ANALYSIS OF OBSERVABILITY OF LINEAR STATIONARY SINGULARLY PERTURBED SYSTEM WITH DELAY ON THE BASIS OF DECOUPLING TRANSFORMATION

O.B. Tsekhan

Yanka Kupala State University of Grodno
22 Ozheshko str., 230023 Grodno, Belarus
tsekhan@grsu.by

Consider the linear stationary singularly perturbed system with delay

$$LSSPSD: \begin{array}{l} \dot{x}(t)=A_{10}x(t)+A_{11}x(t-h)+A_2y(t), x\in\mathbb{R}^{n_1}, y\in\mathbb{R}^{n_2}, \\ \mu\dot{y}(t)=A_{30}x(t)+A_{31}x(t-h)+A_4y(t),\ t\in T=[0,t_1], \\ v(t)=C_1x(t)+C_2y(t), t\in T, v\in\mathbb{R}^m, m\leq n_1+n_2, \\ x(0)=x_0,\ y_0(0)=y_0,\ x(\theta,\mu)=\phi(\theta), \theta\in[-h,0). \end{array} \quad (1)$$

Here A_{ij}, $i=1,3, j=0,1, A_k, k=2,4, C_j, j=1,2$, are real matrices of appropriate dimensions; $0<h$ is constant delay; $x_0\in\mathbb{R}^{n_1}$, $y_0\in\mathbb{R}^{n_2}$, $\varphi(\theta)$ are unknown initial vectors and continuous n_1-vector-function; μ is a small parameter, $\mu\in(0,\mu^0]$, $\mu^0\ll 1$. For a fixed $\mu>0$ denote by

$$\sigma(\mu)=\left\{\lambda\in\mathbb{C}:\ \det\begin{pmatrix}\lambda E_{n_1}-A_{10}-A_{11}e^{-ph} & -A_2 \\ -A_{30}-A_{31}e^{-ph} & \mu\lambda E_{n_2}-A_4\end{pmatrix}=0\right\}$$

the spectrum of (1). By the parameters of LSSPSD (1) we define [1] independent of μ degenerate system (DS) $\{\dot{\bar{x}}(t)=A_s\left(e^{-ph}\right)\bar{x}(t), v_s(t)=C_s\left(e^{-ph}\right)\bar{x}(t), \bar{x}(0)=x_0, \bar{x}(\theta)=\phi(\theta), \theta\in[-h,0)\}$ and boundary layer system (BLS) $\left\{\frac{d\tilde{y}(\tau)}{d\tau}=A_4\tilde{y}(\tau),\ v_f(\tau)=C_2\tilde{y}(\tau), \tau=\frac{t}{\mu}\in\left[0,\frac{t_1}{\mu}\right],\right.$ $\left.\tilde{y}(0)=y_0-A_4^{-1}\left[A_{30}x_0+A_{31}\phi(-h)\right]\right\}$ with the spectrum

$\sigma_s=\{\lambda\in C:\det\left[\lambda E_{n_1}-A_s\left(e^{-ph}\right)\right]=0\}$ and

$\sigma_f=\{\lambda\in C:\det\left[\lambda E_{n_2}-A_4\right]=0\}$, correspondingly.

On the basis of LSSPSD decoupling transformation in [1] the separation (at sufficiently small μ) of the LSSPSD (1) spectrum $\sigma(\mu)$ into two disjoint parts was proved:

the "fast" part $\sigma_y(\mu)=\left\{\tilde{\lambda}_i(\mu)=\frac{1}{\mu}\lambda_i(\mu):\lambda_i(\mu)\underset{\mu\to 0}{\to}\lambda_{fi}\in\sigma_f\right\}$ and

the "slow" part $\sigma_x(\mu)=\left\{\lambda_i(\mu):\lambda_i(\mu)\underset{\mu\to 0}{\to}\lambda_{si}\in\sigma_s\right\}$.

Let us denote by $\Sigma_\lambda(\mu)$ the finite-dimensional system that is the projection of LSSPSD (1) on the generalized proper subspace, associated with its eigenvalue $\lambda\in\sigma(\mu)$.

Definition 1. *At the fixed* $\mu \in (0, \mu^0]$ *LSSPSD (1) is spectrally* $\{x, y\}$*-(x-,y-)observable if any finite-dimensional system* $\Sigma_\lambda(\mu)$*, associated with the eigenvalues* $\lambda \in \sigma(\mu)$ $(\lambda \in \sigma_x(\mu), \lambda \in \sigma_y(\mu))$*, is observable.*

Theorem 1. *If the DS is spectrally observable, i.e.*

$$rank \left[\lambda E_{n_1} - A'_s\left(e^{-\lambda h}\right), \quad C'_s \right] = n_1 \quad \forall \lambda \in \sigma_s, \tag{2}$$

then $\exists \mu_s^* > 0$ *that the LSSPSD (1) is spectrally x-observable for all* $\mu \in (0, \mu_s^*]$*. If the BLS is spectrally observable, i.e.*

$$rank \left[\lambda E_{n_2} - A'_4, \quad B'_2 \right] = n_2, \quad \forall \lambda \in \sigma_f, \tag{3}$$

then $\exists \mu_f^* > 0$ *that the LSSPSD (1) is spectrally y-observable for all* $\mu \in \left(0, \mu_f^*\right]$*. If the conditions (2) and (3) are fulfilled, then LSSPSD (1) is spectrally* $\{x, y\}$*-observable for all* $\mu \in (0, \bar{\mu}]$*,* $\bar{\mu} = \min\{\mu_s^*, \mu_f^*\}$*.*

P r o o f . By applying to (1) the spectral observability condition [2], decoupling transformation [1], taking into account the invariance of the spectrum and preserving the matrix rank under nondegenerate transformations, it is determined that LSSPSD (1) is spectrally $\{x, y\}$-observable at a fixed $\mu > 0$ if and only if the condition is satisfied

$$rank \begin{pmatrix} \lambda E_{n_1} - A_s\left(e^{-\lambda h}\right) + O(\mu) & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_1} & \mu\lambda E_{n_2} - A_4 + O(\mu) \\ C_s\left(e^{-\lambda h}\right) + O(\mu) & C_2 + O(\mu) \end{pmatrix} = n_1 + n_2.$$

Based on above spectrum $\sigma(\mu)$ properties, with the preserving of the matrix rank under small perturbations, of (2) follows the fullness of the last matrix rank $\forall \lambda \in \sigma_x(\mu)$ and the spectral x-observability of (1), analogously of (3) follows spectral y-observability of (1), for sufficiently small $\mu > 0$. Combining, we are convinced of the validity of the statement regarding the spectral $\{x, y\}$-observability.

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References

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STABILIZATION OF NONLINEAR CONTROL-AFFINE SYSTEMS WITH OSCILLATING INPUTS

A. Zuyev

Max Planck Institute for Dynamics of Complex Technical Systems
Sandtorstraße 1, 39106 Magdeburg, Germany
Institute of Applied Mathematics and Mechanics
National Academy of Sciences of Ukraine, G. Batiuka 19, 84116 Sloviansk, Ukraine
zuyev@mpi-magdeburg.mpg.de, zuyev@nas.gov.ua

This talk is devoted to the stabilization problem for nonlinear control-affine systems of the form

$$\dot{x} = f_0(x) + \sum_{j=1}^{m} u_j f_j(x), \tag{1}$$

where the vector fields f_0, f_1,..., f_m are assumed to be smooth in the domain $D \subset \mathbb{R}^n$, $0 \in D$, $f_0(0) = 0$, and the dimension of the state vector $x = (x_1, x_2, ..., x_n)^T$ is strictly less than the dimension of the control $u = (u_1, u_2, ..., u_m)^T$.

We assume that system (1) is small-time locally controllable (STLC) at $x = 0$. Under this kind of controllability assumptions (more precisely, if $x = 0$ is locally continuously reachable in small time with small control and (1) satisfies the strong jet accessibility rank condition $(x, u) = (0, 0)$), it was shown in [1] that system (1) is locally smoothly stabilizable in small time by a periodic time-varying feedback law $u = h(x, t)$, provided that $n \notin \{2, 3\}$. However, the question *how to construct the above controllers* remains open in general case. In this presentation, we propose a control design scheme that allows constructing the stabilizing feedback laws $u = h(x, t)$, provided that the controllability rank condition is satisfied with the iterated Lie brackets up to some fixed order. Our approach extends the idea of [2] for the class of control systems with $f_0 \neq 0$. These results are applied for the stabilization of nonholonomic systems and underactuated mechanical systems with drift.