Theorem 1. *Suppose a controlled object has a description of the (2) form and there is an analytical description of the control target* $\psi(Y(t)) = 0$. *Then variable control* u *from (3) occurs and a control system (2), (3) is asymptotically stable in the mean, with the variables* Y, *and* $\psi(Y(t+1)) + \lambda\psi(Y(t))$ *reaching the minimum possible value of the variance* σ^2.

The report presents the results of comparative modeling of four control synthesis algorithms for nonlinear objects with different forms of uncertainty in their description. We classify them as classical ACAR under nonmodel conditions, nonlinear adaptation method [2] and its modification, ADARs method.

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References

1. *Kolesnikov A.A.* New nonlinear methods of flight control. Fizmatlit, Moscow, 2013.
2. *Kolesnikova S.I.* Multiple control system for a non-linear object with uncertainty // Spiiras Proceedings. 2017. Vol. 55. No. 6. P. 114–133.

ABOUT EXTENSIONS OF THE EXTREMAL PRINCIPLE

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X is an Asplund space, $A, B \subset X$ are closed and $\bar{x} \in A \cap B$. $N_A(\bar{x})$ and $\overline{N}_A(\bar{x})$ stand for the Fréchet and limiting normal cones to A at $\bar{x}$, respectively.

Definition 1 (Extremality [4]) *The pair* $\{A, B\}$ *is* locally extremal *at* $\bar{x}$ *if there exists a* $\rho > 0$ *such that for any* $\varepsilon > 0$ *there are* $u, v \in X$ *satisfying*

$$(A - u) \cap (B - v) \cap \mathbb{B}_\rho(\bar{x}) = \emptyset, \quad \max\{\|u\|, \|v\|\} < \varepsilon.$$

Theorem 1 (Extremal principle [4]) *If the pair* $\{A, B\}$ *is locally extremal at* $\bar{x}$, *then the following two equivalent conditions hold:*

(i) *for any* $\varepsilon > 0$ *there exist points* $a \in A \cap \mathbb{B}_\varepsilon(\bar{x})$, $b \in B \cap \mathbb{B}_\varepsilon(\bar{x})$ *and* $a^* \in X^*$ *such that*

$$\|a^*\| = 1, \quad d(a^*, N_A(a)) < \varepsilon, \quad d(-a^*, N_B(b)) < \varepsilon;$$

(ii) *for any* $\varepsilon > 0$ *there exist points* $a \in A \cap \mathbb{B}_\varepsilon(\bar{x})$, $b \in B \cap \mathbb{B}_\varepsilon(\bar{x})$, $a^* \in N_A(a)$ *and* $b^* \in N_B(b)$ *such that*

$$\|a^*\| + \|b^*\| = 1, \quad \|a^* + b^*\| < \varepsilon. \tag{1}$$

If, additionally, $\dim X < \infty$, *then* $\overline{N}_A(\bar{x}) \cap (-\overline{N}_B(\bar{x})) \neq \{0\}$.

Definition 2 (Stationarity [2]) (i) *The pair* $\{A, B\}$ *is* stationary *at* $\bar{x}$ *if for any* $\varepsilon > 0$ *there exist a* $\rho \in (0, \varepsilon)$ *and* $u, v \in X$ *such that*

$$(A - u) \cap (B - v) \cap \mathbb{B}_\rho(\bar{x}) = \emptyset, \;\; \max\{\|u\|, \|v\|\} < \varepsilon\rho;$$

(ii) *the pair* $\{A, B\}$ *is* approximately stationary *at* $\bar{x}$ *if for any* $\varepsilon > 0$ *there exist* $\rho \in (0, \varepsilon)$, $a \in A \cap \mathbb{B}_\varepsilon(\bar{x})$, $b \in B \cap \mathbb{B}_\varepsilon(\bar{x})$ *and* $u, v \in X$ *such that*

$$(A - a - u) \cap (B - b - v) \cap (\rho\mathbb{B}) = \emptyset, \;\; \max\{\|u\|, \|v\|\} < \varepsilon\rho. \tag{2}$$

Local extremality $\Rightarrow$ Stationarity $\Rightarrow$ Approximate stationarity

Theorem 2 (Extended extremal principle [2]) *The pair* $\{A, B\}$ *is approximately stationary at* $\bar{x}$ *if and only if the two equivalent conditions in Theorem 1 hold true.*

Theorem 3 (Extension 1 [3]) (i) *If points* $a \in A$, $b \in B$ *and* $u, v \in X$ *satisfy conditions* (2) *with some* $\varepsilon > 0$ *and* $\rho > 0$, *then for any* $\delta > \max\{\|a - \bar{x}\|, \|b - \bar{x}\|\} + \rho(\varepsilon + 1)$ *there exist points* $a' \in A \cap \mathbb{B}_\delta(\bar{x})$, $b' \in B \cap \mathbb{B}_\delta(\bar{x})$ *and* $a^* \in N_A(a')$, $b^* \in N_B(b')$ *satisfying conditions* (1).

(ii) *If* $a \in A$, $b \in B$ *and* $a^* \in N_A(a)$, $b^* \in N_B(b)$ *satisfy conditions* (1) *for some* $\varepsilon > 0$, *then for any* $\delta > 0$ *there exists a* $\rho \in (0, \delta)$ *and points* $u, v \in X$ *satisfying conditions* (2).

Theorem 4 (Extension 2 [5]) *Suppose* $A \cap B = \emptyset$. *If* $\|a - b\| < d(A, B) + \varepsilon > 0$, *then, for any* $\lambda > 0$ *and* $\tau \in (0, 1)$, *there exist points* $a' \in A \cap \mathbb{B}_\lambda(a)$, $b' \in B \cap \mathbb{B}_\lambda(b)$ *and* $a^* \in X^*$ *such that*

$$\|a^*\| = 1, \quad d(a^*, N_A(a')) + d(-a^*, N_B(b')) < \varepsilon/\lambda,$$

$$\tau\|a' - b'\| \leq \langle a^*, a' - b' \rangle.$$

In [1], Theorems 3 and 4 are compared and more extensions are provided.

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References

1. *Bui H.T., Kruger A.Y.* About extensions of the extremal principle. // Vietnam J. Math. 2018. DOI 10.1007/s10013-018-0278-y.
2. *Kruger A.Y.* About extremality of systems of sets. // Dokl. Nats. Akad. Nauk Belarusi 1998. Vol. 42. No. 1, P. 24–28.
3. *Kruger A.Y., López, M.A.* Stationarity and regularity of infinite collections of sets. // J. Optim. Theory Appl. 2012. Vol. 154. No. 2. P. 339–369.
4. *Kruger A.Y., Mordukhovich B.S.* Extremal points and the Euler equation in nonsmooth optimization problems // Dokl. Akad. Nauk BSSR 1980. Vol. 24. No. 8. P. 684–687.
5. *Zheng X.Y., Ng K.F.* A unified separation theorem for closed sets in a Banach space and optimality conditions for vector optimization. // SIAM J. Optim. 2011. Vol. 21. No. 3. P. 886–911.

SYNTHESIS OF ROBUST INTERVAL POLYNOMIALS USING THE EXTENDED ROOT LOCUS

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Introduction. Among the modern methods of polynomial synthesis [1–3] the methods related to polynomial families represent a separate group. The most effective solutions for interval families within the algebraic approach were proposed by Kharitonov [4], who offered to consider four specific polynomials of the family with constant coefficients. In [2] the frequency criteria of hurwitz robust stability are considered. Root locus approach is considered in [5].