# APPROXIMATE METHOD FOR FINDING THE SUBOPTIMAL CAC IN MULTI-RATE QUEUES

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An approach to solving the problem of finding the suboptimal call admission control (CAC) in multi-rate queue in which calls of different types require random number of channels is proposed. The goal is maximization of channel utilization. To solve the latter problem the methods of Markov decision process (MDP) theory are employed. The results of numerical experiments are given.

*Keywords:* multi-media traffic, multiple service, call admission control, multi-rate queue, optimization.

## 1. INTRODUCTION

Transmission of different types of calls through the modern multimedia wireless networks (MWN) is carried out by shared use of common radio channels. As a rule, polytypic messages require different quantities of bands during the whole transmission period. Adequate mathematic models for the processing of different types of calls in isolated MWN cells are multi-speed handling systems (Multi Rate Queues - MRQ). Review of the work in this field is available in [1-4].

Optimal CAC have to take into consideration the state of the system at the moment of making the decision. Exact and approximate methods of solution of the problem on the calculation of optimal CAC in general MRQ models with arbitrary number of call types, completely taking into account the state of the system, are shown in works [5-7]. A similar problem for MRQ model with calls of two types was solved in [8], where optimal CAC is searched in a narrow class of access strategies without preemption. It should be noted that in [5-8], the methods used were based on the theory of Markov Decision Processes (MDP). A review of works on application of MDP in problems of CAC calculation in queuing systems is available in [9, 10].

In practice, especially while reseaching general MRQ models with a relatively large number of types of calls, the system state isn't observed completely. Only partial information about the system state is available, namely, only general number of busy (free) channels is observed. Therefore, the desired CAC shall take the decision based on limited information. Let us name optimal (in definite meaning) CAC based only on information about the number of busy (free) channels as suboptimal.

In this work, a method is proposed for calculation of the suboptimal CAC in general MRQ model with pure losses. It is based on the theory of state space merging of semi-Markov systems [11].

## 2. MODEL AND PROBLEM STATEMENT

Poisson flow of the calls of different types with the summary intencity $\Lambda$ is coming to the $N-$channel system, $N > 1$. Any newly received call with $\sigma_i$ probability simultaneously requires $b_i$ channels, $1 \leq b_i \leq N, i = 1, 2, ..., K$, at that $\sigma_1 + \sigma_2 + ... + \sigma_K = 1$. It is supposed that, at the moment of call arrival, the number of the channels, required for its handling becomes clear. Then, it can be considered that Poisson flows of calls of $K$ type come to the of $N-$channel system, at that intensity of the $i-$flow is equal to $\lambda_i = \Lambda\sigma_i$; $i-$type calls require simultaneous $b_i$ channels, $i = 1, 2, ..., K$. At that all $b_i$ channels starts and ends handling of this call at the same time and channel occupancy time is exponentially distributed variable with parameter $\mu_i$ , $i = 1, 2, ..., K$.

Quality of Service (QoS) parameters of the given MRQ essentially depend on accepted CAC and using of Complete Sharing (CS) access strategy doesn't permit optimization upon selected quality criteria. Here, the criterion is maximization of channel occupation. Thus, the problem is defined as calculation of such CAC without preemption, with ability to maximize channel occupation.

## 3. A METHOD FOR PROBLEM SOLUTION

Functioning of the given MRQ is described by $K-$dimensional Markov chain with states of $n = (n_1, ..., n_K)$ type, where $n_i$ denotes number of the $i-$calls in the system, $i = 1, 2, ..., K$. State space of the model is set in the following way:

$$S := \{n : n_i = 0, 1, ..., [N / b_i],\ i = 1, 2, ..., K;\ n \cdot b \leq N\}, \tag{1}$$

where $[x]$ - integer part of $x$; $b = (b_1, ..., b_k)$, $n \cdot b := \sum_{i=1}^{K} n_i b_i$ .

First of all, the problem of calculation of the optimal CAC is examined. With this goal let us examine the moments of $i-$call arrival, assuming that the system at this moment is in $n \in E$ state, in which $f(n) \geq b_i$ , where $f(n) := N - (n \cdot b)$ denotes the number of free channels in $n \in E$ state, (otherwise when $f(n) > b_i$, as mentioned before, $i-$call rejected). In this case, one of two decisions are possible: either (i) $i-$call is accepted, or (ii) it is rejected.

Probabilities of making the mentioned decisions are accordingly indicated through $\alpha_i^+(n)$ and $\alpha_i^-(n)$. Probabilities defined in this way will be named Controllable Situational Parameters (CSP). These parameters meet the following conditions:

$$0 \leq \alpha_i^+(n) \leq 1. \tag{2}$$

$$\alpha_i^+(n) + \alpha_i^-(n) = 1, \ \forall i \in F(n). \tag{3}$$

where $\alpha_i^+(n) + \alpha_i^-(n) = 1, \ \forall i \in F(n) = \{i \in Z_K^+ : f(n) \geq b_i\}, Z_K^+ := \{1, ..., K\}$.

Upon use of the given controlling parameters, the elements of generating matrix of the given MC $q(n, n'), \ n, n' \in S$, are calculated in the following way:

$$q(n,n') = \begin{cases} \lambda_i \alpha_i^+(n), & if \ n' = n + e_i \\ n_i \mu_i, & if \ n' = n - e_i \\ 0 & \text{otherwise}, \end{cases} \tag{4}$$

where $e_i$ - $i-$th ort of $K-$dimention Eucledian space, $i = 1, 2, ..., K$.

Let us denote the stationary probability of state $n \in S$ as $p(n)$. Then, average number of busy channels $(N_{av})$ is calculated in the following way:

$$N_{av} := \sum_{n \in S} (n \cdot b) p(n). \tag{5}$$

Consequently, optimal CAC is found from calculating such an access strategy where

$$N_{av} \to \max. \tag{6}$$

Restrictions of the problem (6) are conditions (2), (3) as well as the system of equilibrium equations for stationary probabilities of states. Formulated problem belongs to the class of nonlinear programming problems, but with the help of substitution of variables it is possible to solve it with the help of linear programming method. It always has optimal solution, according to which $\alpha_i^{\pm}(n)$ value is either 0, or 1 for each $i \in F(n)$. The last circumstance allows creation of algorithm, that realizes calculated optimal non-randomized CAC in the researched system.

The exact approach to the problem solution in calculation of optimal CAC is effective with little values of $N$ and $K$, but following their increasing, dimension of $S$ increases exponentially. Therefore, the problem of calculation of the suboptimal CAC for MRQ that is practically relevant for the system of large dimension is described below.

Since, according to the assumption, only the number of free (busy) channels are observed, let us research the next splitting of $S$:

$$S := \bigcup_{r=0}^{N} S_r, \ S_r \cap S_{r'} = \oslash, \ r \neq r'. \tag{7}$$

where $S_r := \{n \in S : n \cdot b = r\}$, that is class of microstates $S_r$ combines those $n \in S$ states, in which the general number of busy channels equals $r$. Hereinafter each $S_r$ class is described as one merged state (MS), indicated through $< r >, r = 0, ..., N$.

The next merging function is based upon splitting (7):

$$U(n) = < r >, \text{ } if \text{ } n \in S_r, \text{ } r \in Z_N. \tag{8}$$

where $Z_N = \{0, 1, ..., N\}$.

The elements of generating matrix of merged model $q(< r' >, < r'' >), r', r'' \in Z_N$ are defined in the following way:

$$q(< r' >, < r'' >) = \begin{cases} \lambda_i \sum\limits_{n \in S_{r'}} p(n)\alpha_i^+(n), & if \; r'' = r' + b_i, i \in Z_K^+ \\ \mu_i \sum\limits_{n \in S_{r'}} n_i p(n), & if \; r'' = r' - b_i, i \in Z_K^+ \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Stationary probability of state $< r >$, indicated as $\pi(< r >)$, is defined in the following way:

$$\pi(< r >) = \sum_{n \in S_r} p(n), \; r \in Z_N. \tag{10}$$

Taking into account (5) and (10) we can find out that average number of busy channels of the system is expressed through stationary distribution of merged model in the following way:

$$N_{av} := \sum_{r=1}^{N} r\pi(r). \tag{11}$$

The exact values$q(< r' >, < r'' >), r', r'' \in Z_N$ in formula (9) must be approximated for creating the system of equilibrium equations for merged model. This shall be done because these formulae contain unknown stationary distribution of the initial model, as well controlling solutions $\alpha_i^{\pm}(n), n \in S, i \in F(n)$ which are not defined for merged models.

Since, for any microstate $n \in S_r$ upon making a decision about access of arrived $i-$call, $i \in F(n)$, transition into merged $S_{r+b_i}$ state takes place, and virtual transition takes place in case of rejection, then merged models CSP can be defined in the following way:

$$\alpha_i^+(< r >) + \alpha_i^-(< r >) = 1, \; \forall i \in F(r). \tag{12}$$

where $\alpha_i^+(< r >) := P$ (arrived $i-$call is accepted / the system is in MS $< r >$);

$\alpha_i^-(< r >) := P$ (arrived $i-$call is lost / the system is in MS $< r >$);

$F(r) = \{i \in Z_K^+ : b_i \leq N - r\}$.

Then, taking into consideration (9) and (12) we can find that as approximate values of $q(< r' >, < r'' >), r', r'' \in Z_N$ could be used the following correlation estimating them from above:

$$\widetilde{q}(< r' >, < r'' >) = \begin{cases} \approx \lambda_i \alpha_i^+(< r' >), & if \; r'' = r' + b_i, i \in Z_K^+ \\ \approx \left[\frac{r}{b_i}\right] \mu_i, & if \; r'' = r' - b_i, i \in Z_K^+ \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Here $\left[\frac{r}{b_i}\right]$ denotes maximal number of $i$−calls in $n \in S_r$ microstates.

The system of equilibrium equations for merged model is created by taking into consideration correlations (13). Thus the problem of finding of suboptimal CAC in researched multi-rate system is maximization (11) subject to (12) and equilibrium equations for merged model. Like the initial problem of calculation of optimal CAC, it also refers to class ID and the suboptimal non-randomised CAC is found as a result of its solution, that is optimal values $\alpha_i^+(< r >)$, where $r \in Z_N, i \in \widetilde{F}(r)$ are defined.

The optimal values of CSP $\alpha_i^\pm(n)$, $n \in E$, $i \in F(n)$ are defined after solution of the problem of finding the optimal CAC (that is optimal solutions for each microstate are defined), and optimal values of CSP $\alpha_i^\pm(< r >)$, $r \in Z_N$, $i \in \widetilde{F}(r)$ are defined after solution of the problem of finding suboptimal CAC (that is optimal decisions for each merged state are defined). In connection with this it should be noted that in particular cases, optimal and suboptimal strategies might be coincide. Along with this, in general cases it shall not be stated that these strategies will coincide.

In this paper results of appropriate numerical experiments are demonstrated. It should be noted, that this method can be used upon other optimality criterias, as well upon having limitations for probabilities of loss of different types of calls. However having additional limitations for loss probability of different types of calls will lead to nessecity of realization of randomized CAC, which will cause methodical difficulties in real systems.

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