(5) with matrices

$$A = \begin{pmatrix} -0,9 & -6,5 \\ 4,8 & -0,9 \end{pmatrix}, \qquad B = \begin{pmatrix} -1,39 & -0,65 \\ 0,48 & -1,39 \end{pmatrix}$$

is asymptotically stable if $\tau \in (0, \tau_1) \bigcup (\tau_2, \tau_3)$, where $\tau_1 = 0,2862$, $\tau_2 = 0,7141$, $\tau_3 = 1,2142$.

## References


1. *Repin Y.M.* About approximation replacement system with delay by ordinary differential equations // APM. 1965. Vol. 29, No. 2. P. 226–245.
2. *Matviy O.V., Cherevko I.M.* About approximation of system with delay and them stability// Nonlinear oscilations. 2004. Vol. 7, No. 2. P. 208–216.
3. *Ilika S.A., Matviy O.V., Piddubna L.A., Cherevko I.M.* Approximation of differential-functional equations and their application // Bukovinian Mathematical Journal. 2014. Vol. 2, No. 2–3. P. 92–96.


# METHOD OF CODIFFERENTIAL DESCENT FOR GLOBAL D.C. OPTIMIZATION


**M.V. Dolgopolik**

Saint Petersburg State University

7/9 Universitetskaya Ebm., 199034 Saint Petersburg, Russia

maxim.dolgopolik@gmail.com


The method of codifferential descent was developed by professor V.F. Demyanov for solving a large class of nonsmooth nonconvex optimization problem in the mid-1990s [1]. Later on, several modifications of this method designed specifically for solving convex and d.c. optimization problems were proposed [2–4].

Recall that a function $f\colon \mathbb{R}^d \to \mathbb{R}$ is called *codifferentiable* [1] at a point $x \in \mathbb{R}^d$, if there exist compact convex sets $\underline{d}f(x), \overline{d}f(x) \subset \mathbb{R}^{d+1}$ such that $\max_{(a,v)\in\underline{d}f(x)} a = \min_{(b,w)\in\overline{d}f(x)} b = 0$, and for any $\Delta x \in \mathbb{R}^d$ one has

$$f(x+\Delta x) - f(x) = \max_{(a,v)\in\underline{d}f(x)} \big(a + \langle v, \Delta x\rangle\big) + \min_{(b,w)\in\overline{d}f(x)} \big(b + \langle w, \Delta x\rangle\big) + o(\Delta x),$$

where $\langle\cdot,\cdot\rangle$ is the inner product in $\mathbb{R}^d$, and $o(\alpha\Delta x)/\alpha \to 0$ as $\alpha \to +0$. The pair $Df(x) = [\underline{d}f(x), \overline{d}f(x)]$ is referred to as *a codifferential* of $f$ at $x$. Clearly, a codifferential of $f$ at $x$ is not unique.

Let a function $f\colon \mathbb{R}^d \to \mathbb{R}$ be codifferentiable. One can utilize the method of codifferential descent (MCD) in order to minimize this function. The scheme of this method is as follows.

1. Choose $\mu > 0$, $\alpha_* > 0$ and $x_0 \in \mathbb{R}^d$.
2. $k$th iteration ($k \geq 0$).
   (a) Compute $Df(x_k)$.
   (b) For any $z = (b, w) \in \overline{d}_\mu f(x_k)$ compute
   $$\{(a(z), v(z))\} = \arg\min\Big\{\|(a, v)\| \ \Big| \ (a, v) \in \underline{d}f(x_k) + \{z\}\Big\}.$$
   (c) For any $z \in \overline{d}_\mu f(x_k)$ compute
   $$\alpha(z) \in \arg\min\Big\{f\big(x_k - \alpha v(z)\big) \ \Big| \ \alpha \in [0, \alpha_*]\Big\}.$$
   (d) Compute
   $$z_k \in \arg\min\Big\{f\big(x_k - \alpha(z) v(z)\big) \ \Big| \ z \in \overline{d}_\mu f(x_k)\Big\}.$$
   (e) Define $x_{k+1} = x_k - \alpha(z_k) v(z_k)$.

Note that at each iteration of the MCD one must perform line search in several directions. One can verify that at least one of these directions is a descent direction of the function $f$, and $f(x_{k+1}) < f(x_k)$ for all $k \in \mathbb{N}$. On the other hand, some of these directions might not be descent directions, i.e. the function $f$ may first increase and then decrease in these directions. This interesting feature of the MCD allows it to "jump over" some points of local minima of the function $f$, provided the parameter $\mu > 0$ is sufficiently large (for a particular example of this phenomenon see [5]). However, no theoretical results on the convergence of the MCD to a global minimizer of the objective function are known.

In this talk, we will discuss the performance of the MCD in the case when the function $f$ can be represented as the difference of convex functions, and a so-called *global* codifferential of the function $f$ is known. In particular, we will present new necessary and sufficient conditions for a global minimum of a d.c. function in terms of codifferential, and point out their connection with the MCD. With the use of these conditions one can prove the convergence of the MCD to a global minimum of a d.c. function. Furthermore, we will show that if the function $f$ is piecewise affine, then the MCD convergences to a global minimizer of this function in a finite number of steps.

The reported study was partially suported by RFBF, research project No. 18-31-00014.

## References


1. *Demyanov V.F., Rubinov A.M.* Constructive Nonsmooth Analysis. Peter Lang, 1995.
2. *Bagirov A.M., Ganjehlou A.N., Ugon J., Tor A.H.* Truncated codifferential method for nonsmooth convex optimization // Pac. J. Optim. 2010. Vol. 6. No. 3. P. 483–496.
3. *Bagirov A.M., Ugon J.* Codifferential method for minimizing nonsmooth DC functions // J. Glob. Optim. 2011. Vol. 50. No. 1. P. 3–22.
4. *Tor A.H., Bagirov A., Karasözen B.* Aggregate codifferential method for nonsmooth DC optimization // J. Comput. Appl. Math. 2014. Vol. 259. P. 851–867.
5. *Demyanov V.F., Bagirov A.M., Rubinov A.M.* A method of truncated codifferential with applications to some problems of cluster analysis // J. Glob. Optim. 2002. Vol. 23. No. 1. P. 63–80.


## AUTOROTATION OF A VANE WITH VISCOUS FILLING


**M.Z. Dosaev[1], R. Garziera[2]**

[1] Institute of Mechanics, Lomonosov Moscow State University

1 Michurinskiy pr-t, 119192 Moscow, Russia

dosayev@imec.msu.ru

[2] Department of Engineering and Architecture, University of Parma

rinaldo.garziera@gmail.com


We consider the motion around the fixed point $O$ of a dynamically symmetric finned body with mass $M_1$ in a resisting medium (Fig. 1). The flow speed $V$ is constant. The fin assembly consists of four identical blades located symmetrically on the vane. The vane has an axisymmetric cavity filled with uniform uncompressible liquid with mass $M_2$.

For sake of simplicity, it is assumed that the cavity center coincides with the center of mass $C$ of the body. We define the body orientation by Krylov angles. We introduce a coordination system $Cxyz$ that is the principal axes of inertia of the mechanical system. We neglect the gravity and suppose that aerodynamic forces act on blades only. Under these conditions the body can perform an autorotation with some angular velocity around the dynamic symmetry axis $Ox$. The aerodynamic load is represented using the quasi-steady approach as a sum of drag and lift forces.

We assume that liquid can perform the uniform vortex motion. Thus, the state of the filling can be described by the vortex components satisfying the Helmholtz equations. The internal friction between cavity