arbitrary normed spaces. As applications of the above results, we introduce, for nonsmooth functions, a new notion of the Demyanov-Rubinov subdifferential at a given point, and show that it generalizes a number of known notions of subdifferentiability, in particular, the Fenchel-Moreau subdifferential of convex functions and the Dini-Hadamard (directional) subdifferential of directionally differentiable functions. Some applications of Demyanov-Rubinov subdifferentials to extremal problems are considered.

The talk develop and improve the results presented earlier in the papers [1–3].

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ASYMPTOTIC STABILITY PROPERTIES OF NON-AUTONOMOUS SYSTEMS WITH PARAMETRIC EXCITATION

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Consider a class of nonlinear systems with time-varying parameters:

$$\dot{x} = f(x, \gamma(t)), \quad t \in \mathbb{R}^+ = [0, +\infty), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $\gamma : \mathbb{R}^+ \to \bar{\Gamma} \subset \mathbb{R}^m$ is a continuous function, and $f \in C^1(\mathbb{R}^n \times \bar{\Gamma}; \mathbb{R}^n)$. We assume that there exists a function $x^* : \bar{\Gamma} \to \mathbb{R}^n$ such that $f(x^*(\gamma^*), \gamma^*) = 0$ for all $\gamma^* \in \bar{\Gamma}$. Note that, although $x^*(\gamma^*)$ is an equilibrium of system (1) for each fixed γ^*, the function $x^*(\gamma(t))$ is not a solution of (1) in general. In this talk, we propose a novel approach to analyze the asymptotic stability property of system (1) and present sufficient conditions for the convergence of the trajectories of system (1) to a given neighborhood of $x^*(\gamma(t))$. Our approach exploits the concept of stability of a family of sets (cf. [4]).

Definition 1. *A family of sets* $\mathcal{L} = \{\mathcal{L}_t\}_{t\geq 0}$ *(*$\emptyset \neq \mathcal{L}_t \subset \mathbb{R}^n$*) is said to be globally uniformly asymptotically stable for* (1) *if:*
– for each $\varepsilon>0$*, there is a* $\delta>0$ *such that, for all* $t_0 \in \mathbb{R}^+$*, if* $x^0 \in B_\delta(\mathcal{L}_{t_0})$ *then* $x(t) \in \mathcal{B}_\varepsilon(\mathcal{L}_t)$ *for all* $t \geq t_0$ *(uniform stability);*
– for each $\delta > 0$ *and* $\varepsilon>0$*, there is a* $t_1 \in [0,\infty)$ *such that, for all* $t_0 \in \mathbb{R}^+$*, if* $x^0 \in B_\delta(\mathcal{L}_{t_0})$ *then* $x(t) \in B_\varepsilon(\mathcal{L}_t)$ *for all* $t \geq t_0 + t_1$ *(global uniform attractivity).*

Here $B_\varepsilon(\mathcal{L}_t)$ denotes the ε-neigborhood of $\mathcal{L}_t \subset \mathbb{R}^n$. In the paper [1], we have proposed the following asymptotic stability conditions for gradient-like systems.

Theorem 1. *Let* $\gamma \in C^1(\mathbb{R}^+; \mathbb{R}^m)$ *and* $x^* \in C^1(\overline{\Gamma}; \mathbb{R}^n)$*, where* $\overline{\Gamma} \subset \mathbb{R}^m$ *is a closed bounded domain. Assume that there exist functions* $V \in C^2(\mathbb{R}^n \times \mathbb{R}^m; \mathbb{R})$, $w_{11}, w_{12}, w_2 \in \mathcal{K} : [0,\Delta] \to \mathbb{R}^+$ *with some* $\Delta > 0$*, such that the following conditions are satisfied:*
1) $\gamma(t) \in \Gamma$ *for all* $t \geq 0$*, and* $\sup_{t\in\mathbb{R}^+} \|\dot{\gamma}(t)\| \leq \nu$ *with some* $\nu \geq 0$*;*
2) for all $x \in B_\Delta(x^*(g))$, $g \in \Gamma$,

$$w_{11}(\|x - x^*(g)\|) \leq V(x - x^*(g), g) \leq w_{12}(\|x - x^*(g)\|),$$

$$\left\| \frac{\partial V(x - x^*(g), g)}{\partial x} \right\|^2 \geq w_2(\|x - x^*(g)\|);$$

3) there are $L, H>0$, $M \geq 0$ *such that*

$$\left\| \frac{\partial V(x,g)}{\partial x} \right\| \leq L, \quad \left\| \frac{V(x,g)}{\partial g} \right\| \leq M, \quad \left\| \frac{\partial^2 V(x,g)}{\partial x^2} \right\| \leq H,$$

for all $x \in B_\Delta(x^*(g))$ *and for all* $g \in \Gamma$.
Then, for every $\lambda \in (0, w_{11}(\Delta))$*, there exists a* $c > 0$ *such that the family of sets* $\mathcal{L}_{\lambda,t} = \{x \in \mathbb{R}^n : V(x, \gamma(t)) \leq \lambda\}$, $t \geq 0$*, is locally uniformly asymptotically stable for system* $\dot{x} = -c \, \dfrac{\partial V(x, \gamma(t))}{\partial x}$.

It should be emphasized that, in contrast to many other results on the stability of non-autonomous systems (see, e.g., [2,3,5,6]), we do not require that $\lim_{t\to\infty} \|\dot{\gamma}(t)\| \to 0$ to prove the attractivity property. In this talk, we extend the approach of [1] to a general class of non-autonomous systems and discuss applications of the obtained results to such control problems as dynamical optimization and trajectory tracking.

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INCREASING THE ROBUSTNESS OF DEADBEAT CONTROLLER

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Introduction. In modern control systems, almost all algorithms are implemented in digital form. At the same time the PID algorithm has a number of limitations. The problem of integral saturation is emerged if there is limitation of the control action; response speed problem; adapting the settings when changing the dynamic characteristics of the object.

1. Receiving formulas for calculating regulators. Calculation of many digital controllers is much more algorithmized. One such option is a deadbeat controller [1]. This controller is characterized by high response speed. However, this is achieved by forming a high value of the control action, which is difficult to implement under applied conditions.