The system can be described by considering the motion of each particle independently from each other. We assume that sizes of particles tend to zero.

The concentration can be considered a small fixed value and the average distance $\frac{1}{n}$ between particles can be considered significantly larger than sizes of particles, i.e. $\frac{1}{n} \gg 2d$.

The following theorem is true.

Theorem 1. *The limit distribution of the configuration Gibbs distribution in the Boltzmann-Grad limit ($d \to 0$, n is fixed) which describes equilibrium state of one-dimensional symmetric system of particles (hard spheres) is uniform (the speed distribution is the Maxwell one and is unchanged).*

References

1. *Bogolyubov N.N.* Problems of a dynamical theory in statistical physics. Gosudarstv. Izdat., 1946 [in Russian].
2. *Gubal' G.N.* On the existence of weak local in time solutions in the form of a cumulant expansion for a chain of Bogolyubov's equations of a one-dimensional symmetric particle system // Journal of Mathematical Sciences. 2014. Vol. 199. P. 654–666.

CONSTRUCTING A REGULATOR TO OUTPUT A STOCHASTIC OBJECT INTO AN ANALYTICALLY GIVEN SET OF STATES

S.I. Kolesnikova

National Research Tomsk State University

36 Lenina str., 634050 Tomsk, Russia

skolesnikova@yandex.ru

Introduction. A generalization of the method for analytical design of aggregated regulators (ADAR) [1], previously developed only for deterministic objects, is proposed for a nonlinear object with random uncertainty in the description.

1. Control problem statement for a discrete deterministic object in accordance to classical ADAR method. In its statement, the problem of control on manifolds incorporates a) object of control specified by a system of ODEs or a system of difference (nonlinear) equations for continuous and discrete control problems, respectively; b) target of control for an analytically defined equation $\psi(x) = 0$.

ADAR-control will be referred to as the variable $u^A(x(t))$, delivering a solution to the discrete variational problem

$$x(t+1) = f(x(t);\theta) + u(x(t));$$

$$J = \sum_{j=1}^{m}\sum_{t=0}^{\infty}(\omega_j^2\psi_j^2(x(t)) + \Delta\psi_j^2(x(t))) \to \min, \tag{1}$$

with the restrictions $\psi(x) = 0$, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m, m < n$ are vectors of states and control, respectively; $f \in \mathbb{R}^n$ is a nonlinear restricted known vector-function; θ is a parameters vector; $T = \{0, 1, 2, ...\}$. In what follows, the variational problem will be denoted by a pair of symbols $J, \psi(x)$.

2. New statement of control problem for a stochastic object based on ADAR ideology. Let a discrete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 1}, \mathbf{P})$ with filtration be given, where $\mathcal{F}_t = \sigma(\xi(s), s \leq t$ and $(\xi(t)_{t\geq 1}$ is a sequence of independent identically distributed random variables with mean 0 and variance σ^2 . Let us discuss an object with a descrete, most general description

$$Y(t+1) = F(Y(t)) + u(t) + \xi(t+1) + c\xi(t), \tag{2}$$

where $0 < c < 1, t \geq 1, \mathcal{F}_t, u(Y(t))$ are non-linear function and control, respectively.

It is required to carry out control in the state space of the object transferring this object (2) from its given initial state $Y(t_0) = Y_0, t\ ge1$ in the neighborhood of the target manifold $\psi(Y(t)) = 0$ and minimizing both the value of mathematical expectation of the quality functional $E(J)$ from (ref eqJ) and the variance of quantities $Y(t), \psi(Y(t))$.

Note that direct application of the classical ADAR method to the control object (2) is not possible.

3. Problem solution of control construction for a stochastic object. ADARs method. Then follows the formulation of the main result for the problem of object (2) stabilization in the neighborhood of a given value $Y^* = const$. For this purpose we introduce the target macrovariable as follows $\Psi(Y(t)) = Y(t) - Y^*$. Suppose that for $t \geq 1$:

$$u = -F(Y(t) - c\xi(t) + Y^* - \lambda\psi(t). \tag{3}$$

Theorem 1. *Suppose a controlled object has a description of the (2) form and there is an analytical description of the control target* $\psi(Y(t)) = 0$. *Then variable control* u *from (3) occurs and a control system (2), (3) is asymptotically stable in the mean, with the variables* Y, *and* $\psi(Y(t+1)) + \lambda\psi(Y(t))$ *reaching the minimum possible value of the variance* σ^2.

The report presents the results of comparative modeling of four control synthesis algorithms for nonlinear objects with different forms of uncertainty in their description. We classify them as classical ACAR under nonmodel conditions, nonlinear adaptation method [2] and its modification, ADARs method.

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References

1. *Kolesnikov A.A.* New nonlinear methods of flight control. Fizmatlit, Moscow, 2013.
2. *Kolesnikova S.I.* Multiple control system for a non-linear object with uncertainty // Spiiras Proceedings. 2017. Vol. 55. No. 6. P. 114–133.

ABOUT EXTENSIONS OF THE EXTREMAL PRINCIPLE

A.Y. Kruger

Centre for Informatics and Applied Optimisation
Federation University Australia, POB 663, Ballarat, Vic, 3350, Australia
a.kruger@federation.edu.au

X is an Asplund space, $A, B \subset X$ are closed and $\bar{x} \in A \cap B$. $N_A(\bar{x})$ and $\overline{N}_A(\bar{x})$ stand for the Fréchet and limiting normal cones to A at $\bar{x}$, respectively.

Definition 1 (Extremality [4]) *The pair* $\{A, B\}$ *is* locally extremal *at* $\bar{x}$ *if there exists a* $\rho > 0$ *such that for any* $\varepsilon > 0$ *there are* $u, v \in X$ *satisfying*

$$(A - u) \cap (B - v) \cap \mathbb{B}_\rho(\bar{x}) = \emptyset, \quad \max\{\|u\|, \|v\|\} < \varepsilon.$$

Theorem 1 (Extremal principle [4]) *If the pair* $\{A, B\}$ *is locally extremal at* $\bar{x}$, *then the following two equivalent conditions hold:*