ON REPRESENTATION OF REAL-VALUED FUNCTIONS AS THE UPPER ENVELOPE OF MAXIMAL CONCAVE MINORANTS

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The known classical result (see, for instance, [1]) states that a function defined on a metric space is lower (upper) semicontinuous if and only if it can be represented as the upper (lower) envelope of a family of continuous functions. On the other hand, it is also well-known [2], that each lower semicontinuous convex function defined on a normed vector space is the upper envelope of a family of continuous affine function. The latter statement shows that particular classes of semicontinuous functions can be represented as the upper or lower envelope of families of elementary (in some sense) continuous functions. The primary goal of the talk is to establish characteristic properties of (extended) real-valued functions defined on normed vector spaces that admit the representation as the upper envelope of a family of concave functions.

The main result can be formulated as follows [3].

We say that a real-valued function $f : X \to \mathbb{R}$ defined on a real normed space X is Lipschitz bounded from below on X if there exists a Lipschitz continuous on X function $g : X \to \mathbb{R}$ such that $g(x) \le f(x)$ for all $x \in X$.

Теорема. For the function f to be lower semicontinuous and Lipschitz bounded from below on X, it is necessary and sufficient that the family $\Sigma^-_{Lip}(f)$ be nonempty and f admit the following upper envelope representation

$$f(x) = \sup_{h \in \Sigma^-_{Lip}(f)} h(x) \text{ for all } x \in X. \quad (1)$$

Here $\Sigma^-_{Lip}(f)$ stands for the subfamily of all maximal concave minorants of f which are Lipschitz continuous on X.

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Литература

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