RIEMANN BOUNDARY VALUE PROBLEM IN AN EXCEPTIONAL CASE

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Let Γ be a simple smooth closed contour dividing the plane of the complex variable into the interior domain D^+ and the exterior domain D^-, let F be a finite set of points of the contour Γ and let $\alpha = (\alpha_\tau, \tau \in F)$ be a family of complex numbers. Let

$$A(z) = \prod_{\tau \in F} (z-\tau)^{\alpha_\tau}, \quad z \in \overline{D^+}.$$

Let $\lambda^+ = ({\lambda^+}_\tau, \tau \in F)$ and $\lambda^- = ({\lambda^-}_\tau, \tau \in F)$ be families of complex numbers such that $\lambda^+ - \lambda^- = \alpha$. The problem is to find a function $\Phi(z) \in H_{\lambda^\pm}(\overline{D^\pm}, F)$ that is analytic outside Γ, vanishes at infinity, and satisfies the boundary condition

$$\Phi^+(t) - A(t)G_0(t)\Phi^-(t) = g(t), \tag{1}$$

where $g(t) \in H_{\lambda^+}(\Gamma, F)$ and $G_0(t) \in H_0(\Gamma, F)$ is an invertible function.

Theorem. *Let $X(z)$ be specially constructed canonical function* (see [1]), *and let*

$$æ = \sum_{\tau \in F} n_\tau, \quad n_\tau = [\mathrm{Re}\,(\delta_\tau + \alpha_\tau - {\lambda^+}_\tau)], \quad \delta_\tau = \frac{1}{2\pi i}((\ln G_0)(\tau - 0) - (\ln G_0)(\tau + 0)).$$

If $æ \geqslant 0$, then the general solution vanishing at infinity of problem (1) with righthand side $g(t) \in H_{\lambda^+}(\Gamma, F)$ in the class $H_{\lambda^\pm}(\overline{D^\pm}, F)$ is given by the formula

$$\Phi(z) = \begin{cases} A(z)\Psi(z), & z \in D^+, \\ \Psi(z), & z \in D^-, \end{cases} \tag{2}$$

where the function $\Psi(z)$ has the form

$$\Psi(z) = X(z)\left(\frac{1}{2\pi i}\int_\Gamma \frac{g(t)\,dt}{A(t)X^+(t)(t-z)} + P(z)\right),$$

and the degree of the arbitrary polynomial $P(z)$ does not exceed $æ - 1$. If $æ < 0$, then the solution of problem (1) is unique and is given by formula (2) provided that the orthogonality conditions

$$\int_\Gamma \frac{g(t)}{A(t)X^+(t)} t^j\,dt = 0, \quad j = 0, 1, \ldots, -æ - 1,$$

are satisfied. [*For $æ \leqslant 0$ we set $P(z) = 0$.*]

References

1. Urbanovich T. M. *Exceptional Case of the Linear Conjugation Problem in Weighted Hölder Classes* // Differential Equations. 2015. Vol. 51, no. 3. P. 1669–1673.