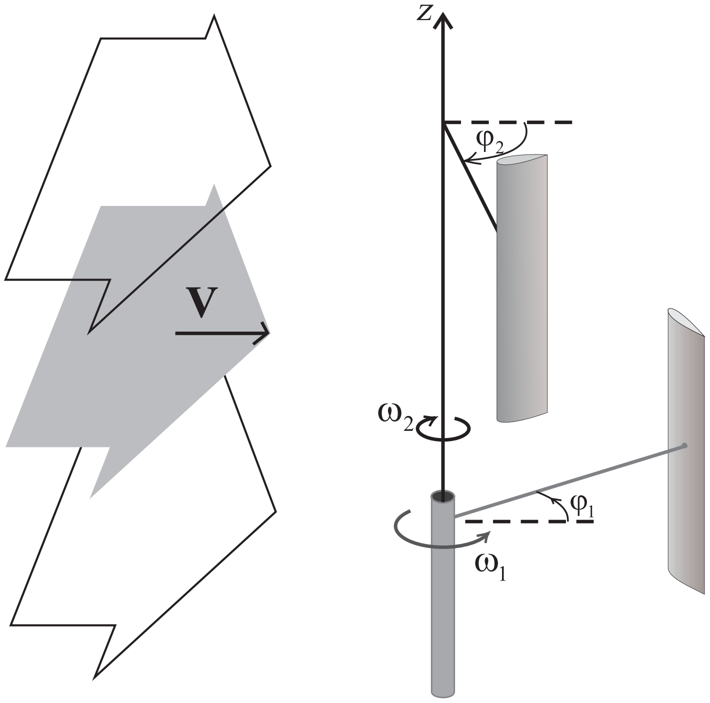

Fig. 1. The scheme of the system

The dynamics of the system can be described by the following dimensionless equations (a derivative with respect to a dimensionless time is denoted by a dot):

$$\begin{cases} \dot\varphi_1 = \omega_1, \\ \dot\varphi_2 = \omega_2, \\ \dot\omega_1 = \varepsilon\big(f(\varphi_1,\omega_1) - c(\omega_1+\omega_2)\big), \\ \dot\omega_2 = \varepsilon\big(f(\varphi_2,\omega_2) - c(\omega_1+\omega_2)\big), \end{cases} \tag{1}$$

where

$$f(\varphi_i,\omega_i) = \sqrt{d_i^2+l_i^2}\,(C_l(\alpha_i)l_i + C_d(\alpha_i)d_i), \quad \varepsilon = \frac{\rho S r^3}{2J}, \quad c = \frac{2A}{V\rho S r^2},$$

$$l_i = \cos(\varphi_i), \quad d_i = (\omega_i + \sin(\varphi_i)), \quad \alpha_i = \arctan\left(\frac{l_i}{d_i}\right), \quad i = 1,2.$$

Here ρ is the air density, r is the radius of each pendulum, S is the characteristic area of each blade, J is the moment of inertia of each pendulum around the axis of rotation, A is the viscous friction coefficient. The angle $\alpha_i, i = 1, 2$ is the instantaneous angle of attack for the blade of the pendulum, $C_d(\alpha_i)$ and $C_l(\alpha_i)$ are the drag and lift aerodynamic coefficients.

The system was analyzed numerically. It was shown that such $c_* > 0$ and $c_{**} > 0$ exist that there are stable self-sustained counter-oscillations if $c \in (c_*, c_{**})$. The corresponding oscillations of each pendulum qualitatively differ from those for a single pendulum described in [4].

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OPTIMIZATION PROBLEM FOR LINEAR DISCRETE 2-D SYSTEMS WITH BOUNDARY CONTROL

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For some problems appeared in the gas network modelling [1, 2] the initial data can be treated as a control parameter for the discrete 2-D system. In particular, it is of interest to determine an optimal