ON GLOBAL EXISTENCE OF SOLUTIONS OF SEMILINEAR PARABOLIC SYSTEMS WITH NONLINEAR NONLOCAL BOUNDARY CONDITIONS

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We consider semilinear parabolic systems with nonlinear nonlocal boundary conditions:

$$\begin{cases} u_t = \triangle u + c_1(x,t)v^p, v_t = \triangle v + c_2(x,t)u^q, & x \in \Omega, t > 0, \\ \dfrac{\partial u}{\partial \nu} = \displaystyle\int_\Omega k_1(x,y,t)u^m(y,t)dy, & x \in \partial\Omega, t > 0, \\ \dfrac{\partial v}{\partial \nu} = \displaystyle\int_\Omega k_2(x,y,t)v^n(y,t)dy, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x), v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1}$$

where p, q, m, n are positive constants, Ω is a bounded domain in $\mathbb{R}^N$ for $N \geq 1$ with smooth boundary $\partial\Omega$, ν is the unit outward normal to $\partial\Omega$, $c_1(x,t), c_2(x,t)$ are nonnegative locally Hölder continuous functions defined for $x \in \overline{\Omega}$ and $t \geq 0$, $k_1(x,y,t), k_2(x,y,t)$ are nonnegative continuous function defined for $x \in \partial\Omega$, $y \in \overline{\Omega}$ and $t \geq 0$. The initial data $u_0(x), v_0(x)$ are nonnegative continuously differentiable function in $\overline{\Omega}$ which satisfy boundary condition for $t = 0$. We prove some global existence results. Criteria on this problem which determine whether the solutions blow up in finite time for large or for all nontrivial initial data are also given. In particular, we prove the following statements.

Theorem 1. Let $\max(pq, m, n) \leq 1$. Then problem (1) has global solutions for any initial data.

Theorem 2. Let $\min(pq, m, n) > 1$,

$$\int_0^\infty \sup_\Omega c_i(x,t)dt < \infty, \quad \int_0^\infty \sup_{\partial\Omega\times\Omega} k_i(x,y,t)dt < \infty, \ i = 1,2,$$

and there exist positive constants α, t_0 and K such that $\alpha > t_0$ and

$$\int_{t-t_0}^t \frac{\sup_{\partial\Omega\times\Omega} k_i(x,y,\tau)}{\sqrt{t-\tau}}d\tau \leq K \text{ for any } t \geq \alpha, i = 1,2.$$

Then problem (1) has bounded global solutions for small initial data.

Problem (1) with Dirichlet boundary condition will be considered also. The results of talk have been published in [1-2].

References

1. *Gladkov A.L., Nikitin A.I.* On the existence of global solutions of a system of semilinear parabolic equations with nonlinear nonlocal boundary conditions. *Differ. Equat.* No 4 (2016), pp. 467—482.

2. *Gladkov A.L., Nikitin A.I.* On global existence of solutions of initial boundary value problem for a system of semilinear parabolic equations with nonlinear nonlocal Neumann boundary conditions. *Differ. Equat.* No 1 (2018), pp. 86–105.