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THEORETICAL AND MATHEMATICAL PHYSICS

Screening of a Low-Frequency Magnetic Field by an Open Thin-Wall Spherical Shell

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Abstract—The solution to the problem of penetration of a low-frequency magnetic field through a semitransparent open spherical shell is reduced to solving the system of second-order Fredholm integral equations. The effect of the opening angle of the open shell and of some geometrical parameters of the screen as well as electrophysical properties of the spherical shell material on the attenuation of the field in the spherical shell is analyzed numerically.

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INTRODUCTION

The problem of formation of an electromagnetic environment ensuring ecological safety and normal operation of various devices is of considerable importance at present. The electromagnetic environment is the aggregate of electromagnetic fields in a given region of space, which may affect the operation of certain technical devices and biological objects [1, 2]. To ensure a favorable electromagnetic environment, screening of electromagnetic fields is used [3–7].

A method for calculating low-frequency magnetic fields in the case of open perfectly conducting screens was proposed in [3, 6, 7]. In this case, the field does not penetrate through the screen walls. In screens with a low conductivity of the material, the field penetrates through the walls of the shell. Such processes are simulated using nonclassical boundary conditions [8].

Here, we will show that the solution of the formulated boundary value problem with nonclassical boundary conditions on a semitransparent open spherical shell can be reduced to solving a system of second-order Fredholm integral equations. In the computational experiment, the values of the screening coefficient for a low-frequency magnetic field in the shell will be obtained.

FORMULATION OF THE PROBLEM

A semitransparent thin-wall open spherical shell Γ of thickness Δ is located in space R^3 with permittivity ε_0 and permeability μ_0. Shell Γ is made of a material with electromagnetic parameters ε, μ, and γ (ε is the permittivity, μ is the permeability, and γ is electrical conductivity). The shell is located on the surface of sphere Γ_1 of radius a with a circular aperture characterized by opening angle θ_0 (Fig. 1).

To solve the problem with point O at the center of sphere Γ_1, we introduce spherical coordinates $\{r, \theta, \varphi\}$:

$$x = r\cos\varphi\sin\theta, \quad 0 \le r \le \infty,$$

$$y = r\sin\varphi\sin\theta, \quad 0 \le \varphi \le 2\pi,$$

$$z = r\cos\theta, \quad 0 \le \theta \le \pi.$$

In this case, idealized shell Γ is described as

$$\Gamma = \{r = a, \theta_0 < \theta \le \pi, 0 \le \varphi \le 2\pi\}.$$

A primary low-frequency magnetic field with potential u_0 and with circular frequency ω propagates in space R^3.

We denote by u_1 the magnetic field potential in the sphere Γ_1 and by $u_2 = u_0 + u_2$ the potential outside the sphere.

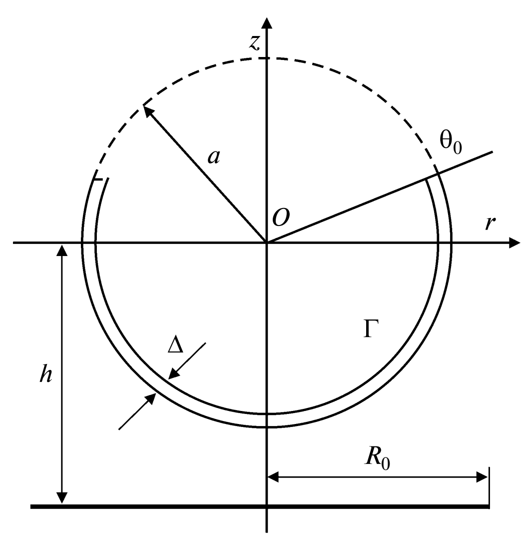

Fig. 1. Axial section of the screen.

To take into account edge effects at the edge of screen Γ,

$$\gamma_k = \{r = a, \theta = \theta 0, 0 \le \varphi \le 2\pi\}$$

we introduce the potential of the sources distributed over the screen (see [12, p. 170]):

$$u^k = \begin{cases} u_1^k = V\sum_{n=0}^{\infty} b_n\left(\frac{r}{a}\right)^n P_n(\cos\theta), & 0 \le r < a \\ u_2^k = V\sum_{n=0}^{\infty} b_n\left(\frac{a}{r}\right)^{n+1} P_n(\cos\theta), & r > a, \end{cases}$$

where

$$b_0 = \frac{1}{\pi}(\theta_2 + \sin\theta_2),$$

$$b_n = \frac{(-1)^n}{\pi}\left(\frac{1}{n}\sin n\theta_2 + \frac{1}{n+1}\sin(n+1)\theta_2\right),$$

$$n \ge 1, \quad \theta_2 = \pi - \theta_0.$$

Potential u^k satisfies the following conditions:

$$u_1^k\big|_\Gamma = u_2^k\big|_\Gamma = V, \quad u_1^k\big|_{\Gamma_1\backslash\Gamma} = u_2^k\big|_{\Gamma_1\backslash\Gamma},$$
$$\left.\frac{\partial u_1^k}{\partial r}\right|_{\Gamma_1\backslash\Gamma} = \left.\frac{\partial u_2^k}{\partial r}\right|_{\Gamma_1\backslash\Gamma}, \quad V = \text{const.} \tag{1}$$

Let us formulate the boundary value problem of screening for the total magnetic potential $u_1^c = u_1 + u_1^k$ in sphere Γ_1 and for the total potential $u_2^c = u_2 + u_2^k$ outside sphere Γ_1 with the special boundary conditions on the surface of screen Γ [8]:

$$\Delta u_1 = 0 \quad \text{in} \quad D_1 = \{0 \le r < a\},$$
$$\Delta \bar{u}_2 = 0 \quad \text{in} \quad D_2 = \{r > a\}, \tag{2}$$

$$u_1^c\big|_{\Gamma_1\backslash\Gamma} = u_2^c\big|_{\Gamma_1\backslash\Gamma}, \quad \left.\frac{\partial u_1^c}{\partial \mathbf{n}}\right|_{\Gamma_1\backslash\Gamma} = \left.\frac{\partial u_2^c}{\partial \mathbf{n}}\right|_{\Gamma_1\backslash\Gamma}, \quad 0 \le \theta < \theta_0,$$

$$\left.\frac{\partial(u_2^c - u_1^c)}{\partial \mathbf{n}}\right|_\Gamma = -apF(u_2^c + u_1^c)\big|_\Gamma, \tag{3}$$

$$\left.\frac{\partial(u_2^c + u_1^c)}{\partial \mathbf{n}}\right|_\Gamma = apF(u_2^c - u_1^c)\big|_\Gamma, \quad \theta_0 < \theta \le \pi,$$

where

$$F(u) = (\mathbf{n}, \text{curl}[\mathbf{n}, \text{grad}u]) = \Delta u - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right),$$

$$p = \frac{\mu\delta}{2\mu_0 a}, \quad q = \frac{2}{\omega^2\varepsilon'\mu_0\delta a}, \quad \delta = \frac{2}{k_\Gamma}\tan\frac{k_\Gamma\Delta}{2},$$

$$\varepsilon' = \varepsilon + i\frac{\gamma}{\omega}, \quad k_\Gamma = \omega\sqrt{\varepsilon'\mu}, \quad 0 \le \arg k_\Gamma < \pi,$$

$\mathbf{n}$ is the outward unit normal to surface Γ,

$$r(\bar{u}_2(r,\theta) + u_2^k(r,\theta)) \longrightarrow 0 \quad \text{at} \quad r \longrightarrow \infty, \tag{4}$$

r being the radial coordinate of an arbitrary point M in space R^3.

The actual magnetic potentials and magnetic fields are defined by the formulas

$$U_j = \text{Re}(u_j^c e^{-i\omega t}), \quad \mathbf{H}_j = -\text{grad}\, U_j,$$

where i is the imaginary unit and $j = 1, 2$.

The first and second boundary conditions in (3) are the continuity conditions imposed on the field and potential in the aperture of spherical shell Γ, while the third and fourth conditions in (3) simulate the magnetic field penetration through thin-wall spherical screen Γ of thickness Δ.

Since operator $F(u) = (\mathbf{n}, \text{curl}[\mathbf{n}, \text{grad}u])$ can be expressed in terms of the tangential derivatives along surface Γ, properties (1) lead to the condition

$$F(u_1^k)\big|_\Gamma = F(u_2^k)\big|_\Gamma = 0.$$

Taking into account the continuity of potential u^k and its derivatives on set $\Gamma_1\backslash\Gamma$, we can write boundary conditions (3) in the form

$$u_1\big|_{\Gamma_1\backslash\Gamma} = u_2\big|_{\Gamma_1\backslash\Gamma}, \quad \left.\frac{\partial u_1}{\partial \mathbf{n}}\right|_{\Gamma_1\backslash\Gamma} = \left.\frac{\partial u_2}{\partial \mathbf{n}}\right|_{\Gamma_1\backslash\Gamma}, \tag{5}$$

$$\left.\frac{\partial(u_2 + u_2^k - u_1 - u_1^k)}{\partial \mathbf{n}}\right|_\Gamma = -apF(u_2 + u_1)\big|_\Gamma,$$

$$\left.\frac{\partial(u_2 + u_2^k + u_1 + u_1^k)}{\partial \mathbf{n}}\right|_\Gamma = apF(u_2 - u_1)\big|_\Gamma.$$

For the primary magnetic field, we can take the field of a circular loop l ($\rho = R_0$, $z = -h$, $0 \le \varphi < 2\pi$) with current I:

$$\mathbf{H}_0(\text{M}) = \frac{I}{4\pi}\int_l \frac{[\mathbf{l}_\text{P}, R_\text{PM}]}{R_\text{PM}^3}dl_\text{P} = -\text{grad}u_0(\text{M}),$$

where $\mathbf{l}_\text{P} = \mathbf{e}_\varphi$ is the unit vector tangential to loop l at point $\text{P} \in l$ and R_PM is the distance between points P and M.

The potential of this field in the vicinity of the spherical shell is given by [9]

$$u_0 = \sum_{n=0}^{\infty} a_n\left(\frac{r}{a}\right)^n P_n(\cos\theta), \quad 0 \le r < r_0, \tag{6}$$

where

$$a_n = \frac{(-1)^{n+1}IR_0}{2nr_0}\left(\frac{a}{r_0}\right)^n P_n^1(\cos\theta_1), \quad a_0 = \frac{I}{2}(1 - h/r_0),$$

$$r_0 = \sqrt{h^2 + R_0^2}, \quad \cos\theta_1 = h/r_0, \quad h > a,$$

$P_n(x)$ are Legendre polynomials and $P_n^k(x)$ are the first-order associated Legendre functions [10, 11].

FULFILLMENT OF BOUNDARY CONDITIONS

We will seek a solution to the boundary value problem stated by (2), (4), and (5) in the form

$$u_1 \in C^2(D_1), \quad \bar{u}_2 \in C^2(D_2).$$

Considering the axisymmetric problem, we write the solution to the problem in the form of series of the solutions to the Laplace equations in the spherical system of coordinates so that condition (4) is satisfied at infinity:

$$u_1 = \sum_{n=0}^{\infty} x_n \left(\frac{r}{a}\right)^n P_n(\cos\theta), \quad r < a,$$

$$\bar{u}_2 = \sum_{n=0}^{\infty} y_n \left(\frac{a}{r}\right)^{n+1} P_n(\cos\theta), \quad r > a,$$

where x_n, y_n are unknown coefficients to be determined from the boundary conditions.

Substituting the expressions for the magnetic potentials into conditions (5), we obtain the following systems of paired summatory equations in the Legendre polynomials:

$$\begin{cases} \sum_{n=0}^{\infty}(x_n - y_n)P_n(\cos\theta) = \sum_{n=0}^{\infty} a_n P_n(\cos\theta), \quad 0 \le \theta < \theta_0; \\ \sum_{n=0}^{\infty}\{n[1+(n+1)p]x_n + (n+1)(1+np)y_n\}P_n(\cos\theta) \\ = \sum_{n=0}^{\infty}[n(1-(n+1)p)a_n - (2n+1)Vb_n]P_n(\cos\theta), \\ \theta_0 < \theta \le \pi; \end{cases} \tag{7}$$

$$\begin{cases} \sum_{n=0}^{\infty}[nx_n + (n+1)y_n]P_n(\cos\theta) = \sum_{n=0}^{\infty} na_n P_n(\cos\theta), \\ 0 \le \theta < \theta_0; \\ \sum_{n=0}^{\infty}\{n[1-(n+1)q]x_n + (n+1)(nq-1)y_n\}P_n(\cos\theta) \\ = \sum_{n=0}^{\infty}\{n[-1-(n+1)q]a_n + Vb_n\}P_n(\cos\theta), \\ \theta_0 < \theta \le \pi. \end{cases} \tag{8}$$

To solve paired equations (7), (8), we introduce new unknown coefficients $T_n^{(1)}$, $T_n^{(2)}$, which are connected with coefficients x_n, y_n by the relations

$$\begin{gathered} T_0^{(1)} = y_0 + Vb_0, \quad T_0^{(2)} = -x_0, \\ T_n^{(1)} = \{n[1+(n+1)p]x_n + (n+1)(1+np)y_n \\ -n[1-(n+1)p]a_n + (2n+1)Vb_n\}/2n+1, \quad n \ge 1, \\ T_n^{(2)} = \{n[1-(n+1)q]x_n + (n+1)(nq-1)y_n \\ +n[1+(n+1)q]a_n - Vb_n\}/2n+1, \quad n \ge 1. \end{gathered} \tag{9}$$

Substituting representations (9) into paired equations (7), (8), we obtain

$$\begin{cases} \sum_{n=0}^{\infty} G_n^{(1)} T_n^{(1)} P_n(\cos\theta) = \sum_{n=0}^{\infty}(A_n^{(1)} T_n^{(2)} + B_n^{(1)} + VM_n^{(1)}) \\ \times P_n(\cos\theta), \quad 0 \le \theta < \theta_0, \\ \sum_{n=0}^{\infty}(2n+1)T_n^{(1)} P_n(\cos\theta) = 0, \quad \theta_0 < \theta \le \pi, \end{cases} \tag{10}$$

where

$$G_0^{(1)} = -1, \quad A_0^{(1)} = 1, \quad B_0^{(1)} = a_0, \quad M_0^{(1)} = -b_0,$$

$$G_n^{(1)} = -\frac{2n+1}{\Delta_n}, \quad A_n^{(1)} = -[2n+1+2n(n+1)p]G_n^{(1)},$$

$$B_n^{(1)} = \left(\frac{-n(n+1)(2n+1)(p+q) - n^2[2+2(n+1)^2pq]}{\Delta_n} + 1\right)a_n,$$

$$M_n^{(1)} = \frac{2n(n+1)}{\Delta_n}pb_n,$$

$$\Delta_n = n(n+1)[(2n+1)(q-p) + 2n(n+1)pq - 2],$$

$$n \ge 1,$$

$$\begin{cases} \sum_{n=1}^{\infty} G_n^{(2)} T_n^{(2)} P_n(\cos\theta) = \sum_{n=0}^{\infty}(A_n^{(2)} T_n^{(1)} + B_n^{(2)} + VM_n^{(2)}) \\ \times P_n(\cos\theta), \quad 0 \le \theta < \theta_0, \\ -T_0^{(1)} - T_0^{(2)} + \sum_{n=0}^{\infty}(2n+1)T_n^{(2)} P_n(\cos\theta) = 0, \\ \theta_0 < \theta \le \pi. \end{cases} \tag{11}$$

Here,

$$G_0^{(2)} = 0, \quad A_0^{(2)} = -1, \quad B_0^{(2)} = 0,$$

$$G_n^{(2)} = \frac{(2n+1)n(n+1)p}{\Delta_n},$$

$$A_n^{(2)} = -n(n+1)[2-(2n+1)q]G_n^{(1)},$$

$$M_0^{(2)} = b_0,$$

$$M_n^{(2)} = \frac{n(n+1)}{\Delta_n}[(2n+1)^2q - p - 2(2n+1)]b_n,$$

$$B_n^{(2)} = \left\{\frac{-n^2(n+1)[(2n+1)(p+q) - 2 - 2(n+1)^2pq]}{\Delta_n} + n\right\}a_n,$$

$$n \ge 1.$$

Coefficients $G_n^{(j)}$ and $A_n^{(j)}$ ($j = 1, 2$) can be written in the form

$$G_n^{(j)} = \alpha^{(j)} + \beta^{(j)}\frac{1}{2n+1} + \gamma_n^{(j)},$$

$$A_n^{(j)} = k^{(j)} + l^{(j)}\frac{1}{2n+1} + m_n^{(j)},$$

$$\alpha^{(1)} = \beta^{(1)} = \alpha^{(2)} = 0, \quad \beta^{(2)} = \frac{2}{q}, \quad \gamma_0^{(1)} = -1,$$

$$\gamma_0^{(2)} = -\frac{2}{q}, \quad k^{(1)} = 0, \quad k^{(2)} = -\frac{2}{p}, \tag{12}$$

$$l^{(1)} = \frac{4}{q}, \quad l^{(2)} = \frac{4}{p^2}, \quad m_0^{(1)} = 1 - \frac{4}{q},$$

$$m_0^{(2)} = -1 + \frac{2}{p} - \frac{4}{p^2},$$

$$\gamma_n^{(j)} = 0(n^{-2}), \quad m_n^{(j)} = 0(n^{-2}) \text{ at } n \longrightarrow \infty.$$

TRANSFORMATION OF PAIRED EQUATIONS

Let us transform the systems of paired summatory equations (10), (11) to a system of second-kind Fredholm integral equations. For this purpose, we introduce new functions $\varphi_1(t)$ and $\varphi_2(t)$ connected with coefficients $T_n^{(1)}$ and $T_n^{(2)}$ by the following relations:

$$T_n^{(1)} = \int_0^{\theta_0} \varphi_1(t)\cos(n+0.5)t\,dt, \quad n = 0, 1, 2, \ldots,$$

$$T_n^{(2)} = \int_0^{\theta_0} \varphi_2(t)\cos(n+0.5)t\,dt, \quad n = 1, 2, \ldots, \tag{13}$$

$$T_0^{(2)} = C + \int_0^{\theta_0} \varphi_2(t)\cos\frac{t}{2}dt,$$

where C is a constant.

Formulas (13) lead to the boundedness condition

$$\left|T_n^{(j)}\right| \le \int_0^{\theta_0} |\varphi_j(t)|\,dt < C_1 = \text{const}, \quad j = 1, 2.$$

Solving system (9) for x_n, y_n and estimating coefficients a_n and b_n, we obtain the inequalities

$$|x_n| < C_2/n, \quad |y_n| < C_2/n, \quad n > 1, \quad C_2 = \text{const},$$

which give $u_1 \in C^2(D_1)$, $\bar{u}_2 \in C^2(D_2)$.

We integrate the right-hand side of the expression for $T_n^{(1)}$ by parts,

$$T_n^{(1)} = \frac{2}{2n+1}\left[\varphi_1(\theta_0)\sin(n+0.5)\theta_0 - \int_0^{\theta_0} \varphi_1'(t)\sin(n+0.5)t\,dt\right]$$

and substitute the resultant representation into the second equation in (10):

$$\begin{gathered} 2\varphi_1(\theta_0)\sum_{n=0}^{\infty}\sin(n+0.5)\theta_0 P_n(\cos\theta) \\ -2\int_0^{\theta_0}\varphi_1'(t)\left[\sum_{n=0}^{\infty}\sin(n+0.5)tP_n(\cos\theta)\right]dt = 0. \end{gathered} \tag{14}$$

Since $t \le \theta_0 < \theta$ in Eq. (14), in accordance with the expansion [12, 13]

$$\sum_{n=0}^{\infty}\sin(n+0.5)tP_n(\cos\theta) = \begin{cases} 0, & 0 \le t < \theta < \pi, \\ (2(\cos\theta - \cos t))^{-1/2}, & 0 < \theta < t \le \pi, \end{cases}$$

the sums of the series are zero. Thus, the second equation in (10) holds identically.

Having performed analogous transformations for coefficients $T_n^{(2)}$ and substituting them into the second equation of system (11), we obtain the condition

$$\int_0^{\theta_0} (\varphi_1(t) + \varphi_2(t))\cos\frac{t}{2}dt = 0. \tag{15}$$

We substitute expressions (13) for $T_n^{(1)}$ and $T_n^{(2)}$ into the first equations in (10), (11) taking into account representations (12) for coefficients $G_n^{(j)}$ and $A_n^{(j)}$ ($j = 1, 2$) of the form

$$G_n^{(j)} = \alpha^{(j)} - \tilde{G}_n^{(j)}, \quad A_n^{(j)} = k^{(j)} - \tilde{A}_n^{(j)} \tag{16}$$

and the Meler–Dirichlet integral representation for Legendre polynomials $P_n(\cos\theta)$ [12, 13]:

$$P_n(\cos\theta) = \frac{2}{\pi}\int_0^{\theta}\frac{\cos((n+0.5)x)dx}{\sqrt{2(\cos x - \cos\theta)}}.$$

As a result, these equations assume the form

$$\int_0^{\theta}\left\{\alpha^{(1)}\varphi_1(x) - k^{(1)}\varphi_{(2)}(x) - \int_0^{\theta_0}\varphi_1(t)K_1(x,t)dt + \int_0^{\theta_0}\varphi_2(t)K_2(x,t)dt\right\}\frac{dx}{\sqrt{2(\cos x - \cos\theta)}}$$

$$= C + \sum_{n=0}^{\infty} (B_n^{(1)} + VM_n^{(1)}) P_n(\cos\theta), \quad 0 \le \theta \le \theta_0,$$

$$\int_0^{\theta} \left\{ \alpha^{(2)} \varphi_2(x) - k^{(2)} \varphi_{(1)}(x) - \int_0^{\theta_0} \varphi_2(t) K_3(x, t) dt + \int_0^{\theta_0} \varphi_1(t) K_4(x, t) dt \right\} \frac{dx}{\sqrt{2(\cos x - \cos\theta)}} \tag{17}$$

$$= \sum_{n=0}^{\infty} (B_n^{(2)} + VM_n^{(2)}) P_n(\cos\theta), \quad 0 \le \theta \le \theta_0,$$

where

$$K_j(x, t) = \frac{2}{\pi} \sum_{n=0}^{\infty} L_n^{(j)} \cos(n + 0.5)t \cos(n + 0.5)x,$$

$$L_n^{(1)} = \tilde{G}_n^{(1)}, \quad L_n^{(2)} = \tilde{A}_n^{(1)}, \quad L_n^{(3)} = \tilde{G}_n^{(2)}, \tag{18}$$

$$L_n^{(4)} = \tilde{A}_n^{(2)}.$$

It is well known that function $\Phi(x)$ satisfying the Abel integral equation

$$\int_0^{\theta} \frac{\Phi(x) dx}{\sqrt{2(\cos x - \cos\theta)}} = f(\theta), \quad 0 \le \theta < \theta_0,$$

is defined by the formula [12]

$$\Phi(x) = \frac{2}{\pi} \frac{d}{dx} \int_0^{x} \frac{f(\theta) \sin\theta d\theta}{\sqrt{2(\cos\theta - \cos x)}}, \quad 0 \le x \le \theta_0. \tag{19}$$

Considering relations (17) and the Abel integral equation, we obtain, in accordance with formula (19), the system of integral equations

$$\int_0^{\theta_0} \varphi_1(t) K_{11}(x, t) dt + \int_0^{\theta_0} \varphi_2(t) K_{12}(x, t) dt$$

$$= f_1(x) + Vg_1(x) + Cf_0(x), \quad 0 \le x \le \theta_0,$$

$$\varphi_1(x) + \int_0^{\theta_0} \varphi_1(t) K_{21}(x, t) dt + \int_0^{\theta_0} \varphi_2(t) K_{22}(x, t) dt \tag{20}$$

$$= f_2(x) + Vg_2(x), \quad 0 \le x \le \theta_0,$$

where

$$K_{11}(x, t) = -K_1(x, t), \quad K_{12}(x, t) = K_2(x, t),$$

$$K_{21}(x, t) = -\frac{1}{k^{(2)}} K_4(x, t), \quad K_{22}(x, t) = \frac{1}{k^{(2)}} K_3(x, t);$$

$$f_0(x) = \frac{2}{\pi} \cos\frac{x}{2}, \quad f_1(x) = \frac{2}{\pi} \sum_{n=0}^{\infty} B_n^{(1)} \cos(n + 0.5)x,$$

$$f_2(x) = \frac{p}{\pi} \sum_{n=0}^{\infty} B_n^{(2)} \cos(n + 0.5)x;$$

$$g_1(x) = \frac{2}{\pi} \sum_{n=0}^{\infty} M_n^{(1)} \cos(n + 0.5)x,$$

$$g_2(x) = \frac{p}{\pi} \sum_{n=0}^{\infty} M_n^{(2)} \cos(n + 0.5)x.$$

In this transformation, we have taken into account the fact that the following relation holds [11, 12]:

$$\frac{2}{\pi} \frac{d}{dx} \int_0^{x} \frac{P_n(\theta) \sin\theta d\theta}{\sqrt{2(\cos\theta - \cos x)}} = \cos(n + 0.5)x.$$

Formulas (12) and (16) lead to the following representation for coefficients $\tilde{G}_n^{(j)}$ and $\tilde{A}_n^{(j)}$, $n = 1, 2$:

$$\tilde{G}_0^{(1)} = 1, \quad \tilde{G}_0^{(2)} = 0, \quad \tilde{A}_0^{(1)} = -1,$$

$$\tilde{A}_0^{(2)} = 1 - \frac{2}{p},$$

$$\tilde{G}_n^{(1)} = -\gamma_n^{(1)}, \quad \tilde{G}_n^{(2)} = -\frac{2}{q(2n+1)} - \gamma_n^{(2)},$$

$$\tilde{A}_n^{(1)} = -\frac{4}{q(2n+1)} - m_n^{(1)}, \quad \tilde{A}_n^{(2)} = -\frac{4}{p^2(2n+1)} - m_n^{(2)},$$

$$n = 1, 2, \dots.$$

Using formulas (18) for representation $K_i(x, t)$, we obtain

$$K_{11}(x, t) = \frac{2}{\pi} \sum_{n=0}^{\infty} \gamma_n^{(1)} C_n(x, t),$$

$$K_{12}(x, t) = -\frac{2}{\pi q} K(x, t) - \frac{2}{\pi} \sum_{n=0}^{\infty} m_n^{(1)} C_n(x, t),$$

$$K_{21}(x, t) = -\frac{1}{\pi p} K(x, t) - \frac{p}{\pi} \sum_{n=0}^{\infty} m_n^{(2)} C_n(x, t),$$

$$K_{22}(x, t) = \frac{p}{2\pi q} K(x, t) + \frac{p}{\pi} \sum_{n=0}^{\infty} \gamma_n^{(2)} C_n(x, t),$$

where

$$C_n(x, t) = \cos(n + 1/2)t \cos(n + 1/2)x,$$

$$K(x, t) = \ln\cot\left(\frac{x+t}{4}\right) + \ln\cot\left(\frac{|x-t|}{4}\right)$$

$$= 4 \sum_{n=0}^{\infty} \frac{C_n(x, t)}{2n+1}.$$

Solution φ_1, φ_2 to system (20) can be written in operator form

$$L\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix} = \begin{pmatrix}f_1 + Vg_1 + Cf_0\\ f_2 + Vg_2\end{pmatrix}. \quad (21)$$

Let us consider the following systems of equations:

$$L\begin{pmatrix}\varphi_1^0\\ \varphi_2^0\end{pmatrix} = \begin{pmatrix}f_1\\ f_2\end{pmatrix},\quad L\begin{pmatrix}\varphi_1^*\\ \varphi_2^*\end{pmatrix} = \begin{pmatrix}f_0\\ 0\end{pmatrix},$$
$$L\begin{pmatrix}\bar{\varphi}_1\\ \bar{\varphi}_2\end{pmatrix} = \begin{pmatrix}g_1\\ g_2\end{pmatrix}. \quad (22)$$

As a result, we find that $\varphi_1 = \varphi_1^0 + C\varphi_1^* + V\bar{\varphi}_1$ and $\varphi_2 = \varphi_2^0 + C\varphi_2^* + V\bar{\varphi}_2$ are the solution to system (21).

It should be noted that condition (4) for potential $\bar{u}_2 + u_2^k$ holds if $y_0 + Vb_0 = 0$ or, in accordance with relations (9),

$$T_0^{(1)} = \int_0^{\theta_0}\varphi_1(t)\cos\frac{t}{2}dt = 0. \quad (23)$$

To satisfy condition (15), we require that the following condition holds:

$$\int_0^{\theta_0}\varphi_2(t)\cos\frac{t}{2}dt = 0. \quad (24)$$

Assuming that solutions to system of equations (22) exist, we obtain from relations (23) and (24) the following system of algebraic equations for determining constants C and V:

$$\begin{cases} C\int_0^{\theta_0}\varphi_1^*(t)\cos\frac{t}{2}dt + V\int_0^{\theta_0}\bar{\varphi}_1(t)\cos\frac{t}{2}dt = -\int_0^{\theta_0}\varphi_1^0(t)\cos\frac{t}{2}dt,\\ C\int_0^{\theta_0}\varphi_2^*(t)\cos\frac{t}{2}dt + V\int_0^{\theta_0}\bar{\varphi}_2(t)\cos\frac{t}{2}dt = -\int_0^{\theta_0}\varphi_2^0(t)\cos\frac{t}{2}dt.\end{cases} \quad (25)$$

Having determined C and V from Eqs. (25), we calculate the total magnetic potential in shell Γ by the formula

$$u_1^c = \sum_{n=0}^{\infty}X_n\left(\frac{r}{a}\right)^n P_n(\cos\theta),\quad X_n = x_n + Vb_n. \quad (26)$$

Taking into account relations (13), we find from Eqs. (9) that coefficients x_n are connected with the solution to system (21) by the formula

$$x_n = \frac{(n+1)(2n+1)}{\Delta_n}\left[(nq-1)\int_0^{\theta_0}\varphi_1(t)\cos(n+0.5)tdt - (1+np)\int_0^{\theta_0}\varphi_2(t)\cos(n+0.5)tdt\right] + \frac{n(n+1)(2n+1)}{\Delta_n}$$
$$\times(p+q)a_n - \frac{(n+1)}{\Delta_n}[2n^2q + n(p+q) - 2n]Vb_n.$$

COMPUTATIONAL EXPERIMENT

The variation of the magnetic field strength at an arbitrary point M_0 of domain D_1 over period $T = 2\pi/\omega$ is described by the formula

$$\mathbf{H}_1(M_0, \bar{t}) = -\frac{1}{a}\sum_{n=1}^{\infty}\mathrm{Re}(X_n\exp(-2\pi i\bar{t}))\left(\frac{r}{a}\right)^{n-1}$$
$$\times(nP_n(\cos\theta)\mathbf{e}_r + P_n^1(\cos\theta)\mathbf{e}_\theta),$$

where $0 \le \bar{t} \le 1$, $\bar{t} = t/T$ being the dimensionless time.

If point M_0 lies on the z axis, we have $|z| < a$, $\theta = 0$ ($\cos\theta = 1$, $P_n(1) = 0$) or $\theta = \pi$ ($\cos\theta = -1$, $P_n(-1) = (-1)^n$), we have

$$\mathbf{H}_1(M_0, \bar{t}) = \begin{cases}\mathbf{H}_1^{(+)}(M_0, \bar{t}), & \text{if } 0 \le z < a,\quad \theta = 0\\ \mathbf{H}_1^{(-)}(M_0, \bar{t}), & \text{if } -a < z \le 0,\quad \theta = \pi,\end{cases}$$

where

$$\mathbf{H}_1^{(+)}(M_0, \bar{t}) = -\frac{1}{a}\sum_{n=1}^{\infty}n\mathrm{Re}(X_n\exp(-2\pi i\bar{t}))\left(\frac{r}{a}\right)^{n-1}\mathbf{e}_r,$$

$$\mathbf{H}_1^{(-)}(M_0, \bar{t}) = -\frac{1}{a}\sum_{n=1}^{\infty}(-1)^n n\mathrm{Re}(X_n\exp(-2\pi i\bar{t}))$$
$$\times\left(\frac{r}{a}\right)^{n-1}\mathbf{e}_r.$$

Coefficients of screening (attenuation) of the field at point M_0 lying on the z axis in domain D_1 can be calculated by the formula

$$K^{(\pm)}(M_0, \bar{t}) = \frac{|\mathbf{H}_1^{(\pm)}(M_0, \bar{t})|}{|\mathbf{H}_0(M_0, \bar{t})|}, \quad (27)$$

where

$$\mathbf{H}_0(M_0, \bar{t}) = -\frac{1}{a}\sum_{n=1}^{\infty}a_n n\cos(2\pi\bar{t})\left(\frac{r}{a}\right)^{n-1}P_n(\cos\theta)\mathbf{e}_r,$$
$$0 \le r < a.$$

We solved integral equations (22) numerically using the collocation method. We divide segment $[0, \theta_0]$ into N partial segments $[\theta_0^0, \theta_0^1]$, $[\theta_0^1, \theta_0^2]$, …, $[\theta_0^{N-1}, \theta_0^N]$ of length $h = \theta_0/N$, $\theta_0^i = ih$, $i = 0, 1, \ldots, N$. We seek the

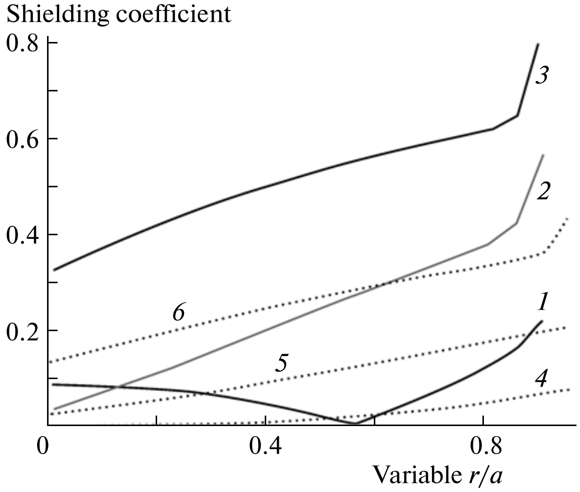

Fig. 2. Dependence of screening coefficients $K^{(+)}(M_0, 0)$ on r/a for an open semitransparent spherical screen (curves *1–3*) for opening angles $\theta_0 = \pi/4$ (*1*, *4*), $\pi/2$ (*2*, *5*), and $2\pi/3$ (*3*, *6*). Curves *4–6* correspond to a perfectly conducting shell.

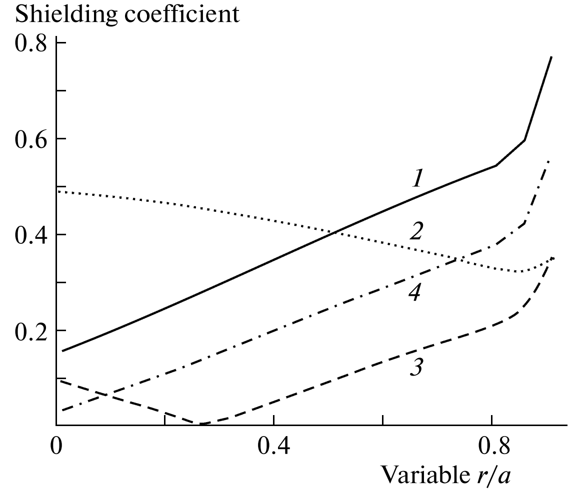

Fig. 3. Dependence of screening coefficients $K^{(+)}(M_0, t)$ on r/a for opening angle $\theta_0 = \pi/2$. The values of $\bar{t} = t/T$ are 0.1 (*1*), 0.3 (*2*), 0.4 (*3*), and 0.5 (*4*).

approximate solution, say, of the first system in (22) in the form of the linear combination

$$\varphi_1^0(t) = \sum_{n=1}^{N} C_n \psi_n(t), \quad \varphi_2^0(t) = \sum_{n=1}^{N} D_n \psi_n(t),$$

where $\psi_n(t)$ are the basis functions.

For the basis functions, we choose the system of Haar functions [14], while the points of collocation are chosen as points $x_m = (\theta_0^{m-1} + \theta_0^m)/2$ corresponding to the middles of the partial segments. This gives the system of linear algebraic equations for coefficients C_n, D_n:

$$\begin{cases} \sum_{n=1}^{N} [C_n A_{nm}^{11} + D_n A_{nm}^{12}] = f_m^1 \\ \sum_{n=1}^{N} [C_n A_{nm}^{12} + D_n A_{nm}^{22}] = f_m^2, \quad m = 1, \dots, N, \end{cases} \tag{28}$$

where $f_m^i = f_i(x_m)$,

$$A_{nm}^{ij} = \begin{cases} \delta_{nm} + \int_{\theta_0^{n-1}}^{\theta_0^n} K_{ij}(x_m, t)dt, & i = 1, \quad j = 2 \\ \int_{\theta_0^{n-1}}^{\theta_0^n} K_{ij}(x_m, t)dt, & \text{for remaining indices}. \end{cases}$$

In addition, to obtain a reliable solution to the system of algebraic equations (28), we must verify the conditionality of the system. The matrix corresponding to the system is assumed to be well-conditioned if the conditionality number of the matrix is greater than or equal to unity [15].

We performed a computational experiment in which the conditionality number of the system of linear algebraic equations in L_1, L_2 [16] did not exceed 80 for the parameters of the problem considered here. In our calculations, the infinite sums appearing in the representation of integral equations (22) was calculated with an error of 10^{-5} with a step $h = 0.05$.

In these calculations, we obtained the values of screening coefficient $K^{(+)}(M_0, 0)$ for some opening angles θ_0 for open semitransparent spherical shell Γ and for the following parameters:

$$a = 1 \text{ m}; \quad R_0 = 0.5 \text{ m}; \quad h = 1.3 \text{ m}; \quad \Delta = 0.01 \text{ m};$$

$$\omega = 2000\pi \text{ Hz}; \quad \varepsilon = \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m};$$

$$\gamma = 10^5 \text{ Sm/m};$$

$$\mu = 100\mu_0; \quad \mu_0 = 4\pi \times 10^{-7} \text{ Hn/m}.$$

The solid curves in Fig. 2 show coefficients $K^{(+)}(M_0, 0)$, $0 < r/a < 1$, for opening angles $\theta_0 = \pi/4$ (*1*), $\pi/2$ (*2*), and $2\pi/3$ (*3*). Dotted curves correspond to $K^{(+)}(M_0, 0)$ for perfectly conducting shell Γ for the same values of the opening angle: $\theta_0 = \pi/4$ (*4*), $\pi/2$ (*5*), and $2\pi/3$ (*6*).

Figure 3 shows coefficients $K^{(+)}(M_0, \bar{t})$, $0 < r/a < 1$, for opening angle $\theta_0 = \pi/2$ and for various values of $\bar{t}$.

CONCLUSIONS

We have proposed the method for solving the problem of screening of a low-frequency magnetic field by an open semitransparent spherical screen. It follows from the computational experiment that the semi-transparent open spherical shell has poorer screening properties as compared to a perfectly conducting shell.

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