PROBLEM OF OPTIMALITY FOR SINGULAR STOCHASTIC CONTROL SYSTEMS

Charkaz Aghayeva

Mus Alparslan University, Mus, Turkey
c.agayeva@alparslan.edu.tr
Institute of Control Systems, Azerbaijan NAS
cherkez.agayeva@gmail.com

This paper is devoted to the optimal control problem of stochastic switching systems. Switching systems form the special class of hybrid systems and have the benefit for description of process with continuous dynamics. The first order necessary condition of optimality is a powerful tool for the study of optimal control problem, but is not always effective. For example, when the solution of adjoint equation is identically zero or the maximum principle is trivial, classified as singular cases, to investigate the corresponding optimal control problem is required additional information. The stochastic optimal control problem of switching system along singular controls is considered. The concept of singularity in maximum principle sense for stochastic switching systems is introduced. Second order necessary optimality condition for above mentioned systems with uncontrolled diffusion coefficients and with the right side restrictions in the form of inclusion is proved. The switching points are obtained. Finally, the transversality conditions for switching law are established.

STOCHASTIC OPTIMAL CONTROL PROBLEM FOR LINEAR SWITCHING SYSTEMS WITH VARIABLE DELAY

Charkaz Aghayeva

Mus Alparslan University, Mus, Turkey
c.agayeva@alparslan.edu.tr
Institute of Control Systems, Azerbaijan NAS
cherkez.agayeva@gmail.com

In this paper the stochastic optimal control problem of linear switching systems with variable delay is investigated. A necessary and sufficient condition of optimality for stochastic linear quadratic control problem is obtained. Linear quadratic controller is determined by means of stochastic Riccati equations.

Statement of Problem. Assume that $w_t^1, w_t^2, ..., w_t^r$ are independent Wiener processes, which generate filtration $F_t^l = \bar{\sigma}(w_t^l, t_{l-1} \leq t \leq t_l)$, $l = 1, 2, ..., r$, $T_0 = t_0 < t_1 < ... < t_r = T_1$. Let (Ω, F, P) be a probability space with filtration $\{F_t, \quad t \in [T_0, T_1]\}$, where $F_t = \bar{\sigma}(F_t^l, l = 1, 2, ..., r)$. $\mathbb{L}^2_{F^l}(a, b; \mathbb{R}^n)$ denotes the space of all predictable processes $x_t(\omega)$ such that $\mathbb{E} \int\limits_a^b |x_t(\omega)|^2 \, dt < +\infty$.

Consider the following stochastic linear control system:

$$\begin{aligned} dx^l(t) &= \left[A^l(t)x^l(t) + A_1^l(t)x^l(t - h(t))) + B^l(t)u^l(t) + g^l(t)\right] dt \\ &+ \left[C^l(t)x^l(t) + D^l(t)u^l(t) + f^l(t)\right] dw^l(t), t \in (t_{l-1}, t_l]; \end{aligned} \tag{1}$$

$$x^l(t) = \Phi^{l-1}x^{l-1}(t_{l-1}) + K^{l-1}, t \in [t_{l-1} - h(t_{l-1}), t_{l-1}], l = 2, 3, ..., r; \tag{2}$$

$$x^1(t) = K^0, t \in [t_0 - h(t_0), t_0]; \tag{3}$$

$$u^l(t) \in U_\partial^l \equiv \left\{u^l(\cdot, \cdot) \in \mathrm{L}^2_{F^l} | u^l(t, \cdot) \in U^l \subset \mathrm{R}^{m_l}, a.c.\right\}. \tag{4}$$

Here $h(t) > 0, t \in [T_0, T_1]$, is continuously differentiable non- random function, such that $\dfrac{dh(t)}{dt} < 1$. Elements of U_∂^l are called admissible controls.

Our goal is to find optimal solution $(x, u) = \left(x^1, x^2, ..., x^r, u^1, u^2, ..., u^r\right)$ and switching sequence $\mathbf{t} = (t_1, t_2, ..., t_r)$, that minimizes the cost functional:

$$J(u) = E\times \tag{5}$$

$$\sum_{l=1}^r \left[\left\langle G^l x^l(t_l), x^l(t_l)\right\rangle + \int\limits_{t_{l-1}}^{t_l} \left(\left\langle M^l(t)x^l(t), x^l(t)\right\rangle + \left\langle N^l(t)u^l(t), u^l(t)\right\rangle\right) dt\right]$$

on the decisions of the system (1)–(3) in the class of admissible controls.

The elements of matrices $A^l, A_1^l, B^l, C^l, D^l, M^l, N^l$ and vectors G^l, g^l, f^l are continuous, bounded functions. Φ^l, G^l, M^l are a positively semi-defined matrices , and N^l are a positively defined matrices.

The necessary and sufficient condition of optimality for stochastic linear control systems (1)–(4) with quadratic cost functional (5) is obtained. Well known that one of the elegant features of the Linear Quadratic theory is the opportunity to give a description for optimal control in a linear state feedback form by Riccati equation. Finally, the explicit representation of the optimal control is defined via a set of stochastic Riccati equations for linear switching systems with variable delay.