Here ρ is the air density, r is the radius of each pendulum, S is the characteristic area of each blade, J is the moment of inertia of each pendulum around the axis of rotation, A is the viscous friction coefficient. The angle $\alpha_i, i = 1, 2$ is the instantaneous angle of attack for the blade of the pendulum, $C_d(\alpha_i)$ and $C_l(\alpha_i)$ are the drag and lift aerodynamic coefficients.

The system was analyzed numerically. It was shown that such $c_* > 0$ and $c_{**} > 0$ exist that there are stable self-sustained counter-oscillations if $c \in (c_*, c_{**})$. The corresponding oscillations of each pendulum qualitatively differ from those for a single pendulum described in [4].

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OPTIMIZATION PROBLEM FOR LINEAR DISCRETE 2-D SYSTEMS WITH BOUNDARY CONTROL

M. Dymkov[1], S. Dymkov[2]

[1] Belarus State Economic University
26 Partizanski ave., 220070 Minsk, Belarus
dymkov_m@bseu.by

[2] Temasek Laboratories, National University of Singapore
T-Lab Building 5A, Engineering Drive 1, 117411 Singapore
tslsmd@nus.edu.sg

For some problems appeared in the gas network modelling [1, 2] the initial data can be treated as a control parameter for the discrete 2-D system. In particular, it is of interest to determine an optimal control programm for gas pressure and gas flow at the pipe when the gas regulation in the time is feasible at a fixed node of the pipe.

The linearised model of this process can be presented by the following discrete 2-D system

$$x(t+1,s) = A_0 x(t,s) + A_1 x(t,s+1) + A_2 x(t,s-1) \tag{1}$$

with initial and boundary control conditions:

$$x(0,s) = \varphi(s), \qquad s \in \mathbb{Z}_+ \setminus \{0\},$$

$$x(t,0) = u(t), \quad t = 0,1,2,...,T-1. \tag{2}$$

The optimization problem is to minimize the cost functional of the form

$$J(u) = \sum_{t=1}^{T} \Big[\sum_{s \in \mathbb{Z}_+} \big(Qx(t,s), x(t,s)\big) + \big(Ru(t), u(t)\big) \Big]. \tag{3}$$

Here $A_i : E \to E$, $i = 0,1,2$ are linear operators, $Q, R : E \to E$ are self-adjoint operators such that $Q \geq 0, R > 0$ where E is finite-dimensional Hilbert space, the inner product in which defined by the symbol $(\cdot,\cdot)$. The state variable $x(t,s), \varphi(s)$ are E-valued functions, $\varphi(s)$ is a given E-valued function, $u(t)$ is E-valued control inputs, $\mathbb{Z}_+$ denotes the set of nonnegative integers, $T > 1$ is a given integer.

In this work the results developed in [3] are extended to the class of 2-D systems with boundary control inputs. In particular, using operator setting approach [3] the following result is obtained.

Theorem 1. *The optimal control u^0 of the problem (1)-(3) is given as*

$$u^0(t) = -R^{-1} A_2^* z^0(t,0),\ t = 0,1,\ldots,T-1, \tag{4}$$

where $z(t,0)$ is determined by the following system of equations

$$z(t,s) \qquad = A_0^* z(t+1,s) + A_1^* z(t+1,s+1) + \tag{5}$$

$$+ A_2^* z(t+1,s-1) + Qx(t+1,s),\ s \in \mathbb{Z}, \tag{6}$$

$$x(t+1,s) = A_0 x(t,s) + A_1 x(t,s+1) + A_2 x(t,s-1) - \tag{7}$$

$$- A_2 R^{-1} A_2^* z(t,0),\ t = 0,1,\ldots,T-1, \tag{8}$$

with the boundary conditions

$$x(0,s) = \varphi(s),\ z(T,s) = 0. \tag{9}$$

It is proved that the boundary problem (5)-(9) has an unique solution.

The subject of ongoing work is also to establish the optimal control in the feedback form [4] in the frequency domain for the important case when $T \to \infty$. Such kind presentation of the control law is of interest for engineering implementation.

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UPPER ENVELOPES OF FAMILIES OF CONTINUOUS CONCAVE FUNCTIONS AND THEIR APPLICATIONS IN NONSMOOTH ANALYSIS

V.V. Gorokhovik

Institute of Mathematics, National Academy of Sciences of Belarus
11 Surganov str., 220072 Minsk, Belarus
gorokh@im.bas-net.by

In the talk we characterize the classes of (extended) real-valued functions defined on normed vector spaces that admit the representation as the upper envelope of their maximal (with respect of the pointwise ordering) concave minorants. The results presented in the talk generalize and extend the well-known Demyanov-Rubinov characterization of lower semicontinuous positively homogeneous functions as the upper envelope of exhaustive families of continuous superlinear functions to more larger classes of (not necessarily positively homogeneous) functions defined on