Theorem 4 (Extension 2 [5]) *Suppose* $A \cap B = \emptyset$. *If* $\|a - b\| < d(A, B) + \varepsilon > 0$, *then, for any* $\lambda > 0$ *and* $\tau \in (0, 1)$, *there exist points* $a' \in A \cap \mathbb{B}_\lambda(a)$, $b' \in B \cap \mathbb{B}_\lambda(b)$ *and* $a^* \in X^*$ *such that*

$$\|a^*\| = 1, \quad d(a^*, N_A(a')) + d(-a^*, N_B(b')) < \varepsilon/\lambda,$$

$$\tau\|a' - b'\| \leq \langle a^*, a' - b' \rangle.$$

In [1], Theorems 3 and 4 are compared and more extensions are provided.

Acknowledgments. The research was partially supported by the Australian Research Council, project DP160100854.

References

1. *Bui H.T., Kruger A.Y.* About extensions of the extremal principle. // Vietnam J. Math. 2018. DOI 10.1007/s10013-018-0278-y.
2. *Kruger A.Y.* About extremality of systems of sets. // Dokl. Nats. Akad. Nauk Belarusi 1998. Vol. 42. No. 1, P. 24–28.
3. *Kruger A.Y., López, M.A.* Stationarity and regularity of infinite collections of sets. // J. Optim. Theory Appl. 2012. Vol. 154. No. 2. P. 339–369.
4. *Kruger A.Y., Mordukhovich B.S.* Extremal points and the Euler equation in non-smooth optimization problems // Dokl. Akad. Nauk BSSR 1980. Vol. 24. No. 8. P. 684–687.
5. *Zheng X.Y., Ng K.F.* A unified separation theorem for closed sets in a Banach space and optimality conditions for vector optimization. // SIAM J. Optim. 2011. Vol. 21. No. 3. P. 886–911.

SYNTHESIS OF ROBUST INTERVAL POLYNOMIALS USING THE EXTENDED ROOT LOCUS

A.A. Nesenchuk

United Institute of Informatics Problems of the National Academy of Sciences of Belarus, 6 Surganov str., 220012 Minsk, Belarus

anes@newman.bas-net.by

Introduction. Among the modern methods of polynomial synthesis [1–3] the methods related to polynomial families represent a separate group. The most effective solutions for interval families within the algebraic approach were proposed by Kharitonov [4], who offered to consider four specific polynomials of the family with constant coefficients. In [2] the frequency criteria of hurwitz robust stability are considered. Root locus approach is considered in [5].

The root locus method is described here for searching intervals of robust stability for coefficients of the given unstable polynomial with constant coefficients subject to perturbations. The method is based on the extended root locus and can be used for both, interval polynomial synthesis and analysis of the polynomial behavior under coefficient perturbations.

Polynomial adjustment is performed by setting up each one of its coefficients separately and sequentially and determining values of coefficient variation intervals (intervals of uncertainty). The influence of each coefficient variation upon the polynomial roots dynamics (behavior) is considered and analyzed separately, and this influence could be observed in the root locus portraits. Root locus method is thus generalized to the cases when the number of polynomial variable coefficients is arbitrary.

1. Problem statement. Define a polynomial like

$$g_n(s) = s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_n, \tag{1}$$

where $a = (a_1, ..., a_{n-1}, a_n)$, $A \subset R^n$, $a \in A$, $\underline{a_j} \leqslant a_j \leqslant \overline{a_j}$, $j = \overline{1, n}$.

Polynomial (1) can be both, nonhurwitz or hurwitz one.

The task is to generalize the root locus method for the cases when the number of variable coefficients is arbitrary and thus to solve the problem of synthesis of the robustly stable uncertain (interval) polynomial by setting up coefficients of the given unstable/stable polynomial (1). The coefficients' values are searched being the nearest to the given ones in terms of their location on the corresponding root locus branches.

2. Polynomial extension and synthesis. Introduce polynomial system (2). Polynomials of (2) have common coefficients, but not common roots.

Definition 1. *System of polynomials (2) name the* ***extension of polynomial*** *(1) or the* ***extended polynomial.***

Definition 2. *Points, where root locus branches begin and where the root locus parameter is equal to zero are called the* ***root locus initial points.***

Definition 3. *The root locus relative to the algebraic equation free term is called the* ***free root locus.***

Definition 4. *Complete set of extension (2) root loci name as the extended root loci of (1).*

$$E_n = \begin{cases} s + a_1 = g_1(s) & (2.1) \\ s^2 + a_1 s + a_2 = g_2(s) & (2.2) \\ ... & \\ s^i + a_1 s^{i-1} + ... + a_{i-1} s + a_i = g_i(s) & (2.i) \\ ... & \\ s^{n-1} + ... + a_{n-2} s + a_{n-1} = g_{n-1}(s) & (2.(n-1)) \\ s^n + a_1 s^{n-1} + ... + a_{n-1} s + a_n = g_n(s) & (2.n), \end{cases} \quad (2)$$

$$g_i(s) = s^i + a_1 s^{i-1} + ... + a_{i-1} s + a_i,\ g_{i-1}(s) = (g_i(s) a_i)/s. \quad (3)$$

Statement 1. *If all initial points of polynomial $g_i(s)$ (3) root locus, excluding a single one at the origin, are located in the left half-plane s, this polynomial is asymptotically stabile, when the following condition holds:*

$$0 < a_i < A_i^+, \quad (4)$$

where is a set of a_i values at cross points of polynomial $g_i(s)$ (3) root locus positive branches with axis $\omega(s = \sigma + \omega)$.

Theorem 1. *For ensuring asymptotic stability of interval polynomial (1) it is enough to*
a) find among polynomials of extension (2) the stable polynomial of degree $i = k$ being the closest one to n;
b) set up sequentially every coefficient a_j of (1), beginning with $a_j = a_k$, within interval $k < j \leqslant n$ by setting up the free term a_i of the corresponding i-th polynomial of extension (2) as per condition (4) assuming $i = j$.

It is worth mentioning here, that among polynomials $g_i(s)$, $i = 1, 2, ..., n$, of extension (2) it is always could be found at least one stable polynomial, as the 2-nd degree polynomial with positive coefficients is always asymptotically stable.

On the basis of Theorem 1 and Statement 1 an easy to use algorithm for the robustly stable interval polynomial synthesis has been worked out.

Conclusion. A method has been worked out for synthesis of asymptotically stable interval polynomial on the basis of the given unstable polynomial with constant coefficients by setting up coefficients of the unstable one. The root locus approach is used. The task is solved by introduction of the polynomial extended root locus notion, which allows to obtain a descriptive picture of the polynomial roots dynamics under coefficient variations and to disclose on this basis the cause of instability. The intervals of uncertainty for each coefficient being set up are specified along the root locus branches. The nearest stable polynomial to the given unstable one in terms of a distance along the root locus branches is being found. Solving the task of ensuring the required quality (e.g. the polynomial stability margin) could be one of the possible directions for further development of the method.

The developed method is new and allows to extend the application sphere of the root locus method, which is traditionally considered to be the method of systems synthesis by only a single parameter (coefficient) variation and with only one variable parameter (coefficient), in both directions, systems synthesis by many parameters variation and systems synthesis with many parameters variation.

References

1. *Ackermann J.* Robust Control: The Parameter Space Approach. London: Springer Verlag, 2002.
2. *Polyak B.T., Scherbakov P.S.* Robust Stability and Control [in Russian]. M.: Nauka, 2002.
3. *Dorf R., Bishop R.* Modern Control Systems. N.Y.: Prentice Hall, 2011.
4. *Kharitonov V.L.* About asymptotic stability of equilibrium of the linear differential equations systems family [in Russian] // Differential Equations. 1978. Vol. XI, No. 11. P. 2086–2088.
5. *Nesenchuk A.A.* The root locus method of synthesis of stable polynomials by adjustment of all coefficients // Automation and Remote Control. 2010. Vol. 71. No. 8. P. 1515–1525.

ROBUST SYNTHESIS METHOD FOR LINEAR CASCADED SYSTEM WITH LOW ORDER CONTROLLER

O.F. Opeiko

Belarusian National Technical University
65 Nezalejnosti av., 220013 Minsk, Belarus
oopeiko@bntu.by

The controllers synthesis with determined structure, for instance, proportional-integral differential (PID), is an important problem for the