A GRAY-SCALE OBJECT CORRELATION DETECTION

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Abstract. This paper presented a novel algorithm for computational complexity reduction in gray-scale image processing. We developed a correlation algorithm of the detection for gray-scale objects that is invariant with respect to rotation by 180° relative to the horizontal axis. It is possible to detect the object by analyzing the correlation coefficients lying at the diagonals of the matrix.

1. Introduction

The detection of an object in an image is one of the key problems in visual data processing applications [1-3]. Conventional procedures for detecting objects employ correlation algorithms realizing all the advantages of the maximum-likelihood method. All the objects in the image are compared to the template by scanning the image, usually from left to right and from top to bottom. The estimate is based on mutual correlation of the input and template images [1, 2]. A correlation function then characterizes the degree of matching of the objects compared [4, 5].

There are two main well known strategies are used for correlation image processing [1, 6]. The first strategy is the direct spatial domain correlation, where the calculation of the correlation coefficients is based on direct computation of vector-matrix multiplication [6, 7]. The second strategy utilises two-dimensional Fourier transform. This strategy is based on the fact that the correlation between image and object template is expressed as a simple multiplication of their spectrums in frequency domain [4,6].

In practical application, it is often necessary to detect for an object rotated by 180^0 relative to the horizontal axis. In this work, we present a method of the correlation detection for image, which does not require additional calculation of the elements of the correlation matrix and enables one to detect an object identical to the template and rotated by 180^0 relative to the horizontal axis.

2. Correlation image processing

Correlation approach is based on pixel-to-pixel comparison of analyzed image and object template. In this case misalignment will determine the level of correlation function between two images, and the correlation will be maximum at minimum misalignment.

Thus correlation image processing task proposes the correlation function calculation between two images, one of which is utilized as an object template and another as an analyzed image. The discrete correlation function between images S and X will be computed as [4]:

$$R_{S,X}[\tau_1,\tau_2]=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} s_{m,n}x_{m-\tau1,n-\tau2}\,, \qquad (1)$$

for $\tau_1 = 0,1,...., M-1$ and $\tau_2 = 0,1,...., N-1$.

However, equation (1) may be modified to yield a de-meaned, normalized correlation coefficient:

$$R_{S,X} = \frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} s_{m,n} x_{m,n}}{\sqrt{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} s_{m,n}^2}\sqrt{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x_{m,n}^2}}. \quad (2)$$

A normalized coefficient is a more robust measure of correlation between two images since it accounts for intensity variation between images. This equation can be used to express the correlation between subimages and object.

3. Correlation computation based on direct method

Definitions

Let the *p*th leading diagonal of a rectangular matrix [A]={a_{ij}} $i=\overline{1,M}$, $j=\overline{1,N}$, M<N, p=(0,…,N-M) is the diagonal formed by the elements $(a_{i,j=i+p})$.

Let the *p*th secondary diagonal of a rectangular matrix [A]={a_{ij}} $i=\overline{1,M}$, $j=\overline{1,N}$, M<N, p=(0,…,N-M) is the diagonal formed by the elements $(a_{M-i+1,j=i+p})$.

Consider a problem of the correlative detection for a given object A={a_{ij}} of size nxn in an image D={d_{ij}} of size NxN. An elementwise comparison of the pixels of the object and the image involves an extraction of the group of elements in the image of the size Nxn. Then, the detection of an object *A* in the extracted block D_1^T={d_{ij}}, $i=\overline{1,n}$, and $j=\overline{1,N}$, is reduced to determining the correlation of the rows of the object and the image. Mathematically, such a procedure represents an analysis of the matrix H={h_{ij}} $i=\overline{1,n}$, $j=\overline{1,N}$, where *H* is the matrix of the correlation coefficients of the rows in the extracted fragment and the object.

$$h_{i,j+p} = \frac{\sum_{i=1}^{n} a_{ij} d_{i,j+p}}{\sqrt{\sum_{i=1}^{n} a_{ij}^2}\sqrt{\sum_{i=1}^{n} d_{i,j+p}^2}}. \quad (3)$$

The aim is the detection of the object as a whole. Therefore, an object *A* is detected *in the extracted block D1 if n rows in [A] sequentially* coincide with *n* rows in [*D1*]. The above condition is satisfied if the *maximum correlation coefficients* are obtained. These coefficients *must* lie *at one of the leading diagonals* of the matrix [*H*], which formally follows from the definition of the product of two matrices. The starting position of *A* in *D*1 depends on the position of the first element of the *p*th diagonal.

The selection of the *p*th diagonal with maximum correlation elements in the matrix *H* is equivalent to analyzing the elements of the vector represented as *X*:

$$X_p = \prod_{i=1}^{n} h^2{}_{i,j=i+p}. \quad (4)$$

A decision regarding the presence of the object *A* in the *p*th zone of the block *D*1 is made by comparing X_p with a threshold level related to the signal-to-noise ratio in the image *D*.

In practical applications, it is often necessary to detect an object rotated by 180^0 relative to the horizontal axis. In matrix representation, a rotation of the object *A* by 180^0 relative to the horizontal axis is equivalent to a simple permutation of rows. For an object of size $m \times n$ occupying the rows ranging from i to k in an image of size $M \times N$, the rotation is equivalent to substituting the kth, ($k-1$)th, ($k-2$), etc., rows for the i th, ($i-1$)th, ($i-2$), etc., rows, respectively. If $m \neq 0$, the row with the number $(i+k)/2$ remains unchanged.

In the matrix form, the transposition of a row from the ith position to the jth one can be done by multiplying on the left the original matrix by a permutation matrix given by

$$P = \begin{array}{c} \left[\begin{array}{ccccccc} 1 & & & & & & \\ & 1 & & & & & \\ & & 0 & & & 1 & \\ & & & 1 & & & \\ & & & & 1 & & \\ & & 1 & & & 0 & \\ & & & & & & 1 \end{array}\right] \begin{array}{l} \\ \\ i\text{th row} \\ \\ \\ j\text{th row} \\ \\ \end{array} \\ \begin{array}{ccccccc} & & i\text{th column} & & & j\text{th column} & \end{array} \end{array}$$

Fig. 1 Permutation matrix

Hence, the correlation matrix $[H_1^R]$ of the object A and a fragment $D1$ of the image is calculated with allowance for the rotation of the object by 180° relative to the horizontal axis as

$$[H_1^R] = [A \cdot (P_1 \cdot P_2 \cdot ... P_m \cdot D_1)^T]. \tag{5}$$

It is known [8] that $[F \cdot G]^T = [G^T] \cdot [F^T]$; therefore,

$$[H_1^R] = [A \cdot D_1^T \cdot P_1^T \cdot P_2^T \cdot ... \cdot P_{m/2}^T] = [H_1 \cdot P_1^T \cdot P_2^T \cdot ... \cdot P_{m/2}^T]. \tag{6}$$

Hence, it is possible to detect the rotated object by analyzing the correlation coefficients lying at the pth secondary diagonals of the matrix $[H1]$. The selection of the pth secondary diagonal in the matrix H is equivalent to analyzing the elements of the vector represented as Y:

$$Y_p = \prod_{i=n}^{1} h^2_{i,j=i+p}. \tag{7}$$

Maximal value of correlation coefficient Y_p will indicate where the rotated object $A=\{a_{ij}\}$ of size nxn in a image $D=\{d_{ij}\}$ of size NxN is situated.

4. Object detection algorithm

The algorithm of object detection based on correlation computation can be written as the following.

1. Extraction of the first fragment $D_1 = d_{ij}$ of size Mxn, $i = \overline{1,M}$, $j = \overline{1,n}$ in the left-hand side of the raster image.
2. Computing of the elements of the correlation matrix $[H]=\{h_{i,j}\}$ by determining the correlation of the rows of the object and the image using correlation coefficient.

3. Computing values X and Y using expressions (4) and (7).
4. Examining values X and Y to decide the object existence. Maximal correlation values indicate object position in the fragment.
5. A shift to the right by one element in the image, selection of the next fragment of the size $n \times M$ and transfer to step (2) if the number of shifts is less than $(N-n)$. In the opposite case, the procedure is terminated. The total number of the fragments analysed is $(N-n+1)$.

For example, consider two matrixes **A** and **X** defined as

$$A = \begin{bmatrix} 48 & 50 & 36 & 48 & 121 \\ 127 & 185 & 179 & 69 & 115 \\ 84 & 183 & 161 & 84 & 106 \\ 199 & 185 & 178 & 130 & 59 \\ 124 & 146 & 111 & 10 & 15 \\ 125 & 178 & 103 & 19 & 49 \end{bmatrix} \tag{8}$$

$$D^T = \begin{bmatrix} 133 & 45 & 45 & 58 & 71 & \mathbf{48} & \mathbf{127} & \mathbf{84} & \mathbf{199} & \mathbf{124} & \mathbf{125} & 128 & \boldsymbol{\mathit{125}} & \boldsymbol{\mathit{124}} & \boldsymbol{\mathit{199}} & \boldsymbol{\mathit{84}} & \boldsymbol{\mathit{127}} & \boldsymbol{\mathit{48}} \\ 133 & 45 & 45 & 58 & 71 & \mathbf{50} & \mathbf{185} & \mathbf{183} & \mathbf{185} & \mathbf{146} & \mathbf{178} & 255 & \boldsymbol{\mathit{178}} & \boldsymbol{\mathit{146}} & \boldsymbol{\mathit{185}} & \boldsymbol{\mathit{83}} & \boldsymbol{\mathit{185}} & \boldsymbol{\mathit{50}} \\ 40 & 51 & 51 & 68 & 128 & \mathbf{36} & \mathbf{179} & \mathbf{161} & \mathbf{178} & \mathbf{111} & \mathbf{103} & 128 & \boldsymbol{\mathit{103}} & \boldsymbol{\mathit{111}} & \boldsymbol{\mathit{178}} & \boldsymbol{\mathit{161}} & \boldsymbol{\mathit{179}} & \boldsymbol{\mathit{36}} \\ 35 & 49 & 49 & 79 & 188 & \mathbf{48} & \mathbf{69} & \mathbf{84} & \mathbf{130} & \mathbf{10} & \mathbf{19} & 255 & \boldsymbol{\mathit{19}} & \boldsymbol{\mathit{10}} & \boldsymbol{\mathit{130}} & \boldsymbol{\mathit{84}} & \boldsymbol{\mathit{69}} & \boldsymbol{\mathit{48}} \\ 35 & 49 & 49 & 79 & 188 & \mathbf{121} & \mathbf{115} & \mathbf{106} & \mathbf{59} & \mathbf{15} & \mathbf{49} & 128 & \boldsymbol{\mathit{49}} & \boldsymbol{\mathit{15}} & \boldsymbol{\mathit{59}} & \boldsymbol{\mathit{106}} & \boldsymbol{\mathit{115}} & \boldsymbol{\mathit{121}} \end{bmatrix} \tag{9}$$

Then, forA and D, we obtain H:

$$H^T = \begin{bmatrix} 0.68 & 0.89 & 0.89 & 0.92 & 0.92 & \mathbf{1} & 0.81 & 0.81 & 0.72 & 0.6 & 0.68 & 0.8 & 0.68 & 0.6 & 0.72 & 0.81 & 0.81 & \mathit{1} \\ 0.86 & 0.95 & 0.95 & 0.92 & 0.81 & 0.81 & \mathbf{1} & 0.99 & 0.95 & 0.92 & 0.95 & 0.88 & 0.95 & 0.92 & 0.95 & 0.99 & \mathit{1} & 0.81 \\ 0.84 & 0.95 & 0.95 & 0.92 & 0.82 & 0.8 & 0.99 & \mathbf{1} & 0.93 & 0.89 & 0.93 & 0.92 & 0.93 & 0.89 & 0.93 & \mathit{1} & 0.99 & 0.8 \\ 0.92 & 0.94 & 0.94 & 0.9 & 0.77 & 0.72 & 0.95 & 0.93 & \mathbf{1} & 093 & 0.93 & 0.9 & 0.93 & 0.93 & \mathit{1} & 0.93 & 0.95 & 0.72 \\ 0.93 & 0.8 & 0.8 & 0.73 & 0.55 & 0.6 & 0.92 & 0.89 & 0.93 & \mathbf{1} & 0.98 & 0.76 & 0.98 & \mathit{1} & 0.93 & 0.89 & 0.92 & 0.6 \\ 0.96 & 0.84 & 0.84 & 0.79 & 0.62 & 0.68 & 0.95 & 0.93 & 0.93 & 0.98 & \mathbf{1} & 0.82 & \mathit{1} & 0.98 & 0.93 & 0.93 & 0.95 & 0.68 \end{bmatrix} \tag{10}$$

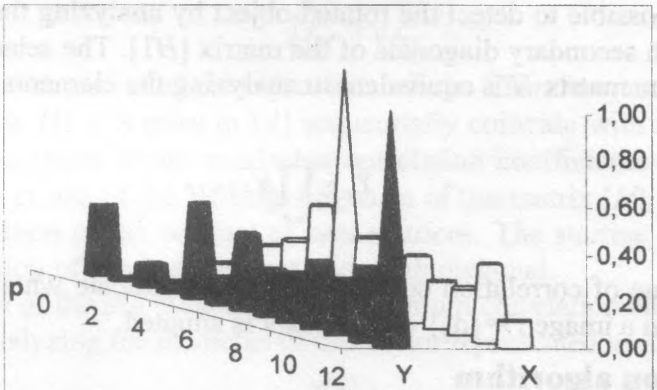

Fig.2 Correlation functions for matrix A and D

The analysis of the correlation functions shows that the object identical to the template and the object rotated relative to the horizontal axis start from the sixth and eighth rows of the fragment processed, respectively.

5. Conclusion

Substantial computational complexity and time costs are typical for correlation image processing based on direct method, therefore eliminating unnecessary calculations is particularly desirable for this method of correlation. We showed the possibility to detect the gray-scale object, had rotated by rotation by 180^0 relative to the horizontal axis with out additional computation.

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