МЕТОДИКА ЛОКАЛЬНОЙ ВТОРИЧНОЙ ДЕКОМПОЗИЦИИ В РЕШЕНИИ ДВУХМЕРНЫХ ЭЛЕКТРОДИНАМИЧЕСКИХ ЗАДАЧ МЕТОДОМ МИНИМАЛЬНЫХ АВТОНОМНЫХ БЛОКОВ

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Разработана методика, которая позволяет использовать локальную вторичную декомпозицию в двумерной версии метода минимальных автономных блоков (МАБ). Методика использована для задачи дифракции плоской электромагнитной волны на периодической решетке из металлических полос-диэлектриков. Разработанная методика позволяет значительно уменьшить вычислительные ресурсы, необходимые для решения многомасштабных проблем методом МАБ.

Ключевые слова: вычислительный электромагнетизм; параметры рассеяния.

TECHNIQUE OF LOCAL SECONDARY DECOMPOSITION IN SOLVING TWO-DIMENSIONAL ELECTROMAGNETIC PROBLEMS BY MINIMAL AUTONOMOUS BLOCKS METHOD

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A technique which makes it possible to use local secondary decomposition in two-dimensional version of minimal autonomous blocks (MAB) method is developed. The technique is validated for the problem of a plane electromagnetic wave diffraction on a periodic grating made from metal-dielectric strips. Developed technique can significantly decrease computational resources required for solving multiscale problems by the MAB method.

Keywords: computational electromagnetics; scattering parameters.

INTRODUCTION

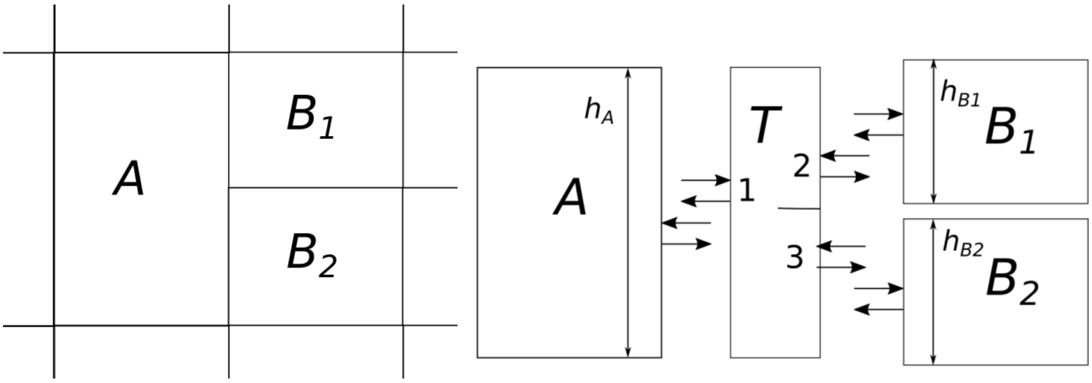
Minimal autonomous blocks method (MAB, also used acronym is MAD) is successfully used for solving various electromagnetic problems [1–3]. Unlike the finite element method (FEM), the MAB method uses exact solution of Maxwell equations inside a mesh cell [2].

The MAB method uses rectilinear grid and well suited for solving multiscale problems due to the technique of averaged scattering matrices [3]. But if the technique is not applicable for modeled structure, then accounting of the structure's local features will lead to decreasing sizes of mesh cells for entire modeling domain.

Use of local secondary decomposition makes it possible to decrease sizes of mesh cells only near to the modeled structure [4].

LOCAL DECOMPOSITION IN MAB METHOD

To solve problem of stitching grids with different cell sizes, we will use special transmission blocks, similarly to [4] (fig. 1). Every such block is connected at one side to the channel of block A of the main grid, and at another side is connected to the channels of local grid blocks B_i, $i = 1,..,k$ (k is ratio of blocks in grids along border between grids).For purposes of simplicity, it is assumed that the same medium is placed on both sides of transmission blocks (otherwise it is required to consider transmission blocks between mediums [1]). Let us assume that two blocks of local grid are connected to each block of main grid (k = 2). Size h_{Bi} of blocks B_i along border between grids is equal to half of size h_A of block A along this border.



a *b*

Fig. 1. Stitching of grids: *a* – part of total grid; *b* – transmission block with channel waves between grids

It is required to determine transmission matrix S_T of introduced transmission block T between grids. Channels numbering was performed in the following way (see fig. 1): 1 is number of channel connected to the block A, 2 is number of channel connected to the block B_1, 3 is number of channel connected to the block B_2. It is adopted that diagonal elements $S_{T\,i,i}$ responsible to reflections are equal to zero (reflection phenomena will be accounted correctly due to appearance of reflected waves from blocks A and B_i and propagation of these waves through introduced transmission block T). We also adopt that $S_{T\,2,3} = S_{T\,3,2} = 0$ (interactions between channels of blocks B_i connected to the block T will take place only due to reflections from block A).

One can derive remaining elements of transmission matrix S_T when propagation of plane wave perpendicularly to the boundary between grids is considered. In this case presence of the transmission block should not influence propagation of the wave. If incident wave have unit amplitude at the first channel of the block T (amplitudes of another incidence waves are zeros), waves with unit amplitudes must exit from the second and third channels. Therefore, $S_{T\,i,1} = 1$, $i = 2,3$.

In case of incidence of waves with unit amplitudes on the second and third channels of the introduced transmission block T (amplitude of incidence wave for the first channel is zero) wave with unit amplitude must exit from the first channel:

$$1 = S_{T\,1,2} + S_{T\,1,3}. \tag{1}$$

In the case of blocks B_1 and B_2 having equal sizes $S_{T\,1,2} = S_{T\,1,3}$. Then:

$$S_{T\,1,2} = S_{T\,1,3} = 0{,}5. \tag{2}$$

Obtained matrix will have the following form:

$$S_T = \begin{pmatrix} 0 & 0{,}5 & 0{,}5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{3}$$

INCIDENCE OF PLANE WAVE ON GRATING MADE FROM METAL-DIELECTRIC STRIPS

Validation of grids stitching technique

Capabilities of the developed technique are shown by example of numerical solution for problem of plane linearly polarized electromagnetic wave diffraction on periodic grating made by metal-dielectric strips (fig. 2). Parameters of the grating: thickness of strips is h = 0,5 mm, width of strips is d = 5 mm, distance between strips is $w = d$. Dielectric permittivity is equal to 10,0 and corresponds to material Arlon AR 1000. Case of normal incidence is considered. Magnetic component of electromagnetic field is parallel to strips. Initial diffraction problem is reduced to equivalent waveguide containing one period of the grating. Periodic boundary conditions are defined at sides of the waveguide. Part of the waveguide with length of 6 wavelengths containing inhomogeneity is divided into N blocks along the grating plane (x axis) and into 29 blocks perpendicularly to the grating plane (y axis). Dielectric part of the grating is divided into 3 blocks along y axis, metallic coating of the grating is modeled by metallization blocks with infinite conductivity [1]. Sizes of blocks along y axis placed in 4 rows adjacent to the grating from every side are equal to the thickness of strips (see fig. 2). Coefficients of reflection and transmission are computed with accounting of single-mode scattering by using iterative algorithm for the MAB (calculation of complex amplitudes of incident and reflected waves for all blocks by sequential iterations with accounting of sources and boundary conditions [3]) with 20 000 iterations.

Dependencies of absolute values of transmission coefficients for plane wave fallen on the grating versus wave sizes of strips calculated by the developed technique for various k and number of blocks along x axis for local grid $M = kN$ (local grid contains blocks corresponding to the grating internals and blocks placed in two rows adjacent to the grating) are shown in fig. 3.

According to fig. 3, frequency of resonance is less and less changed with increase of blocks number. This is connected with gradual convergence of the MAB to the exact solution (in order to retrieve exact solution, it is required also to reduce discretization along y axis, but it is not the purpose of the paper).

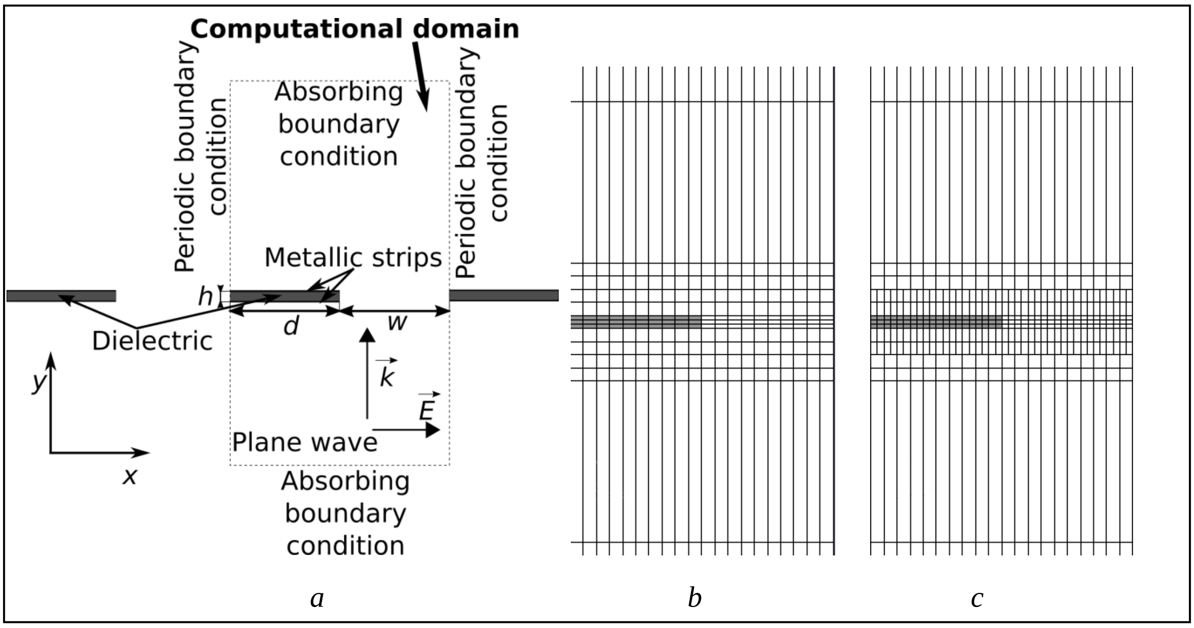


Fig. 2. *a* – Modelled structure: infinite periodic grating made from metal-dielectric strips. Dielectric part of strips is coated at both sides by infinitely thin metal with ideal conductivity; *b* – Part of rectiliear grid; *c* – Grid with local secondary decomposition used for modeling (N = 20, k = 2, M = 40)

One can see from fig. 3 that transmission coefficient is mostly determined by number M of blocks along x axis near to the grating: difference between transmission coefficients calculated using local decomposition and without it but with the same M is negligibly small. This proves validity of transmission matrix S_T used for transmission block T .

Maximal size of the MAB block is limited to 0,25 of wave length [1]. Because we investigate the structure at frequencies near to the frequency of first resonance in strips (half of wave length in dielectric is nearly equal to width of the strips), minimal number of blocks along the grating period M is 4. For grid without local decomposition $N = M$, therefore using N equal to 2 is not possible without local decomposition (see fig. 3, *a*), curve «N = 2, k = 20, M = 40»).

INCREASING OF COMPUTATIONAL EFFICIENCY DUE TO LOCAL DECOMPOSITION

If iterative algorithm is used, then time required for calculation of one iteration and memory required for storing vectors of complex amplitudes of incident and reflected waves are proportional to number of used blocks. For the structure examined in Section «Validation of grids stitching technique», total number of blocks (with accounting of introduced transmission block) is equal to $K_{subgrids} = 29N + 7(k-1)N + 2N$. In order to achieve the same precision without local decomposition, it is required to use number of blocks along the grating nearly equal to kN (see Section «Validation of grids stitching technique»). Total number of blocks in this case is $K = 29kN$. Expenses of time and memory are reduced by using local decomposition $K / K_{subgrids}$ times:

$$K / K_{subgrids} = \frac{29kN}{29N + 7(k-1)N + 2N} = \frac{29k}{29 + 7(k-1) + 2}; \tag{4}$$

So, for grid with N =2 blocks and with local grid containing 6 blocks along the grating ($M = 6$, k = 3) gain will be $K / K_{subgrids}\big|_{k=3} \approx 1{,}9$. For k =20 we get $K / K_{subgrids}\big|_{k=20} \approx 3{,}5$. Maximally possible gain is 29/7≈4,1.

Gain of computational efficiency in three-dimensional case should be more valuable.

Note that number of iterations required to retrieve converged solution is decreased with decreasing number of blocks. But given effect requires separate study.

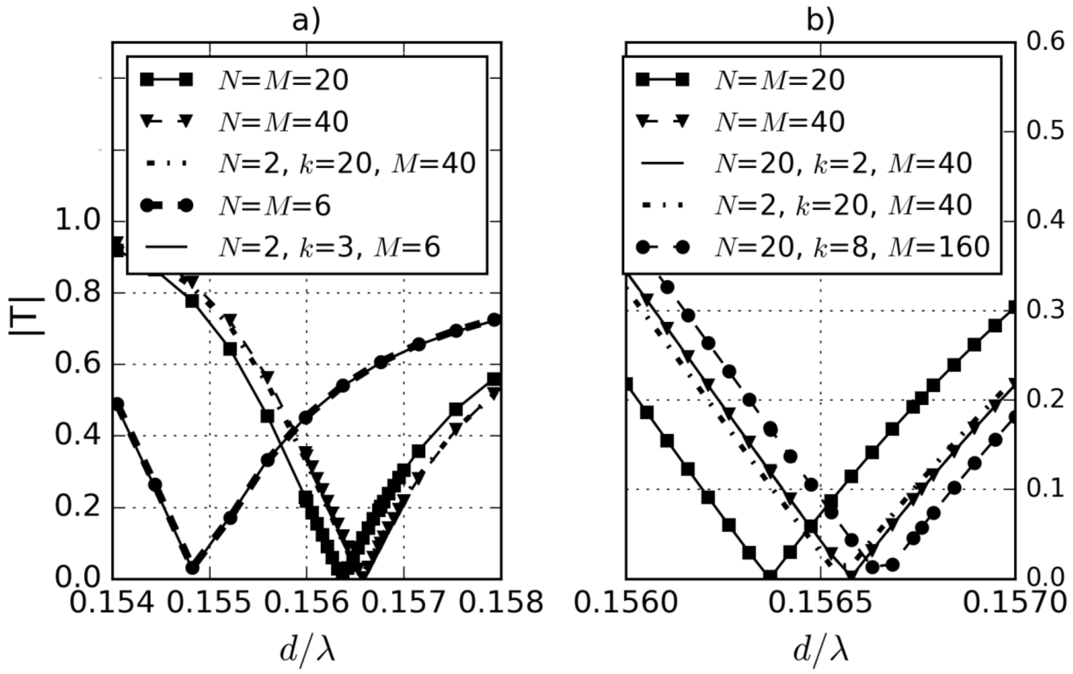


Fig. 3. Transmission coefficient of plane wave through the grating made from metal-dielectric strips (magnetic polarisation)

CONCLUSION

A technique which makes it possible to use local decomposition in two-dimensional version of the minimal autonomous blocks method is described. Efficiency of the developed technique is shown for problem of plane wave diffraction at periodic grating made from metal-dielectric strips.

Directions of further development include expansion of the developed technique: 1) on three-dimensional case, 2) on case of local decomposition with $h_{B1} \neq h_{B2}$ (see Section «Local decomposition in MAB method»).

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