STABLE HAMILTONIAN CONSTRUCTIONS IN DYNAMIC RECONSTRUCTION PROBLEMS

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We present a new method to solve inverse problems with the help of dynamic optimization [1]–[5] of a regularized integral negative discrepancy.

1. Dynamic reconstruction problem. We consider controlled dynamical systems of the form

$$\frac{dx(t)}{dt} = f(t, x(t)) + G(t, x(t))u(t), \quad t \in [0, T], \tag{1}$$

where $x \in \mathbb{R}^n$ are state variables, controls $u \in \mathbb{R}^n$ are restricted:

$$u \in U = \{u_i \in [a_i^-, a_i^+],\ a_i^- < a_i^+,\ i \in \overline{1, n}\}. \tag{2}$$

We know inaccurate measurements $x(t_i)$, $t_i = t_{i-1} + \Delta t$, $i \in \overline{1, N}$, $t_0 = 0, t_N = T$, of a realizing (basic) solution $x^*(t)$ system (1)-(2) under a measurable control $u^*(t)$, such that

$$|x(t_i) - x^*(t)| \leq \delta, \tag{3}$$

where $\delta \in (0, \delta_0]$ estimates errors of measurement.

The dynamic reconstruction (DR) problem is: at current instant $t \geq t_2$, using smooth interpolations $y^\delta(\cdot) : [0, t] \to \mathbb{R}^n$ of measurements, reconstruct a piecewise continuous control $u^\delta(\cdot) : [0, t - \Delta t] \to U$ generating the trajectory $x^\delta(\cdot) : [0, t - \Delta t] \to \mathbb{R}^n$ of system (1)-(2), such that

$$\|x^\delta(\cdot) - x^*(\cdot)\|_C = \max_{\tau \in [0, T - \Delta t]} \|x^\delta(\tau) - x^*(\tau)\| \to 0, \tag{4}$$

$$\|u^\delta(\cdot) - u^*(\cdot)\|_{L_2} = \int_0^{T - \Delta t} \|u^\delta(\tau) - u^*(\tau)\|^2 d\tau \to 0, \tag{5}$$

hold, as $\delta \to 0$, $\Delta t \to 0$.

2. Auxiliary problems of calculus of variations. To solve this inverse DR problem we introduce auxiliary problems of calculus of variations (CV). For each $[t_{i-1}, ti], i \in \overline{2, N}$ we consider the cost functional

$$I_{t_{i-2},x_0}(u(\cdot)) = \int_{t_{i-2}}^{t_i} \Big[- \frac{\|x(\tau) - y^\delta(\tau)\|^2}{2} + \frac{\alpha}{2}\|v(\tau)\|^2 \Big] d\tau, \qquad (6)$$

where $\alpha > 0$ — is a small parameter of regularization, $x_0 \in \mathbb{R}^n$.

The auxiliary calculus of variations problem is: minimize the cost functional (6) over all solutions $x(\cdot)$ of system (1) satisfied the boundary conditions

$$x(t_{i-2}) = x_0 = y^\delta(t_{i-2}), \quad \frac{dx(t_{i-2})}{dt} = \frac{dy^\delta(t_{i-2})}{dt}. \qquad (7)$$

We get solutions of hamiltonian systems (state and conjugate characteristics $x(\cdot), s(\cdot) : [t_{i-2}, t_i] \to \mathbb{R}^n \times \mathbb{R}^n$) obtained with the help of optimality conditions for CV problems (1), (6), (7). The characteristics are stable relative to $y^\delta(\cdot)$. The obtained constructions $x(t),\ u(t) = u(x(t), s(t))$ are close to $x^*(t), u^*(t)$. Namely, they are solutions of DRP (1)-(5) if parameters $\delta, \alpha, \Delta t$ tend to zero in a necessary concordance.

Discussion of current results on the subject is given [6], [7].

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