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GLOBAL OPTIMALITY CONDITIONS AND NUMERICAL METHODS

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1. Problem Statement. Consider the following problem:

$$
(\mathcal{P}): \quad \begin{cases} f_0(x) := g_0(x) - h_0(x) \downarrow \min\limits_x, \quad x \in S, \\ f_i(x) := g_i(x) - h_i(x) \le 0, \quad i \in I = \{1, \dots, m\}, \\ f_i(x) := g_i(x) - h_i(x) = 0, \quad i \in \mathcal{E} = \{m+1, \dots, l\}; \end{cases} \tag{1}
$$

where the functions $g_i(\cdot)$, $h_i(\cdot)$, $i \in \{0\} \cup I \cup \mathcal{E}$, are convex on $I\!R^n$, so that the functions $f_i(\cdot)$, $i \in \{0\} \cup I \cup \mathcal{E}$, are the d.c. functions [1–5]. Besides, assume that the set $S \subset I\!R^n$ is convex and compact.

Furthermore, suppose that the set $Sol(\mathcal{P})$ of global solutions to Problem $(\mathcal{P})$ and the feasible set $\mathcal{F}$ of Problem $(\mathcal{P})$ are non-empty. Additionally, in what follows the optimal value $\mathcal{V}(\mathcal{P})$ of Problem $(\mathcal{P})$ is supposed to be finite:

$$
\mathcal{V}(\mathcal{P}) := \inf(f_0, \mathcal{F}) := \inf_x \{ f_0(x) \mid x \in \mathcal{F}) \} > -\infty.
$$

Further, we introduce the following penalty function $W(x) := \max\{0, f_1(x), \dots, f_m(x)\} + \sum\limits_{j \in \mathcal{E}} |f_j(x)|$, and consider the penalized problem as follows:

$$
(\mathcal{P}_\sigma): \quad \theta_\sigma(x) := f_0(x) + \sigma W(x) = G_\sigma(x) - H_\sigma(x) \downarrow \min_x, \quad x \in S, \tag{2}
$$

where $\sigma \ge 0$ is a penalty parameter, $G_\sigma(\cdot)$ and $H_\sigma(\cdot)$ can be shown to be convex functions.

2. Global Optimality Conditions.

Theorem 1. *Let a feasible point $z \in \mathcal{F}$, $\zeta := f_0(z)$, be a solution to Problem $(\mathcal{P})$ and $\sigma \geq \sigma_* > 0$, where $\sigma_* \geq 0$ is a threshold value of the penalty parameter such that $Sol(\mathcal{P}) = Sol(\mathcal{P}_\sigma)$ $\forall \sigma \geq \sigma_*$.*

Then, for every pair $(y, \beta) \in I\!R^n \times I\!R$ such that

$$H_\sigma(y) = \beta - \zeta, \tag{3}$$

the following inequality holds

$$G_\sigma(x) - \beta \geq \langle H'_\sigma(y), x - y \rangle \quad \forall x \in S, \tag{4}$$

for every subgradient $H'_\sigma(y) \in \partial H_\sigma(y)$ of the function $H_\sigma(\cdot)$ at the point y.

Under supplementary conditions the Global Optimality Conditions (GOCs) of Theorem 1 become sufficient for a feasible point $z \in \mathcal{F}$ to be a global solution.

Moreover, it shown that GOCs are related to the Classical Optimization Theory and possess the constructive (algorithmic) property (if GOCs are violated, one can find a feasible (in the original problem) vector which is better than the point in question). Using this property of the GOCs we develop new local and global search algorithms for the original problem and study its convergence. Computational testing witnessed on numerical effectiveness of the developed approach.

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