## References

- 1. Demyanov V.F., Rubinov A.M. Constructive Nonsmooth Analysis. Peter Lang, 1995.
- 2. Bagirov A.M., Ganjehlou A.N., Ugon J., Tor A.H. Truncated codifferential method for nonsmooth convex optimization // Pac. J. Optim. 2010. Vol. 6. No. 3. P. 483–496.
- 3. Bagirov A.M., Ugon J. Codifferential method for minimizing nonsmooth DC functions // J. Glob. Optim. 2011. Vol. 50. No. 1. P. 3–22.
- 4. Tor A.H., Bagirov A., Karasözen B. Aggregate codifferential method for nonsmooth DC optimization // J. Comput. Appl. Math. 2014. Vol. 259. P. 851–867.
- 5. Demyanov V.F., Bagirov A.M., Rubinov A.M. A method of truncated codifferential with applications to some problems of cluster analysis // J. Glob. Optim. 2002. Vol. 23. No. 1. P. 63–80.

## AUTOROTATION OF A VANE WITH VISCOUS FILLING

M.Z. Dosaev<sup>1</sup>, R. Garziera<sup>2</sup>

<sup>1</sup> Institute of Mechanics, Lomonosov Moscow State University <sup>1</sup> Michurinskiy pr-t, 119192 Moscow, Russia dosayev@imec.msu.ru

<sup>2</sup> Department of Engineering and Architecture, University of Parma rinaldo.garziera@gmail.com

We consider the motion around the fixed point O of a dynamically symmetric finned body with mass  $M_1$  in a resisting medium (Fig. 1). The flow speed V is constant. The fin assembly consists of four identical blades located symmetrically on the vane. The vane has an axisymmetric cavity filled with uniform uncompressible liquid with mass  $M_2$ .

For sake of simplicity, it is assumed that the cavity center coincides with the center of mass C of the body. We define the body orientation by Krylov angles. We introduce a coordination system Cxyz that is the principal axes of inertia of the mechanical system. We neglect the gravity and suppose that aerodynamic forces act on blades only. Under these conditions the body can perform an autorotation with some angular velocity around the dynamic symmetry axis Ox. The aerodynamic load is represented using the quasi-steady approach as a sum of drag and lift forces.

We assume that liquid can perform the uniform vortex motion. Thus, the state of the filling can be described by the vortex components satisfying the Helmholtz equations. The internal friction between cavity

walls and filling is modeled as a linear function of difference between the angular speed of the body and the vortex vector of the filling.

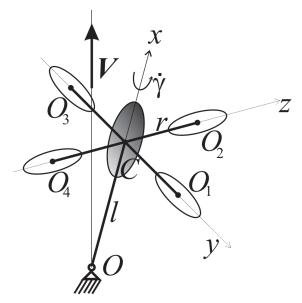


Fig. 1. Vane with cavity in a flow

The nonlinear dynamic system describing motion of the mechanical system is composed of 3 first-order and 3 second-order equations for 9 variables. The dynamic system was numerically integrated for chosen set of parameters and wide range of values of coefficient of interior friction. Two types of body (oblate and oblong) as well as two types of cavity were considered. Calculations show that the mechanical system in consideration has several different modes of motion: steady rotation, damped oscillations, precessions, and so on. It is shown that the mechanical system has variable dissipation.

This work was supported by the Russian Foundation for Basic Research (project NN 17-08-01366, 18-01-00538).

## STABLE SELF-SUSTAINED COUNTER-OSCILLATIONS OF AERODYNAMIC PENDULA

M.Z. Dosaev, L.A. Klimina, E.S. Shalimova

Institute of Mechanics, Lomonosov Moscow State University
1 Michurinsky prosp., 119192 Moscow, Russia
{dosayev,klimina}@imec.msu.ru, ekateryna-shalimova@yandex.ru

The mechanical system consists of two aerodynamic pendula that are intended for rotation around a fixed axis z in the opposite directions