REGENERATIVE ESTIMATOR FOR EFFECTIVE BANDWIDTH

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The effective bandwidth (EB) estimation is considered for a station with regenerative input. It is assumed that regenerative approach can be useful to overcome a drawback of the traditionally used batch-mean approach, where batch size is taken arbitrarily and dependence between different batches may arise.

Key words: large deviations, regenerative processes, batch means, tandem networks.

1. INTRODUCTION

It is known that for a wide class of queuing systems the probability that stationary workload W exceeds a high level x (*large deviation*) has an exponential decay, that is

$$P(W > x) \approx e^{-\delta x}. \tag{1}$$

Assume constant service rate C and that $A(t) = \sum_{i=1}^{t} X_i$, where X_i is the number of arrivals during the ith unit of time, $t \in \{1, 2, \dots\}$. A key role to calculate the exponent δ plays the limiting scaled cumulant generating function (SCGF)

$$\Lambda(\theta) = \lim_{t\to\infty} \frac{1}{t} \log E e^{\theta A(t)}$$

(provided that the limit exists) because (see [1])

$$\delta \equiv \delta(C) = \sup\{\theta > 0 : \Lambda(\theta) \le C\theta\}. \tag{2}$$

The equation (1) leads to the following approximation of the overflow probability in the systems with fixed buffer-size b,

$$\Gamma \approx P(W > b) \approx e^{-\delta(C)b}.$$

Since the overflow probability represents an upper bound for a given loss probability Γ in the corresponding system with a finite buffer (of size b), the large deviation approximation (1) leads to the following estimation of the loss rate using effective bandwidth (EB)

$$C_\Gamma = \min\{C : e^{-\delta(C)b} \le \Gamma\}.$$

Assuming equation $e^{-\delta(C)b} = \Gamma$ has a unique root $\delta(C_\Gamma)$ we obtain

$$\theta^* \equiv \delta(C_\Gamma) = -\frac{\log \Gamma}{b},$$

and it follows from (2) and definition of C_Γ that

$$C_\Gamma = \frac{\Lambda(\theta^*)}{\theta^*}.$$

Thus, to estimate C_Γ we need to estimate $\Lambda(\theta^*)$.

2. EB ESTIMATION

The standard approach for the estimation of the SCGF relies on the assumption that the input data form a stationary mixing sequence. Under this hypothesis, a partition of the given input sequence into blocks of fixed size B

$$\hat{X}_j = \sum_{i=1+(j-1)B}^{jB} X_i, \quad j \geq 1,$$

is used to construct a sample mean estimate of the SCGF.

In other words, under the implicit assumption that for large values of B the blocks constitute i.i.d. random variables, the batch-mean method is used for the estimation, see for instance [2]. However, it is unclear how to choose an appropriate block size B to obtain an effective estimation of $\Lambda(\theta)$.

In this work, we present a refined EB approximation, which is based on a regenerative structure of the input sequence. We restrict our attention to a tandem network with two single server stations, with a renewal input to the first station, and a constant service rate at the second station. Then (under well-known stability conditions) the second station is fed by a positive recurrent regenerative input, the output from the first station.

Hence, regenerations of the input (to the 2nd station) are generated by the customers non waiting at the first station. Then we define a partition of the input sequence into i.i.d. blocks of random (instead of fixed) length, delimited by the boundaries of the regenerative cycles. More exactly, for an input data $\{X_i\}$ with regeneration points β_n and regeneration periods $\alpha_n = \beta_{n+1} - \beta_n$, we put

$$\hat{X}_k = \sum_{i=\beta_k}^{\beta_{k+1}-1} X_i, \; k \geq 1. \tag{3}$$

Assume that $Ee^{\theta\hat{X}} < \infty$ and $E\alpha < \infty$. Because random variables $\hat{X}_k$ are i.i.d., then we expect that for a large number of observations n, the function

$$\frac{1}{n}\log Ee^{\theta^* \sum_{i=1}^{n} X_i}$$

behaves like

$$\frac{1}{\beta_k}\log Ee^{\theta^*\sum_{i=1}^{k}\hat{X}_i}=\frac{k}{\beta_k}\log Ee^{\theta^*\hat{X}}$$

for a large number of regenerative blocks k. This implies the following convergence

$$\frac{1}{n}\log Ee^{\theta^*\sum_{i=1}^{n}X_i}\to\frac{1}{E\alpha}\log Ee^{\theta^*\hat{X}}\equiv\Lambda(\theta^*), \quad (4)$$

that leads to the following definition of the EB estimator

$$\hat{\Lambda}_k(\theta^*)=\frac{k}{\beta_k}\log\frac{1}{k}\sum_{i=1}^{k}e^{\theta^*\hat{X}_i}\,. \quad (5)$$

Note that by the Strong Law of Large Numbers, (5) is the strongly consistent estimator of $\Lambda(\theta^*)$.

Simulation trials carried out for different values of the system parameters (buffer sizes at the two queues, the arrival and service rates at the first queue as well as the target loss probability) highlighted that the new estimator significantly outperforms the traditional fixed block size approach in terms of estimation variance.

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