REGULAR AND SINGULAR PERTURBATION OF MATRIX-FUNCTIONS WITH UNSTABLE PARTIAL INDICES

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We deal with factorization of matrix-functions, i.e. representation

$$G(x) = G^-(x)\Lambda(x)G^+(x), \tag{1}$$

with continuous invertible factors $G^{\pm}(x), (G^{\pm})^{-1}(x)$, possessing analytic continuation into the corresponding half-plane $\Pi^{\pm} = \{z = x + iy : \mathrm{Im} \pm z < 0\}$, and

$$\Lambda(x) = \mathrm{diag}\left(\left(\frac{x-i}{x+i}\right)^{\varkappa_1}, \ldots, \left(\frac{x-i}{x+i}\right)^{\varkappa_n}\right), \ \varkappa_1, \ldots, \varkappa_n \in \mathbb{Z}. \tag{2}$$

It is said that a non-singular matrix function $G(x)$ has a *stable set of partial indices* if there exists $\delta > 0$ such that any matrix function $F(x)$ from the δ-neighbourhood of $G(x)$ (i.e. $\|F - G\| < \delta$) has the same set of partial indices (right or left). If not, then $G(x)$ has an *unstable set of partial indices.* It has known that a set of partial indices $\varkappa_1, \ldots, \varkappa_n$ is stable if and only if $\varkappa_1 - \varkappa_n \leq 1$.

Any matrix $G_\varepsilon \in \mathcal{GH}_\mu(\mathbb{R})^{n\times n}$ that satisfies the following asymptotic relation

$$\|G_\varepsilon(x) - G(x)\| = O(\varepsilon), \ \ \varepsilon \to 0,$$

will be called a *perturbation* of the matrix G. If the matrix $G \in \mathcal{GH}_\mu(\mathbb{R})^{n\times n}$ is factorizable matrix. Its perturbation G_ε is considered *"regular"*, if there exists $\varepsilon_0 > 0$ such that the matrix G_ε possesses a bounded factorization (i.e. $|G_\varepsilon^{\pm}(z)| \leq M$ for all $\varepsilon \in [0, \varepsilon_0)$ and $z \in \Pi^{\pm}$). Otherwise the perturbation is considered *"singular"*.

We discuss whether, and under which conditions, it is possible to find an $n \times n$ matrix function $G_\varepsilon^*(x), x \in \mathbb{R}$, sufficiently close to a given regular perturbation $G_\varepsilon(x)$ of the matrix function $G_0(x)$, and possessing an unstable set of partial indices. More exactly, we ask when it is possible to find $G_\varepsilon^*(x)$ while preserving the partial indices of $G_\varepsilon(x)$? To reach an answer to this question in the case of unstable partial indices, a new definition of the asymptotic factorization is given and applied. In [1], the method, as proposed in [2], [3], is generalized and employed. We find conditions under which our asymptotic procedure is effective. Its properties and details are illustrated by examples.

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References

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