A METHOD TO STUDY IEEE 802.11 BROADBAND WIRELESS NETWORK WITH FINITE BUFFERS

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We consider IEEE 802.11 broadband wireless network with DCF (distributed coordination function) and finite buffers. The network operation is described in terms of a Markov process which allows to calculate performance characteristics.

Keywords: broadband wireless network, IEEE 802.11, DCF, matrix-analytical approach.

1. INTRODUCTION

IEEE 802.11 wireless networks with DCF (distributed coordination function) use carrier sense multiple access with collision avoidance (CSMA/CA). The modelling of the network performance is essentially different for cases of heavy traffic (a station always has a packet to transmit) and low traffic (a station queue can be empty). The first case is investigated in details in the literature, see e.g. [1, 2]. The case of low or normal traffic is analyzed in [3, 4]. The model of a network with normal traffic proposed in [5] describes two significant features of DCF: 1) asynchronous transmission which is made without a backoff and starts immediately when a packet arrives to an idle station at the moment the channel is idle; 2) final backoff implying that a station enters the backoff stage every time it finishes transmission even if its queue is empty. In the paper, we analyze the case of finite buffers and for brevity, we suppose the packets have the same size and consider the basic access mechanism only. As it is made in [5], the results can be easily generalized to the case when packets have arbitrary size and RTS/CTS mechanism is used.

2. ANALYSIS OF IEEE 802.11 DCF WITH FINITE BUFFERS

Stochastic behavior of a station is described by a Markov chain where a station can be in one of the following states: idle, backoff, transmission (collision or successful) and final backoff.

Procedure of backoff and contention window increasing is implemented correspondingly to CSMA/CA mechanism. Note, that a station enters a backoff stage after every transmission even if there are no packets in the queue. Two types of transmission are considered: 1) *synchronous* transmission that is made after a backoff, and 2) *asynchronous* one that is made without a backoff when a packet arrives to the idle station and the channel is not busy.

Synchronous transmissions are subjected to collisions since stations count down their backoffs synchronously, and the backoff counters can drop to zero simultaneously. Whereas, a packet transmitted asynchronously rarely suffers from collision. A station can start an asynchronous transmission at any moment of time slot σ, and the probability that it starts at the end of the slot or along with the other asynchronous transmission is negligible. The network consists of n stations.

Let $x(t)$, $t \geq 0$ be the number of packets in a station queue at the time t, $x(t) \in \{0, 1, 2, \dots, K\}$; $s(t)$ be the backoff stage at the time t, $s(t) \in \{0, 1, \dots, I\}$. The value of $s(t)$ characterizes the number of unsuccessful attempts to transmit a packet. If transmission fails at the stage I, the packet is discarded. Let also $b(t)$ be the backoff counter at the time t, $b(t) \in \{0, 1, \dots, W_i - 1\}$, where $W_i = 2^{\min\{i,m\}}W$, $0 \leq i \leq I$, $W = CW_{min}$, and m is determined by $CW_{max} = 2^m CW_{min}$.

As in [1], we suppose that at each transmission attempt, and regardless of the number of retransmissions suffered, each packet collides with constant and independent probability p. Let also τ (τ_a) be the probability that a station transmits a packet synchronously (asynchronously). The packets are supposed to have the same size and arrive accordingly to a Poisson process with parameter λ.

A time slot can be one of the following types: *empty slot* of size σ when no station transmits; *successful slot* of size T_s when only one station transmits synchronously; *collision slot* of size T_c when two or more stations transmit synchronously; *asynchronous slot* of the mean size $T_a = T_s + \sigma/2$.

Let t_k, $k \geq 1$, be the k-th epoch when the backoff counter changes its value, and we consider behavior of the station at the moments t_k, $k \geq 1$. The process $\xi_k = (x(t_k), s(t_k), b(t_k))$, $k \geq 1$, is a Markov chain. Introduce into consideration the following condition probabilities (given that the tagged station does not transmit):

- P_e, the probability that an arbitrary slot is empty (the other $n-1$ stations do not transmit), $P_e = (1 - \tau - \tau_a)^{n-1}$;
- P_s, the probability that an arbitrary slot is successful (only one of $n-1$ stations transmits), $P_s = (n-1)\tau(1-\tau)^{n-2}$;
- P_s, the probability that an arbitrary slot is asynchronous (only one of $n-1$ stations transmits synchronously, and the rest $n-2$ stations do not transmit), $P_a = (n-1)\tau_a(1-\tau)^{n-2}$;
- P_s, the probability that an arbitrary slot is collision one, $P_c = 1 - P_e - P_s - P_a$;
- r_i, s_i, a_i and t_i are the probabilities that i packets arrive to the tagged station during an empty, successful, asynchronous and collision slot, correspondingly,

$$r_i = \frac{(\lambda\sigma)^i}{i!}e^{-\lambda\sigma}, \quad a_i = s_i = \frac{(\lambda T_s)^i}{i!}e^{-\lambda T_s}, \quad t_i = \frac{(\lambda T_c)^i}{i!}e^{-\lambda T_c}.$$

  Since $n\lambda\sigma << 1$, we suppose that no more than one packet arrives to the station during a slot of size σ, i.e. $r_1 \approx \lambda\sigma e^{-\lambda\sigma} \approx \lambda\sigma$ and $r_i \approx 0$ if $i > 1$. For the same reason, we have $a_i \approx s_i$.
- q_i, the probability that i packets arrive to the station during an arbitrary slot $q_i = P_e r_i + P_s s_i + P_a s_i + P_c t_i$.

One-step transition probabilities of the Markov chain ξ_t, $t \geq 0$, are the following:

$$P\{(0,0)|(0,0)\} = q_0 + s_0 r_1 P_e/W, \quad P\{(0,l)|(0,0)\} = s_0 r_1 P_e/W,$$

$$P\{(k,0,j)|(0,0)\} = [s_k(r_1 P_e + P_s + P_a) + t_k P_c]/W, \quad k = \overline{1,K-1},$$

$$P^*\{(K,0,j)|(0,0)\} = \left[(1 - \sum_{k=0}^{K-1} s_k)(r_1 P_e + P_s + P_a) + (1 - \sum_{k=0}^{K-1} t_k) P_c\right] /W,$$

$$P\{(k,0,l-1)|(0,l)\} = q_k,\ k = \overline{1,K-1},\ P^*\{(K,0,l-1)|(0,l)\} = 1 - \sum_{k=0}^{K-1} q_k,$$

$$P\{(0,l-1)|(0,l)\} = q_0, \quad P\{(0,j)|(1,i,0)\} = [(1-p)s_0 + pt_0 e\{i = I\}]/W,\ i = \overline{0,I},$$

$$P\{(k+m,i,l_i-1)|(k,i,l_i)\} = q_m,\ k = \overline{1,K-1},\ m = \overline{0,K-k-1},\ i = \overline{0,I},$$

$$P^*\{(K,i,l_i-1)|(k,i,l_i)\} = 1 - \sum_{m=0}^{K-k-1} q_m,\ k = \overline{1,K},\ i = \overline{0,I},$$

$$P\{(k+m-1,0,j)|(k,i,0)\} = (1-p)s_m/W,\ k = \overline{1,K},\ m = \overline{0,K-k},\ i = \overline{0,I-1},$$

$$P^*\{(K,0,j)|(k,i,0)\} = (1-p)\left(1 - \sum_{m=0}^{K-k} s_m\right)/W,\ k = \overline{1,K},\ i = \overline{0,I-1},$$

$$P\{(k+m-1,0,j)|(k,I,0)\} = [(1-p)s_m + pt_m]/W,\ k = \overline{1,K},\ m = \overline{0,K-k},$$

$$P^*\{(K,0,j)|(k,I,0)\} = \left[(1-p)(1 - \sum_{m=0}^{K-k} s_m) + p(1 - \sum_{m=0}^{K-k} t_m)\right]/W,\ k = \overline{1,K},$$

$$P\{(k+m,i+1,l_i)|(k,i,0)\} = \frac{pt_m}{W_{i+1}}, k = \overline{1,K-1}, m = \overline{0,K-k-1}, i = \overline{0,I-1},$$

$$P^*\{(K,i+1,l_{i+1})|(k,i,0)\} = p\left(1 - \sum_{m=0}^{K-k-1} t_m\right)/W_{i+1}, k = \overline{1,K},\ i = \overline{0,I-1},$$

$j = \overline{0,W-1}$, $l = \overline{1,W-1}$, $l_i = \overline{1,W_i-1}$, $e\{A\}$ is an indicator function, $e\{A\} = 1$ if A is true, and $e\{A\} = 0$ otherwise.

The matrix $\boldsymbol{P}$ of one-step transition probabilities of the Markov chain ξ_t, $t \geq 0$ has the block structure

$$\boldsymbol{P} = \begin{bmatrix} C_0 & A_1 & A_2 & A_3 & A_4 & \cdots & A_K \\ C_1 & B_1 & B_2 & B_3 & B_4 & \cdots & B_K^* \\ O & C_2 & B_1 & B_2 & B_3 & \cdots & B_{K-1}^* \\ O & O & C_2 & B_1 & B_2 & \cdots & B_{K-2}^* \\ O & O & O & C_2 & B_1 & \cdots & B_{K-3}^* \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ O & O & O & O & \cdots & C_2 & B_1^* \end{bmatrix} \tag{1}$$

of size $(W + \beta K) \times (W + \beta K)$, where $\beta = \sum_{i=0}^{I} W_i = (2^{m+1} - 1 + 2^m(I - m)e\{m < I\})W$.

Below we describe the blocks of matrix $\boldsymbol{P}$. The matrices C_i, $i = 0, 1, 2$, are defined as

$$C_0 = \begin{bmatrix} q_0 + \frac{s_0 r_1 P_e}{W} & \frac{s_0 r_1 P_e}{W} & \cdots & \frac{s_0 r_1 P_e}{W} & \frac{s_0 r_1 P_e}{W} \\ q_0 & 0 & \cdots & 0 & 0 \\ 0 & q_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & q_0 & 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} C_{10} \\ C_{11} \\ C_{12} \\ \vdots \\ C_{1I} \end{bmatrix},$$

where
$C_{1i} = G_{W_i,W}\left(\frac{(1-p)s_0}{W}, 0\right)$, $0 \le i \le I-1$, $C_{1I} = G_{W_I,W}\left(\frac{(1-p)s_0 + pt_0}{W}, 0\right)$, $C_2 = \begin{bmatrix} C_1 & O \end{bmatrix}$,
$A_i = \begin{bmatrix} G_{W,W}(x_i, z_i) & O \end{bmatrix}$ where $x_i = \frac{s_i\left(r_1 P_e + P_s + P_a\right) + t_i P_c}{W}$, and $z_i = q_i$ if $i < K$;
$x_K = P^*\{(K,0,j)|(0,0)\}$ and $z_K = P^*\{(K,0,j-1)|(0,j)\}$,

$$G_{a,b}(x,y) = \begin{bmatrix} x & x & \cdots & x & x \\ y & 0 & \cdots & 0 & 0 \\ 0 & y & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & y & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} D & E_1 & 0 & \cdots & 0 & 0 \\ D_1 & F_1 & E_2 & \cdots & 0 & 0 \\ D_2 & 0 & F_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ D_{I-1} & 0 & 0 & \cdots & F_{I-1} & E_I \\ D_I & 0 & 0 & \cdots & 0 & F_I \end{bmatrix},$$

blocks of the matrix B_i^* are defined similarly and marked by asterisk.

$D = G_{W,W}\left(\frac{(1-p)s_i}{W}, q_{i-1}\right), D^* = G_{W,W}(u_i, m_i)$,

$D_j = G_{W_j,W}\left(\frac{(1-p)s_i}{W}, 0\right), D_j^* = G_{W_j,W}(u_i, 0)$, $j = \overline{1, I-1}$,

$D_I = G_{W_I,W}\left(\frac{(1-p)s_i + pt_i}{W}, 0\right), D_I^* = G_{W_I,W}(g, 0)$,

$E_j = G_{W_{j-1},W_j}\left(\frac{pt_{i-1}}{W_j}, 0\right), E_j^* = G_{W_{j-1},W_j}(h_i, 0)$, $j = \overline{1, I}$,

$F_j = G_{W_j,W_j}(0, q_{i-1}), F_j^* = G_{W_j,W_j}(0, m_i)$, $j = \overline{1, I}$,

where $u_i = (1-p)(1 - \sum_{l=0}^{i-1} s_l)/W$, $m_i = 1 - \sum_{l=0}^{i-2} q_l$, $g = P^*\{(K,0,j)|(K+1-i, I, 0)\}$, $h_i = p(1 - \sum_{l=0}^{l=i-2} t_l)/W_j$.

Let $\pi_{l,i,j} = \lim_{k\to\infty} P\{(x(t_k), s(t_k), b(t_k)) = (l,i,j)\}$ and $\pi_{0,j} = \lim_{t\to\infty} P\{x(t) = 0, b(t) = j\}$ be the stationary state probabilities of the Markov chain ξ_k, $k \ge 1$. Consider the vectors $\pi_l = (\pi_{l,0,0}, \pi_{l,0,1}, \ldots, \pi_{l,I,W_I-1})$, $l = \overline{1, K}$, $\pi_0 = (\pi_{0,0}, \ldots, \pi_{0,W-1})$. The stationary state distribution π_l, $l \ge 0$, can be obtained by the matrix-analytical approach, see e.g. [5, 6].

Having the vectors π_l, $l \ge 0$, calculated, we can obtain the probabilities τ and τ_a that a station transmits synchronously and asynchronously in an arbitrary slot:

$$\tau = \sum_{k=1}^{K} \sum_{i=0}^{I} \pi_{k,i,0}, \quad \tau_a = \pi_{0,0} r_1 P_e, \quad p = 1 - (1-\tau)^{n-1}. \tag{2}$$

The set of equations (2) with unknown τ, τ_a and p can be solved numerically.

3. NUMERICAL RESULTS

In this section, we compare the analytical results with simulation ones obtained using the general-purpose simulation system GPSS World, [7]. We consider point-to-point channel with symmetric traffic under ideal channel conditions. Initial data are taken from IEEE 802.11 standard with the nominal rate 54 Mbit/sec. We compare fractions of synchronous packets and the mea [illegible] r both models with limited and unlimite [illegible] for the case when contention window i [illegible]

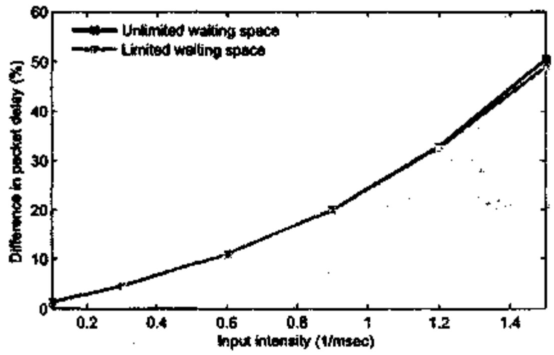

Fig. 1. [illegible] = 16

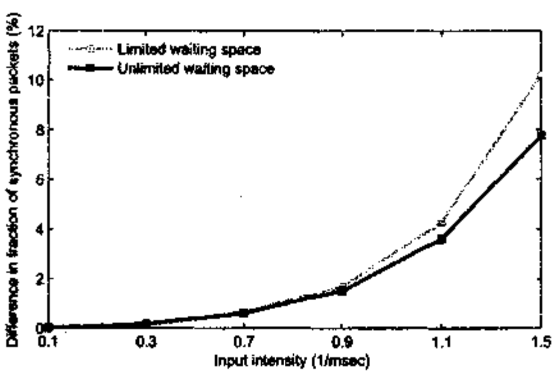

Fig. 2. Difference in packet delay for $W = 16$

As it is seen from figures, both models with limited and unlimited (see [5]) waiting spaces give close results which coincide with simulation in case of low traffic. But when the input intensity increases, the deviation of results for the model with limited waiting space stops increasing in contrast to the model with unlimited waiting space, that significantly enlarges an area of the model applicability in terms of fraction of synchronous packets transmitted. The mean packet delay has the essential comparison error even when the traffic is low, but for the model with limited waiting space it does not exceed 20%.

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