ON IMPULSIVE RELAXATION OF CONTROL CONTINUITY EQUATIONS WITH UNBOUNDED VECTOR FIELDS

M.V. Staritsyn

Matrosov Institute for System Dynamics and Control Theory of SB RAS

134 Lermontov str., 664033 Irkutsk, Russia

starmaxmath@gmail.com

1. Object of the study. We address a class of control problems for the continuity equation driven by a control-affine vector field:

$$\mu_0 = \vartheta; \quad \partial_t \, \mu_t + \nabla \cdot (\mu_t \, f_t) = 0, \quad t \in \mathcal{T} \doteq [0, T], \tag{1}$$

where ϑ is a fixed probability measure on $\mathbb{R}^n$,

$$t \mapsto f_t(x) \doteq f\big(x, u(t)\big), \quad x \in \mathbb{R}^n; \quad f(x, u) = g(x) + H(x)\, u;$$

$g : \mathbb{R}^n \to \mathbb{R}^n$ and $H : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ are given vector and matrix functions. As control inputs we admit Borel measurable functions $u(\cdot) : \mathcal{T} \mapsto \mathbb{R}^m$ satisfying the constraint:

$$u(\cdot) \in \mathcal{U} = \mathcal{U}(M) \doteq \{u \in L_\infty(\mathcal{T}, \mathbb{R}^m) \,\big|\, \|u\|_{L_1(\mathcal{T}, \mathbb{R}^m)} = M\}. \tag{2}$$

Here, $M > 0$ represents the available resource of the guide.

Due to the input-affine structure of the system and the obvious unboundedness of the maps $f\big(x, u(t)\big)$ in the pointwise sense, the arcs $t \mapsto \mu_t$ of system (1), (2) can become somehow close to discontinuous measure-valued functions. In view of this, one should not expect the desired compactness of the tube of solutions to (1), (2) in the natural topology. As a consequence of this fact, one looses to guarantee the well-posedness of related optimal control problems.

2. Content of the talk. In our study, we elaborate a constructive relaxation (compactification) of the funnel of solutions to the continuity equation in a coarse topology of the space of measure-valued functions with bounded variation. For this relaxation, we derive a constructive representation in terms of a characteristic measure differential equation (being an impulsive extension of the characteristic ordinary differential equation). Finally, we consider an optimal impulsive control problem of steering the initial distribution to a given target set. For this problem, we prove the existence of a solution.

Our study extends some results from [1] and calls up papers [2, 3].

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References

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GLOBAL OPTIMALITY CONDITIONS AND NUMERICAL METHODS

A.S. Strekalovsky

Matrosov Institute for System Dynamics and Control Theory of SB RAS

134 Lermontov str., 664033 Irkutsk, Russia

strekal@icc.ru

1. Problem Statement. Consider the following problem:

$$(\mathcal{P}):\quad \begin{cases} f_0(x) := g_0(x) - h_0(x) \downarrow \min\limits_x, \quad x \in S, \\ f_i(x) := g_i(x) - h_i(x) \le 0, \quad i \in I = \{1, \dots, m\}, \\ f_i(x) := g_i(x) - h_i(x) = 0, \quad i \in \mathcal{E} = \{m+1, \dots, l\}; \end{cases} \tag{1}$$

where the functions $g_i(\cdot)$, $h_i(\cdot)$, $i \in \{0\} \cup I \cup \mathcal{E}$, are convex on $I\!R^n$, so that the functions $f_i(\cdot)$, $i \in \{0\} \cup I \cup \mathcal{E}$, are the d.c. functions [1–5]. Besides, assume that the set $S \subset I\!R^n$ is convex and compact.

Furthermore, suppose that the set $Sol(\mathcal{P})$ of global solutions to Problem $(\mathcal{P})$ and the feasible set $\mathcal{F}$ of Problem $(\mathcal{P})$ are non-empty. Additionally, in what follows the optimal value $\mathcal{V}(\mathcal{P})$ of Problem $(\mathcal{P})$ is supposed to be finite:

$$\mathcal{V}(\mathcal{P}) := \inf(f_0, \mathcal{F}) := \inf_x \{f_0(x) \mid x \in \mathcal{F})\} > -\infty.$$

Further, we introduce the following penalty function $W(x) := \max\{0, f_1(x), \dots, f_m(x)\} + \sum\limits_{j \in \mathcal{E}} |f_j(x)|$, and consider the penalized problem as follows:

$$(\mathcal{P}_\sigma):\quad \theta_\sigma(x) := f_0(x) + \sigma W(x) = G_\sigma(x) - H_\sigma(x) \downarrow \min_x, \quad x \in S, \tag{2}$$

where $\sigma \ge 0$ is a penalty parameter, $G_\sigma(\cdot)$ and $H_\sigma(\cdot)$ can be shown to be convex functions.