SOME RESULTS OF ANALYSIS AND OPTIMIZATION OF HM-NETWORKS AND THEIR APPLICATION

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The methods of finding of expected incomes in systems of HM-network of arbitrary topology are observed. There are examples of applying such networks as mathematical models of different objects.

Keywords: HM-queueing networks, expected incomes.

1. INTRODUCTION

As it is known, functioning of any Markov queueing network (QN) is described with the help of Markovian chains with continuous time. In Howard's work [1] the conception of Markovian chains with incomes which appear to be constants was introduced and it was proposed to use method of Laplace transformation and method of z-transformation for analysis of such chains with small number of states. This concept laid down in a basis of definition of Markov QN with incomes that were examined in works [2]-[4] at first. Usually the closed networks with large number of states are observed, during the investigation of which the problem of dimension appears. Open networks with countable number of states are also observed. First open and closed networks with central queueing system (QS) were investigated in cases when A) network states transition incomes depend on states and time or B) incomes are random variables (RV) with the set moments of the first and the second orders. Later these results were used in case of networks of arbitrary topology. As observed in cases with Markov networks with negative and positive messages that were first investegated by E. Gelenbe were named G-networks, QN with incomes are recently refered to as HM (Howard-Matalytski)-networks. Detailed bibliography of HM-networks dated 2003-2008 is available in [9, 12].

Let us examine open exponential QN with one type messages which consists of n QS $S_1, S_2, ..., S_n$. Vector $k(t) = (k, t) = (k_1, k_2, ..., k_n, t)$ is state of network where k_i is number of messages in system S_i at the moment t, $t \in [t_0, +\infty)$, $i = \overline{1, n}$. Let us introduce system S_0 (outside medium) for the unification of designation, from which the Poisson flow of messages with arrival rate λ enters into network. The service rate of messages μ_i in system S_i depends on the number of messages in it, $i = \overline{1, n}$. Let's assume that p_{0j} is probability of message entry from system S_0 to system S_j, $\sum_{j=1}^{n} p_{0j} = 1$; Let's assume that p_{ij} is probability of message transition to system S_j after its service in system S_i , $\sum_{j=0}^{n} p_{ij} = 1$, $i = \overline{1, n}$. Message brings to system S_j some income during its transition from system S_i to system S_j and income of system S_i is reduced by the amount of said income, $i, j = \overline{0, n}$. Basically, results can be also spread in case when message while leaving the QS also brings to it some income. It is necessary to find the expected (average) incomes of network systems during time t considering that network state in the starting time t_0 is known.

2. METHODS OF ANALYSIS AND OPTIMIZATION OF HM-NETWORKS IN CASE A)

Let's assume that $v_i(k, t)$ is a complete expected income that system S_i is receiving during time t, if network at the initial time is in state (k, t_0) ; $r_i(k)$ – income of system S_i in time unit when network is in state k; I_i – n-vector with zero components with the exception of component with number i that equals 1; $r_{0i}(k + I_i, t)$ – income of system S_i, when network makes transition from state (k, t) to state $(k + I_i, t + \Delta t)$ during time Δt; $-R_{i0}(k - I_i, t)$ - income of that system if network makes transition from state (k, t) to state $(k - I_i, t + \Delta t)$; $r_{ij}\left(k + I_i - I_j, t\right)$ – income of system S_i (expense or loss of system S_j), when network changes its state from (k, t) to $\left(k + I_i - I_j, t + \Delta t\right)$ during time Δt, $i, j = \overline{1, n}$; $u(x)$ is Heavyside function. For the expected income of system S_i we can get the system of difference-differential equation (DDE)[7]:

$$\frac{dv_i(k, t)}{dt} = -\left[\lambda + \sum_{j=1}^{n} \mu_j(k_j)u(k_j)\right] v_i(k, t) +$$

$$+ \sum_{j=1}^{n} \left[\lambda p_{0j} v_i(k + I_j, t) + \mu_j(k_j)u(k_j)p_{j0}v_i(k - I_j, t)\right] +$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[\mu_j(k_j)u(k_j)p_{ji}v_i(k + I_i - I_j, t) + \mu_i(k_i)u(k_i)p_{ij}v_i(k - I_i + I_j, t)\right] +$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[\mu_j(k_j)u(k_j)p_{ji}r_{ij}(k + I_i - I_j, t) - \mu_i(k_i)u(k_i)p_{ij}r_{ij}(k - I_i + I_j, t)\right] +$$

$$+\sum_{\substack{c,s=1\\ c,s\neq i,j}}^{n}\mu_s(k_s)p_{sc}v_i(k+I_c-I_s,t)+\lambda p_{0i}r_{0i}(k+I_i,t)-$$

$$-\mu_i(k_i)u(k_i)p_{i0}R_{i0}(k-I_i,t)+r_i(k). \qquad (1)$$

The equation number equals the number of network states in this system.

System of equations (1) for closed networks can be represented as system of finite number linear non-homogeneous ordinary differential equations (ODE) with constant coefficients which can be solved by applying direct approach (matrix exponent) or transfiguration method of Laplace. It is important to remember that state number of closed QN equals $L = C_{n+K-1}^{n-1}$, where K – number of messages that are serviced in network, where the state number is large enough with comparatively small n and K. Computer calculations showed that such methods are sufficient enough for finding the expected incomes for network systems with comparatively small state space ($L < 100$); the direct approach is more useful for networks with bigger dimension then transfiguration method of Laplace. Numerical methods can be applied for finding the expected incomes also.

For open networks when equation number in system (1) is infinite, expected incomes in network systems can be found by using method of multidimensional z-transformations if incomes from transitions between network states depend on their states and don't depend on time . Let $z \in \left\{(z_1, z_2, ..., z_n)/\,|z_k| < 1,\ k = \overline{1,n}\right\}$, let's introduce multidimensional z-transformation for expected income of system S_i:

$$\varphi_i(z,t)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\cdots\sum_{k_n=0}^{\infty}v_i(k_1,k_2,\dots,k_n,t)z_1^{k_1}z_2^{k_2}\cdot\ldots\cdot z_n^{k_n}=\sum_{\substack{k_i=0,\\ i=\overline{1,n}}}^{\infty}v_i(k,t)\prod_{l=1}^{n}z_l^{k_l}.$$

In [8, 14] relations for $\varphi_i(z,t)$ were obtained , $i=\overline{1,n}$. In [8] algorithm of finding the expected incomes by using these relations was suggested and it is also too complicated for computer realization.

Let us describe another general method that permits us to find expected incomes of network QS in this case. Let us examine HM-network of arbitrary topology, m_i queues are generated for service in i-th QS, $i=\overline{1,n}$. Condition of queue α of i-th QS at the moment t is characterized by vector $(\overline{k}_{i\alpha}, t) = (k_{i\alpha 11}, k_{i\alpha 21}, \dots, k_{i\alpha r_1 1}, k_{i\alpha 12}, k_{i\alpha 22}, \dots, k_{i\alpha r_2 2}, \dots, k_{i\alpha 1g}, k_{i\alpha 2g}, \dots, k_{i\alpha r_g g}, \dots, k_{i\alpha 1h}, k_{i\alpha 2h}, \dots, k_{i\alpha r_h h}, t)$, where $k_{i\alpha cg}$ is the message number of c type and g class in queue α of i-th QS, $i=\overline{1,n}$, $\alpha=\overline{1,m_i}$, $c=\overline{1,r_g}$, $g=\overline{1,h}$, condition of i-th QS is characterized by vector $(\overline{k}_i, t) = (\overline{k}_{i1}, \overline{k}_{i2}, \dots, \overline{k}_{im_i}, t)$, $i=\overline{1,n}$, and network condition – by vector $(k, t) = (\overline{k}_1, \overline{k}_2, \dots, \overline{k}_n, t)$. In general case, when functioning of network is described with Markovian process $k(t)$, $t \geq 0$, system DDE for expected income of some QS $v(k,t)$ can be set down as

$$\frac{dv(k,t)}{dt} = A(k) - \Lambda(k)v(k,t) + \sum \Phi_{i\alpha cgj\beta sq}(k)v(k + I_{i\alpha cg} - I_{j\beta sq}, t), \quad (2)$$

$\Lambda(k)$, $\Phi_{i\alpha cgj\beta sq}(k)$ are some limited nonnegative functions, $\sum = \sum_{i,j=1}^{n}\sum_{\alpha=1}^{m_i}\sum_{\beta=1}^{m_j}\sum_{c=1}^{r_g}\sum_{s=1}^{r_q}\sum_{g,q=1}^{h}$, $I_{i\alpha cg}$ is vector of dimension $n\sum_{i=1}^{n} m_i \sum_{g=1}^{h} r_g$ with zero components with the exception of the component with number $n\sum_{g=1}^{h} r_g \left(\sum_{j=1}^{i-1} m_j + \alpha - 1\right) + c$, that equals 1. We suggest that incomes from transitions between network states don't depend on time.

Let $v_m(k,t)$ is approximation of income $v(k,t)$ at m-th iteration, $v_{m+1}(k,t)$ is the solution of the system (2), that was obtained by method of successive approximations, $m = 0, 1, 2, \ldots$. Then

$$v_{m+1}(k,t) = e^{-\Lambda(k)t}\left\{v(k,0) + \int_0^t e^{\Lambda(k)x} \sum \Phi_{i\alpha cgj\beta sq}(k) v_m(k + I_{i\alpha cg} - I_{j\beta sq}, x)dx\right\} + \frac{A(k)}{\Lambda(k)}\left[1 - e^{-\Lambda(k)t}\right]. \quad (3)$$

It is evident that $v_m(k,0) = v(k,0)$, and let also

$$v_0(k,t) = v(k) = \lim_{t\to\infty} v(k,t) = \frac{1}{\Lambda(k)}\left\{A(k) + \sum \Phi_{i\alpha cgj\beta sq}(k)v(k + I_{i\alpha cg} - I_{j\beta sq})\right\}.$$

In [13] it was proved that successive approximations $v_m(k,t)$, $m = 1, 2, \ldots$, converge to stationary solution of the system (2) when $t \to \infty$ if it exists, and sequence $\{v_m(k,t)\}$, $m = 0, 1, 2, \ldots$, which was built by scheme (3), converges to single solution of equation system (2) for any limited by t zero approximation $v_0(k,t)$ when $m \to \infty$. Statement [13] is important for practical application of method under consideration. Any successive approximation $v_m(k,t)$, $m \geq 1$, can be represented as power series that converges for any limited $t > 0$, coefficients of series answer the recurrence relations.

This statement permits us to find expected incomes of network systems with a large number of states with a help of computer during acceptable processing time. Ought to notice, that it will take more time to solve the system (2) provided that it will be reduced to system of linear heterogeneous GDE and solved by direct method.

It's important to notice that work [6] is devoted to simulation modeling of incomes in HM-networks with a central QS when service times of messages in their systems are distributed by random law.

3. ANALYSIS OF EXPECTED INCOMES IN CASE B)

Let's examine the dynamics of income changing of some system S_i of HM-network. Let's denote income at the moment t as $V_i(t)$. Let the income of this system at the moment

t_0 equals v_{i0}. We will be interested in system income $V_i(t_0+t)$ at the moment t_0+t. Interval $[t_0;\ t_0+t]$ can be subset on m parts of length $\Delta t = \frac{t}{m}$, and rushing m to $+\infty$. Let's denote income changing of i-th system on l-th time interval of range Δt as $d_{il}(\Delta t)$:

$$d_{il}(\Delta t) = \begin{cases} r_{0i} + r_i\Delta t \ \text{ with probability } \lambda p_{0i}\Delta t + o(\Delta t), \\ -R_{i0} + r_i\Delta t \ \text{ with probability } \mu_i(k_i(l))u(k_i(l))p_{i0}\Delta t + o(\Delta t), \\ r_{ji} + r_i\Delta t \ \text{ with probability } \mu_j(k_j(l))u(k_j(l))p_{ji}\Delta t + o(\Delta t),\ j = \overline{1,n},\ j \neq i, \\ -R_{ij} + r_i\Delta t \ \text{ with probability } \mu_i(k_i(l))u(k_i(l))p_{ij}\Delta t + o(\Delta t),\ j = \overline{1,n},\ j \neq i, \\ r_i\Delta t \ \text{ with probability } 1 - (\lambda p_{0i} + \mu_i(k_i(l))u(k_i(l)))\,\Delta t + o(\Delta t), \end{cases}$$

where r_{0i}, R_{i0}, r_{ji}, R_{ij}, r_i are random variables with expectation values:

$$M\{r_{ji}\} = a_{ji},\quad M\{R_{ij}\} = b_{ij},\quad M\{r_{0i}\} = a_{0i},\quad M\{r_i\} = c_i,\quad i,j = \overline{1,n},$$

$$M\{R_{i0}\} = b_{i0},\quad a_{ji} = b_{ji},\quad i,j = \overline{1,n}.$$

Summing the expected incomes of different time intervals we will obtain the next approximate relations [10, 14]:

$$v_i(t) = M\{V_i(t)\} = v_{i0} + (c_i + a_{0i}\lambda p_{0i})\,t +$$

$$+\sum_{j=1}^{n}\mu_j a_{ji} p_{ji}\int_0^t \min(N_j(s), m_j)\,ds - \mu_i\int_0^t \min(N_i(s), m_i)\,ds\sum_{j=0}^{n} b_{ij}p_{ij},\quad i = \overline{1,n},$$

$$M\left\{\sum_{i=1}^{n} V_i(t)\right\} = \sum_{i=1}^{n}\left[v_{i0} + (c_i + \lambda a_{0i}p_{0i})\,t - \mu_i b_{i0}p_{i0}\int_0^t \min(N_i(s), m_i)\,ds\right], \tag{4}$$

where $N_i(s)$ is average number of messages in system S_i on time interval $[0,\ s]$. It can be obtained by using recurrence by the time moments method of analysis of average values for open QN which was developed in [11]. In case when RV r_{ji}, R_{ij}, r_{0i}, R_{i0} are independent relative to RV r_i, $i,j = \overline{1,n}$, in [10, 14] relations for variations of network system incomes are also obtained.

4. ABOUT PROBLEMS OF OPTIMIZATION, CONTROL AND APPLICATIONS OF HM-NETWORKS

For considering open exponential network two optimization problems can be formulated. They are connected with maximization of network incomes in a whole and separately (for example QS S_j):

$$\begin{cases} W(T, m_1, ..., m_n) = \frac{1}{T}\int\limits_0^T \sum\limits_{i=1}^{n} (Mv_i(t) - d_iN_i(t) - E_im_i)dt \to \max\limits_{m_1, m_2, ..., m_n}, \\ m_i \le M_i, \ i = \overline{1, n}, \end{cases} \quad (5)$$

$$\begin{cases} W_j(T, m_1, ..., m_n) = \frac{1}{T}\int\limits_0^T \left(v_j(t) - d_jN_j(t) - E_jm_j\right) dt \to \max\limits_{m_1, m_2, ..., m_n}, \\ m_i \le M_i, \ i = \overline{1, n}, \end{cases} \quad (6)$$

where M_i – some integer numbers; d_i – maintenance costs of one message in i-th QS (in queue and service); E_i – maintenance coasts of one service device, $v_i(t)$ is satisfied to relation (4), $i = \overline{1, n}$. Solution of problems (5), (6) is described in [10].

Optimum control problem for HM-network is described in [12].

Examples of HM-network applications for estimation and forecasting of expected incomes in Internet, Internet-shops, insurance companies, logistic transports systems were described in [12]. Analysis of expected incomes of interbank payments in banking networks and working expenses of soft calculating cluster with help of HM-networks can be found in [15, 16]. Let describe another examples of application.

Example (forecasting of enterprise incomes from production realization). S_n enterprise produces production of h classes that can be delivered to customers $S_1, S_2, ..., S_{n-1}$ by r_g types of transports. For each delivery of production of class g by transport of type c customer transmits sum $r(i, n, c, g)$ to enterprise bank account; such delivery brings to customer income $r(n, i, c, g)$, $i = \overline{1, n-1}$, $c = \overline{1, r_g}$, $g = \overline{1, h}$. For forecasting of enterprise incomes closed network with central QS S_n and peripheral QS S_i, $i = \overline{1, n-1}$, can be used. Message of type c and class $g$in this case is delivery of production of class g by transport of type c.

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