ANALYSIS OF THE SINGLE SERVER RETRIAL QUEUE WITH BREAKDOWNS

N. Djellab, L. Boutarfa
University of Annaba
Annaba, Algeria
djellab@yahoo.fr

In this work, we consider an M/G/1 retrial queue where the customer service is subject to interruptions and the customer, whose service is interrupted, has to either leave the system forever or join the retrial group. We first present a recursive procedure to calculate the steady-state joint distribution of the server state and the number of customers in the orbit. Then, for the same distribution, we obtain the closed form expressions when the service times follow an exponential law.

Keywords: breakdown, retrial queue, generating function, steady-state distribution, hyper geometric series.

1. INTRODUCTION

Retrial queues with unreliable servers are of great importance because they occur in many practical situations. We distinguish between the failure/repair behaviour of the server when it is idle (passive breakdown) and when it is busy (active breakdown). In the latter case, the customer whose service is interrupted can leave the service area or stay at the server until the repair is done and then restart his service. Under the first assumption, the customer in question can leave the system forever or join the retrial group. Under the second assumption, we can handle the following situations: pre-emptive resume – the service starts from where it was interrupted; pre-emptive repeat identical – the service starts from scratch with the same service time demanded again; pre-emptive repeat different – the service starts all over again with a resampled service requirement. These models were discussed in the literature[1, 3, 4, 5, 7].

In this work, we consider an M/G/1 retrial queue where the customer service is subject to interruptions and the customer, whose service is interrupted, has to either leave the system forever or join the orbit. We first present a recursive procedure to find the steady-state joint distribution of the server state and the number of customers in the orbit. Then, for the same distribution, we obtain the closed form expressions when the service times follow an exponential law.

2. MODEL DESCRIPTION

We consider a single server queueing system with no waiting space. Primary customers arrive at the service area according to a Poisson process with rate $\lambda > 0$. An arriving customer receives immediate service if the server is able to start a service time; otherwise he leaves the service area to join the orbit. Successive inter-retrial times of any orbiting customer follow an exponential distribution with parameter $\theta > 0$. Thus, we have a classical retrial discipline with intensity $j\theta$, where j is the number of customers in the orbit. The service times follow a general distribution with distribution function $B(x)$ having finite mean $1/\mu$ and Laplace-Stieltjes transform $\widetilde{B}(s)$. We assume that the customer service can be interrupted by active breakdowns but the server is restarted instantaneously. The customer whose service time is interrupted has to either leave the system forever or join the orbit. Therefore, we have two types of breakdown governed by exponential laws: the first - with rate α_1(customer leaves the system); the second – with rate α_2(customer goes to the orbit). Finally, we admit the hypothesis of mutual independence between all random variables defined above.

The state of the system at time t can be described by means of the process $\{C(t), N_o(t), \xi(t), t \geq 0\}$, where $C(t)$ is 0 or 1 depending on whether the server is idle or busy, $N_o(t)$ is the number of customers in the orbit. If $C(t) = 1$, $\xi(t)$ represents the elapsed service time of the customer being served at time t.

3. STEADY-STATE JOINT DISTRIBUTION OF THE SERVER STATE AND THE NUMBER OF CUSTOMERS IN THE ORBIT

We note that our model can be viewed as a special case of the model studied in [2] by using the tools of Markov regenerative processes [6]. Hence, the considered system is in steady state if

$$\rho = \frac{\lambda\left(1 - \widetilde{B}(\alpha_1 + \alpha_2)\right)}{\alpha_1 + \alpha_2 \widetilde{B}(\alpha_1 + \alpha_2)} < 1. \tag{1}$$

Moreover, we adapt the presented recursive procedure to obtain the steady-state distribution $p_{ij} = \lim_{t\to\infty} P(C(t) = i, N_o(t) = j)$ ($i = 0, 1$ and $j = 0, 1, 2, \ldots$) for our model.

Now, we suppose that $B(x) = 1 - e^{-\mu t}, \mu > 0, t \geq 0$. Therefore, we have a Markov process

$$\{C(t), N_o(t), t \geq 0\} \tag{2}$$

with state space $S = \{0, 1\} \times N$.

The condition (1) becomes

$$\rho = \frac{\lambda}{\alpha_1 + \mu} < 1.$$

Theorem 1. *Let $\rho < 1$. The steady-state distribution of the process (2) is defined by*

$$p_{0j} = \lim_{t\to\infty} P(C(t) = 0, N_o(t) = j) = \frac{(\lambda + \alpha_2)}{\theta} \frac{\left(1 + \frac{(\lambda+\alpha_2)}{\theta}\right)_{j-1}}{(1)_j} \rho^j p_{00}, \ j \geq 1;$$

$$p_{1j} = \lim_{t\to\infty} P(C(t) = 1, N_o(t) = j) = \frac{\left(1 + \frac{(\lambda+\alpha_2)}{\theta}\right)_{j-1}}{(1)_j} \rho^j p_{10}, \ j \geq 1;$$

$$p_{00} = \lim_{t\to\infty} P\left(C\left(t\right)=0, N_o\left(t\right)=0\right) = \rho^{-1} p_{10} = \left[F\left(1, 1+\frac{\lambda+\alpha_2}{\theta}, 1; \rho\right)\right]^{-1}.$$

$$where\ (x)_n = \begin{cases} 1 & n=0 \\ x(x+1)...(x+n-1) & n\geq 1 \end{cases}; F\left(a,b,c;z\right) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}.$$

REFERENCES

1. *Aissani A., Artalejo J. R.* On the single server retrial queue subject to breakdowns // Queueing Systems. 1998. V. 30. P. 309–321.
2. *Artalejo J. R., Gomez Corral A.* Unreliable retrial queues due to service interruptions arising from facsimile networks // Belgian Journal of Operations Research,Statistics and Computer Science 1998. V. 38. P. 31–41.
3. *Djellab N. V.* On the single server retrial queue // YUJOR. 2006. V. 16. № 1. P. 45–53.
4. *Krishna Kumar B., Pavai Madheswari S., Vijayakumar A.* The M/G/1 retrial queue with feedback and starting failures // Applied Mathematical Modelling 2002. V. 26. P. 1057–1075.
5. *Kulkarni V. G., Choi B. D.* Retrial queue with server subject to breakdowns and repairs // Queueing Systems. 1990. V. 7. P. 191–208.
6. *Tijms H. C.* Stochastic Models: An Algorithmic Approach // John Wiley & Sons. 1994.
7. *Wang J., Cao J., Li Q.* Reliability Analysis of the retrial queue with server breakdowns and repairs // Queueing Systems. 2001. V. 38. P. 363–380.