NECESSARY AND SUFFICIENT CONDITIONS FOR NONDIFFERENTIABLE PROGRAMMING PROBLEMS

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In the paper, using the classes of $S-(\alpha,\beta,\nu,\delta,\omega)$ and $S-(\beta,\delta)$ locally Lipschitz mappings at the point, higher order necessary and sufficient conditions of the extremum are received for extreme problems in the presence of restrictions.

Introduction. Let X and Y be Banach spaces, $C \subset X$, $F : X \to Y$, $S : X \to Y$, $f : X \to \mathbb{R}$, $\varphi : X \to \mathbb{R}$, $\alpha > 0$, $\nu > 0$, $\beta \geq \alpha\nu$, $K > 0$, $\delta > 0$, $o : \mathbb{R}_+ \to \mathbb{R}_+$, $\tilde{o} : \mathbb{R}_+ \to \mathbb{R}_+$, $o(0) = \tilde{o}(0) = 0$, $\mathbb{R}_+ = [0, +\infty)$. Let's put $B = \{z \in X : \|z\| \leq 1\}$, $B(x,\delta) = \{z \in X : \|z - x\| \leq \delta\}$.

Let $o : \mathbb{R}_+ \to \mathbb{R}_+$, where $\lim\limits_{t\downarrow 0} \frac{o(t)}{t} = 0$. The mapping F is said to be $S - (\alpha,\beta,\nu,\delta,o(\beta))$ locally Lipschitz with the constant K at the point $\bar{x} \in X$, if F satisfies the condition

$$\|\mathrm{F}(\bar{\mathrm{x}} + \mathrm{x} + \mathrm{z}) - \mathrm{F}(\bar{\mathrm{x}} + \mathrm{x}) - S(\mathrm{x} + \mathrm{z}) + S(\mathrm{x})\| \leq$$

$$\leq K\|z\|^{\nu} (\|\mathrm{x}\|^{\beta-\alpha\nu} + \|z\|^{\frac{\beta-\alpha\nu}{\alpha}}) + o(\|x\|^{\beta})$$

at $x, z \in \delta B$. If $S(x) \equiv 0$, then the mapping F is said to be $(\alpha,\beta,\nu,\delta,o(\beta))$ locally Lipschitz with the constant K at the point $\bar{x}$.

We call the mapping $F : X \to Y$ satisfying the condition $\|\mathrm{F}(\bar{\mathrm{x}} + \mathrm{z}) - \mathrm{F}(\bar{\mathrm{x}}) - S(\mathrm{z})\| \leq K\|z\|^{\beta}$ at $z \in \delta B$, $S - (\beta,\delta)$ locally Lipschitz with the constant K at the point $\bar{x}$.

If the function $f : X \to \mathbb{R}$ satisfies the condition $\mathrm{f}(\bar{\mathrm{x}} + \mathrm{z}) - f(\bar{\mathrm{x}}) - \varphi(\mathrm{z}) \leq K\|z\|^{\beta}$ at $z \in \delta B$, we call the function f $\varphi - (\beta,\delta)$ locally semi-Lipschitz with the constant K at the point $\bar{x}$.

If $F(x) = f(x)$, we put $S(x) = \varphi(x)$. Further we consider that $S(0) = 0$ and $\varphi(0) = 0$. Let's designate $I = \{0, 1, \ldots, m\}$ and $J = \{1, \ldots, m\}$.

1. Necessary condition of the higher order. Let X and Y be Banach spaces, $f_i : X \to \mathbb{R}$, $i \in I$, $F : X \to Y$, $C \subset X$.

Let's consider the problem

$$f_0(x) \to \min, \qquad f_i(x) \leq 0, i = 1, \ldots, m, F(x) = 0, \ x \in C. \qquad (1)$$

If $C \subset X$ is a convex set and $x_0 \in C$, we will designate that $\tilde{C} = intC \bigcup \{x_0\}$, $S_{\tilde{C}}(x_0) = \bigcup_{\lambda>0} \frac{\tilde{C}-x_0}{\lambda}$ and $T_C(x_0) = cl \bigcup_{\lambda>0} \frac{C-x_0}{\lambda}$.

Theorem 1. *If x_0 be the local minimum point in the problem (1), $\beta > 1$, the function f_i satisfy $\varphi_i - (\beta, \delta)$ locally semi-Lipschitz condition with the constant K at the point x_0, where $i \in I$, $\varphi_i : X \to \mathbb{R}$ sublinear continuous functions at $i \in I$, $f_j(x_0) = 0$ at $j \in J$, the operator $F : X \to Y$ be strictly differentiable at the point x_0 and $F'(x_0)X = Y$, C be a convex set, $intC \neq \emptyset$, then there exist simultaneously non-zero $\alpha_0 \geq 0$, $\alpha_1 \geq 0, \ldots, \alpha_m \geq 0$ and $y^* \in Y^*$ such that $\sum_{i=0}^{m} \alpha_i \varphi_i(x) + \langle y^*, F'(x_0)x\rangle \geq 0$ at $x \in T_C(x_0)$.*

Let's put $g_{(a,y)}(x) = \max\limits_{0\leq i\leq m} (\varphi_i^1(x) + a_i) + \delta_{T_C(x_0)\bigcap \Lambda^{-1}(-y)}(x)$, where $\varphi_i^1 : X \to \mathbb{R}$, $\Lambda x = F'(x_0)x$, $(a, y) \in \mathbb{R}^{m+1} \times Y$, and $H = \{x \in X : \varphi_i^1(x) \leq 0,\ i \in I,\ \Lambda x = 0,\ x \in S_{\tilde{C}}(x_0)\}$.

Theorem 2. *If x_0 is the local minimum point in problem (1), $f_i(x_0) = 0$ at $i \in I$, $\beta > 2$, the functions f_i, $i \in I$, satisfy $\varphi_i^1 + \varphi_i^2 - (\beta, \delta)$ locally semi-Lipschitz condition with the constant K at the point x_0, $\varphi_i^1 : X \to \mathbb{R}$ are sublinear continuous functions at $i \in I$, $\varphi_i^2 : X \to \mathbb{R}$ are positive homogeneous functions of degree 2 and satisfy $(1, 2, 1, \delta, o(2))$ locally Lipschitz condition with the constant K at the zero point at $i \in I$, the mapping $F : X \to Y$ is strictly differentiable at the point x_0, $F'(x_0)X = Y$ and F satisfies $F'(x_0)x + S(x) - (\beta, \delta)$ locally Lipschitz condition with the constant K at the point x_0, $S : X \to Y$ is a positive homogeneous operator of degree 2 and satisfies $(1, 2, 1, \delta, \tilde{o}(2))$ locally Lipschitz condition with the constant K at the zero point, C is a convex set, $\partial g^*_{(a,y)}(0) \neq \emptyset$ at $(a, y) \in \mathbb{R}^{m+1} \times Y$ and $int\, T_C(x_0) \bigcap Ker\, \Lambda \neq \emptyset$, then*

$$E(h) = \sup(\sum_{i=0}^{m} \lambda_i \varphi_i^2(h) + \langle y^*, S(h)\rangle : \lambda_i \geq 0, \sum_{i=0}^{m} \lambda_i = 1,\ p_i \in \partial\varphi_i^1(0),$$
$$x^* \in N_C(x_0), y^* \in Y^*, \sum_{i=0}^{m} \lambda_i p_i + \Lambda^* y^* + x^* = 0) \geq 0$$

at $h \in H$.

A number of strengthens of Theorem 1 and 2 are also obtained.

Let's note that the strengthening of the condition of Theorem 2 is also a sufficient condition for the local extremum.