Application of Fuzzy Decision Trees in Construction of Mathematical Model for Reliability Analysis based on Uncertain Data

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Abstract: *Data mining (DM) is a collection of algorithms that are used to find some novel, useful and interesting knowledge in databases. Some methods of these fields can be used to find hidden relation between data, what can be used to create models that predict some behavior or describe some common properties of analyzed objects. In this paper, we combine methods of DM with tools of reliability analysis to evaluate availability of investigated object. An important step in reliability evaluation of any object is selection of an appropriate mathematical representation. One of the possible mathematical representations is structure function that expresses dependency of system state on states of its components. In this paper, we propose a new method for construction of the structure function from uncertain or incomplete data. This method is developed based on application of Fuzzy Decision Tree.*

Keywords**:** Fuzzy Decision Tree, Multi-State System, Structure Function, Uncertainty

1. INTRODUCTION

The reliability is important characteristic of any object/system in step of its development and exploitation. The reliability analysis of any object is possible based on the mathematical representation (description). The mathematical representation of an initial object (system) and estimation of its reliability properties includes next steps [1]:

1. the definition of number of performance levels for a system model;
2. the mathematical representation of system model;
3. the quantification of the system model (calculation of indices and measures);
4. the measuring of the system behavior.

The first step in reliability analysis agrees with the definition of mathematical model type depending on the number of performance levels. Two types of models can be recognized. These models are named as *Binary-State Systems* (BSSs) and *Multi-State Systems* (MSSs).

A BSS admits only two states in investigation of the system and its components: perfect functioning and complete failure. However, in practice, many systems can go through different performance levels between these two extreme states [1, 2]. A MSS is a mathematical model that is used to describe such systems since it allows defining more than two levels of performance [2, 3, 4].

The second step supposes the definition of the representation type of mathematical model. There are different types of mathematical representations of a system. In reliability engineering, structure function, fault trees, reliability block diagrams, Markov models and Petri nets are typically used for the mathematical representation of real systems under study.

The definition of number of performance levels and representation type of mathematical model cause the quantification analysis of investigated object/system at the third step. As a rule the reliability is estimated by set of specific indices and measures [1]. Some most used of them are system availability, reliability function, mean time to failure (repair), mean time between failures, faults frequency, importance measures. There are different methods and algorithms in reliability engineering to calculate these and other indices and measures and their values are used for estimation of investigated object/system behavior in point of view of reliability at the fourth step.

MSS structure function is one of possible mathematical model of investigated object/system. In this case, a system is modeled as a mapping that assigns system state to all possible combinations of component states. MSS allows describing of the system behavior in detail and taking into account preceding states before failure. This mathematical model permits to represent the system with any topological complexity and structure by application of the structure function. The exactness and undependability on complexity are principal advantages of MSS structure function. But the structure function is constructed based on complete information about the system structure and possible components states. However, there are a lot of practical problems when the complete information is not available because data from which it can be derived cannot be collected. As a rule, other mathematical representations and methods for evaluation of system reliability are used in these situations [5, 6, 7]. In this paper, we propose a new method for construction of the structure function from uncertain or incomplete data.

There are two principal factors of uncertain data in structure function construction. The first are ambiguity and vagueness of initial data. It means that initial data about the system operation are collected based on (a) measurement that can be inaccurate and with an error or (b) experts that can have different opinions on one situation. Therefore, values of states of the components or system performance level cannot be indicated as exact (integers). The fuzzy logic makes it possible to define the structure function in a more flexible form for such data than the probabilistic approach. So, non-exact values are the first factor of the uncertainty of initial data, and it can be expressed using fuzzy values [5, 6, 8].

Secondly, situations in which it is impossible to indicate some values of the system components states or performance level can exist. For example, it can be very expensive, or it needs unacceptable long time. This implies that some information about the system behavior can be absent. Therefore, the data are incomplete.

In this paper, we propose a method based on the application of an *Fuzzy Decision Tree* (FDT) for construction of the structure function. FDTs allow taking into account uncertainties of two types [9]. The first of them is ambiguity of initial data. This can occur when it is expensive to obtain all data about real system behavior, or there are poorly documented data. This type of uncertainty is covered by fuzzy values in an FDT. The second type of uncertainty agrees with incompletely specified initial data. As a rule, if the exact values of the actual data about the system behavior cannot be determined, we need to rely on more data to get additional information necessary to correct the used theoretical model [6, 12]. An FDT allows reconstructing these data with different levels of the confidence [10, 11].

2. MULTI-STATE SYSTEM STRUCTURE FUNCTION AND AVAILABILITY

As a rule two types of models are used in reliability analysis. The first one is known as a BSS. This model is based on the assumption that the system and all its components can be in one of only two possible states – functioning (labelled by number 1) and failure (represented by number 0). A general MSS permit defining different number of states for the system and for its components. Let us suppose that the system can be divided into n components (subsystems). If we assume that the system has M possible states and its i-th component, for $i = 1,\ldots,n$, can be in one of m_i states.

The dependency between states of individual system components and system state is expressed by a special relation that is known as structure function. The structure function as a mathematical model was introduced in reliability engineering as one of the firsts [13].

The structure function $\phi(\boldsymbol{x}) = \phi(x_1,\ldots, x_n)$ of a MSS has the following [14]:

$$\phi(\boldsymbol{x}): \{0,\ldots, m_1\text{-}1\}\times\ldots\times\{0,\ldots,m_n\text{-}1\}\rightarrow\{0,\ldots,M\text{-}1\}, \quad (1)$$

where $\phi(\boldsymbol{x})$ defines system state from complete failure ($\phi(\boldsymbol{x}) = 0$) to perfect functioning ($\phi(\boldsymbol{x}) = M$ -1); $\boldsymbol{x} = (x_1,\ldots, x_n)$ is a state vector; x_i is the i-th component state that changes from complete failure ($x_i = 0$) to perfect functioning ($x_i = m_i$ -1).

A special type of MSSs is a homogenous system, in which $m_1 = \ldots = m_n = M$. The structure function of BSS based on (1) is defined if $m_1 = \ldots = m_n = M = 2$.

Typically investigated system is coherent and its component failure doesn't cause the system functioning improving [3, 4, 13]. This means: (a) the system structure function is monotone: $\phi(x_i, \boldsymbol{x}) \le \phi(x_j, \boldsymbol{x})$ for any $x_i \le x_j$; and (b) there are no irrelevant components in the system.

For example, consider a twin-engine jet and it representation as MSS structure function [15]. It can land normally if one engine is at full power and the other engine is at half power. It can land on a foamed runway if one engine is at full power or if both engines are at half power. It will crash if one engine is at half power and the other engine is failed. This jet's structure function is is function of two variables ($n = 2$) with three values ($m = 3$) and is defined in Table 1.

Table 1. Truth table of the structure function

$\boldsymbol{x_1}$	**0**	**0**	**0**	**1**	**1**	**1**	**2**	**2**	**2**
$\boldsymbol{x_2}$	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
$\boldsymbol{\phi(x)}$	0	0	1	0	1	2	1	2	2

The structure function (1) allows calculation some of reliability indices and measures. One of them is system availability and unavailability. The system unavailability (5) for MSS is considered as probability of system failure and is defined identically [3, 14]:

$$U = \Pr\{\phi(\boldsymbol{x}) = 0\}. \quad (2)$$

Availability of MSS must be considered for some different performance levels and the availability (5) can be transformed into two types of measures for MSS [3, 4, 13]: system availability and probability of system performance level. The probability of system performance level is defined for every performance level as:

$$A_j = \Pr\{\phi(\boldsymbol{x}) = j\},\ j = 1,\ldots, M\text{ -1}. \quad (3)$$

MSS availability is defined as follows [3, 4, 13]:

$$A(j) = \Pr\{\phi(\boldsymbol{x}) \ge j\},\ j = 1,\ldots, M\text{ -1} \quad (4)$$

The probability of the system performance levels according (3) is initial measure that allows computing the system availability and unavailability. In papers [3, 4, 13] authors shown that any system state j ($j = 1,\ldots, M$ -1) for fixed components state vector of a coherent MSS according to the assumption (b) can be calculated as the product of probabilities of components states:

$$p_{is} = \Pr\{x_i = s\},\ s = 0,\ \ldots,\ m_i\text{ -1} \quad (5)$$

Consider a twin-engine jet availability and unavailability. The MSS structure function of this object is defined in Table 1. The unavailability (2) of this twin-engine jet is:

$$\mathrm{U} = p_{10}{\cdot}p_{20} + p_{10}{\cdot}p_{21} + p_{11}{\cdot}p_{20}, \quad (6)$$

and its probabilities for performance levels "1" and "2" according to (3) are:

$$A_1 = p_{10}{\cdot}p_{22} + p_{11}{\cdot}p_{21} + p_{12}{\cdot}p_{20},$$
$$A_2 = p_{11}{\cdot}p_{22} + p_{12}{\cdot}p_{21} + p_{12}{\cdot}p_{22}, \quad (7)$$

and its availabilities for performance levels "1" and "2" according to (4) are calculated as:

$$A(1) = A_1 + A_2 = p_{10}{\cdot}p_{22} + p_{11}{\cdot}p_{21} + p_{11}{\cdot}p_{22} + p_{12} \quad (8)$$
$$A(2) = A_2 = p_{11}{\cdot}p_{22} + p_{12}{\cdot}p_{21} + p_{12}{\cdot}p_{22}.$$

Suppose that this system has equal component probabilities that are defined as: $p_{10} = p_{20}$ =0.1, $p_{11} = p_{21}$ =0.2 and $p_{12} = p_{22}$ =0.7. The system unavailability for this data is U = 0.05, probabilities of the system performance levels are A_1 = 0.18 and A_2 = 0.77, and this MSS availabilities for two performance levels are $A(1)$ = 0.95 and $A(1)$ = 0.77.

The structure function also allows calculating the boundary system states [14], minimal cut/path sets [15] and importance measures [16]. However, defining structure function as equation (1) for a real application can be a difficult problem.

3. STRUCTURE FUNCTION CONSTRUCTION BASED ON UNCERTAIN DATA

As a rule, the structure function can be defined as a result of the system structure analysis or based on expert data [12, 17]. In system structure analysis, the system is interpreted as a set of components (subsystems) with correlations. These correlations can be defined by functional relations that are interpreted as the structure function (1). For example, such correlations are defined for a twin-engine jet structure unction (Table 1). However, there are many structure-complex systems for which correlations and/or connections of components are hidden or uncertain (e.g. power systems, network systems). As a rule, other methods are used in reliability estimation for such systems [5, 18]. Construction of a structure function based on the expert data requires special analysis and transformation of initial data [12, 19]. We suggest the new method for construction of the structure function (1) that is based on the application of an FDT.

In terms of Data Mining, the structure function can be interpreted as a table of decisions [9, 20], where state vector $\boldsymbol{x} = (x_1,\ldots, x_n)$ is interpreted as a set of input attributes and value of the structure function as an output attribute. This table of decisions can be constructed based on an FDT for all combinations of the input attributes. So, values of the structure function can be defined for all combinations of component states using the FDT: component states are interpreted as FDT attributes, and the structure function value agrees with one of M values (classes) representing system performance levels. The FDT is inducted based on some samples (not all) of the inputs and output attributes. In case of construction of the structure function, the samples are state vectors with the corresponding function value. These samples have to be collected as initial information about the system.

The method proposed in this paper includes the following steps:

- collection of data into the repository according to requests of FDT induction;
- representation of the system model in the form of an FDT that classifies components states according to the system performance levels;
- construction of the structure function as a decision table that is created by inducted FDT.

Collection of data in the form of a repository is provided by the monitoring of values of system component states and system performance level. This repository can be presented in the form of a table where the columns agree with the input and output attributes. The number of the input attributes is n and the i-th has m_i possible values (the i-th column includes m_i sub-columns). Every row contains a real sample of components states and the corresponding system performance level.

For example, let us consider the offshore electrical power generation system presented in [2]. The purpose of this system (Fig. 1) is to supply two nearby oilrigs with electric power. The system includes 3 generators: two main generators A_1 and A_3, and standby generator A_2. Both main generators are at oilrigs. In addition, oilrig 1 has generator A_2 that is switched into the network in case of outage of A_1 or A_3. The control unit U continuously supervises the supply from each of the generators with automatic control of the switches. If, for instance, the supply from A_3 to oilrig 2 is not sufficient, whereas the supply from A_1 to oilrig 1 is sufficient, U can activate A_2 to supply oilrig 2 with electric power through the standby subsea cables L. This implies that the system consists of 5 relevant components (n = 5): generators A_1, A_2, and A_3, control unit U, and the standby subsea cables L. Furthermore, according to the description of the system activity in [2], we assume that the system and all its components have 3 states/performance levels (M = 3 and m_i = 3, for i = 1,…,5). Next, let us denote variables defining states of the system components in the following way: main generators A_1 and A_3 as x_1 and x_3 respectively, standby generator A_2 as x_2, and control unit U and standby subsea cables L as x_4 and x_5 respectively.

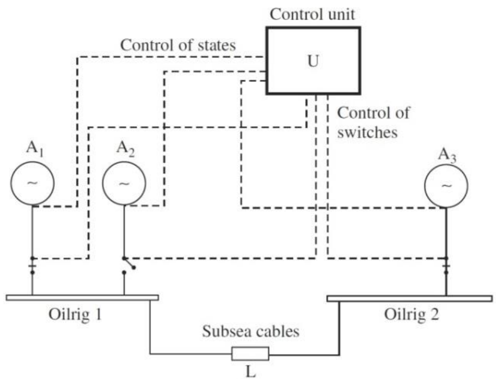

Fig.1 – Outline of the offshore electrical power generation system [2]

Let us suppose monitoring of the offshore power generation system that allowed collecting 108 (from 243 possible) samples of the system behaviour. Some of them are shown in Table 2. The monitoring of this system permitted obtaining information about some combinations of component states and the corresponding performance levels of the system. However, this information is not complete. This uncertainty is caused by the ambiguity of classification of component states and system performance levels into classes of exact values [12, 20]. Therefore, these data is interpreted as quasi-fuzzy data.

Table 2. Data obtained based on the monitoring of the offshore electrical power generation system

No	x_1			x_2			x_3			x_4			x_5			$\phi(x)$		
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
1	0.8	0.2	0.0	0.8	0.1	0.1	0.7	0.2	0.1	0.8	0.2	0.0	0.7	0.3	0.0	**0.7**	**0.3**	**0.0**
2	0.8	0.1	0.1	0.7	0.1	0.2	0.6	0.2	0.2	0.8	0.2	0.0	0.0	1.0	0.0	**0.8**	**0.1**	**0.1**
3	1.0	0.0	0.0	0.7	0.3	0.0	0.9	0.1	0.0	0.0	0.9	0.1	0.7	0.2	0.1	**1.0**	**0.0**	**0.0**
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	**…**	**…**	
15	0.0	0.2	0.8	0.9	0.1	0.0	0.2	0.8	0.0	0.0	0.1	0.9	0.0	0.1	0.9	**0.0**	**0.6**	**0.4**
16	0.0	0.1	0.9	1.0	0.0	0.0	0.0	0.1	0.9	0.1	0.6	0.3	0.0	0.2	0.8	**0.2**	**0.5**	**0.3**
17	0.0	0.2	0.8	0.1	0.6	0.3	0.2	0.5	0.3	0.2	0.7	0.1	0.0	0.3	0.7	**0.1**	**0.1**	**0.8**
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	**…**	**…**	
108	0.0	0.0	1.0	0.0	0.1	0.9	0.0	0.1	0.9	0.0	0.1	0.9	0.0	0.8	0.2	**0.0**	**0.0**	**1.0**

For example, the first row in Table 2 indicates the nonworking (x_1 = 0) and insufficient (x_1 = 1) states of generator A_1 with possibility of 0.8 and 0.2 respectively, while the possibility of the working state (x_1 = 2) is 0. In case of stable generator A_2, the state is indicated as nonworking (x_2 = 0) with possibility of 0.8 and as other values (x_2 = 1 and x_2 = 2) with possibilities of 0.1. States of main generator A_3, control unit U and the standby subsea cables L are defined similarly. The system state is interpreted as a failure for this components states with the possibility 0.7 ($\phi(\boldsymbol{x}) = 0$) and as the sufficient state ($\phi(\boldsymbol{x})$ = 1) with the possibility 0.3, while the state of perfect operation ($\phi(\boldsymbol{x}) = 2$) is not indicated since its possibility is 0.

The data obtained based on the monitoring and presented in Table 2 is interpreted as fuzzy data [21]. This data is incompletely specified because we have 108 of all 243 combinations of components states. In this paper, we suggest the new method for construction of the structure function based on an FDT. This method allows reducing indeterminate values and obtaining a completely specified structure function.

Therefore next step of the method is induction of FDT for representation of system mathematical model. A decision tree is a formalism for expressing mappings of input attributes (components states) to output attribute/attributes (system performance level), consisting of an analysis of attribute nodes (input attributes) linked to two or more sub-trees and leafs or decision nodes labeled with classes of the output attribute (in our case, a class agrees with a system performance level) [21]. An FDT is one of the possible types of decision trees that permit operating with fuzzy data (attributes) and that use methods of fuzzy logic. The uncertainty may be present in obtaining numeric values of the attributes (system components states) or in obtaining the exact class (system performance level) where the instance belongs to.

There are different methods for inducting an FDT [10, 22, 23]. An FDT induction is implemented by the definition of the correlation between n input attributes $\{A_1,\ldots, A_n\}$ and an output attribute B. The construction of the system structure function supposes that the system performance level is the output attribute and component states defined by a state vector are input attributes. Each input attribute (component state) A_i ($1 \le i \le n$) is measured by a group of discrete values ranging from 0 to m_i -1, which agree with the values of states of the i-th component: $\{A_{i,0},\ldots, A_{i,j},\ldots, A_{i,m_i-1}\}$. An FDT assumes that the input set $\mathbf{A} = \{A_1,\ldots, A_n\}$ is classified as one of the values of output attribute B. Value B_w of output attribute B agrees with one of the system performance levels and is defined as M values ranging from 0 to M -1 (w = 0,…, M -1). The correlation between the terminologies and basic concepts of FDTs and reliability analysis are shown in Table 3.

A fuzzy set A with respect to a universe U is characterized by a *membership function* $\mu_A : U \to [0,1]$, which assign an A-membership degree, $\mu_A(u)$, to each element u in U. $\mu_A(u)$ gives us an estimation that u belongs to A. The *cardinality measure* of the fuzzy set A is defined by $M(A) = \Sigma_{u\in U}\ \mu_A(u)$, and it is measure of size of set A. For $u \in U$, $\mu_A(u) = 1$ means that u is definitely a member of A and $\mu_A(u) = 0$ means that u is definitely not a member of A, while $0 < \mu_A(u) < 1$ means that u is a partial member of A. If either $\mu_A(u) = 0$ or $\mu_A(u) = 1$ for all $u \in U$, A is a crisp set. The set of input attributes A is crisp if $\mu_A(u) = 0$ or $\mu_A(u) = 1$.

For example, let us consider input attributes $\mathbf{A} = \{A_1, A_2, A_3, A_4, A_5\}$ and the output attribute B for the offshore electrical power generation system in Fig. 1. This system is represented by 5 input attributes. Each input attribute is defined as: $A_i = \{A_{i,0}, A_{i,1}, A_{i,2}\}$, for i = 1,…, 5, and the output attribute is $B = \{B_0, B_1, B_2\}$. The values of the input attributes and the output attribute are obtained based on the data from Table 2 and are used for the FDT construction as a training test. We propose to induct the FDT using the method based on the *cumulative information estimates* proposed in [20, 22]. These estimations allow inducting FDTs with various properties. Criteria for building non-ordered, ordered or stable FDTs, as well as, development of this method have been considered in [24].

Table 3. Correlation between the terminologies of FDTs and reliability analysis

FDT	System reliability
Number of input attributes: n	Number of the system components: n
Attribute A_i (i = 1,…, n)	System component x_i (i = 1,…, n)
Values of attribute A_i: $\{A_{i,0},\ldots, A_{i,j},\ldots, A_{i,m_i-1}\}$	State of component i: $\{0, \ldots, m_i\text{-}1\}$
Output attribute B	System performance level $\phi(\boldsymbol{x})$
Values of output attribute B: $\{B_0, \ldots, B_{M-1}\}$	Values of system performance level: $\{0, \ldots, M\text{-}1\}$
Decision table	Structure function

The FDT resulted from the training set presented in Table 2 has been inducted by application of the cumulative information estimates using the method in [23]. This FDT is presented in Fig. 2. The nodes of this FDT agree with the input attributes. Every node has 3 branches according to the values of the corresponding input attribute from the training test (Table 2). Every branch correlates with some values of the output attribute. The set of output attribute values in a branch is named as a leaf if the analysis finish and one of the values of the output attribute can be chosen according to algorithms proposed in [20, 24].

This FDT can be used for the analysis of all possible states of system components to construct the structure function of the offshore electrical power generation system. This process is considered below.

The construction of the structure function based on FDT is provided by the induction of decision table. According to [20], FDTs allow developing fuzzy decision rules or a decision table. A decision table contains all possible values of input attributes and the corresponding values of the output attribute that is calculated using the FDT. Such decision table agrees with the structure function. This implies that all possible combinations of values of the component states (all state vectors) have to be analyzed by the FDT to classify state vectors into M classes of the system performance levels.

Each non-leaf node is associated with an attribute

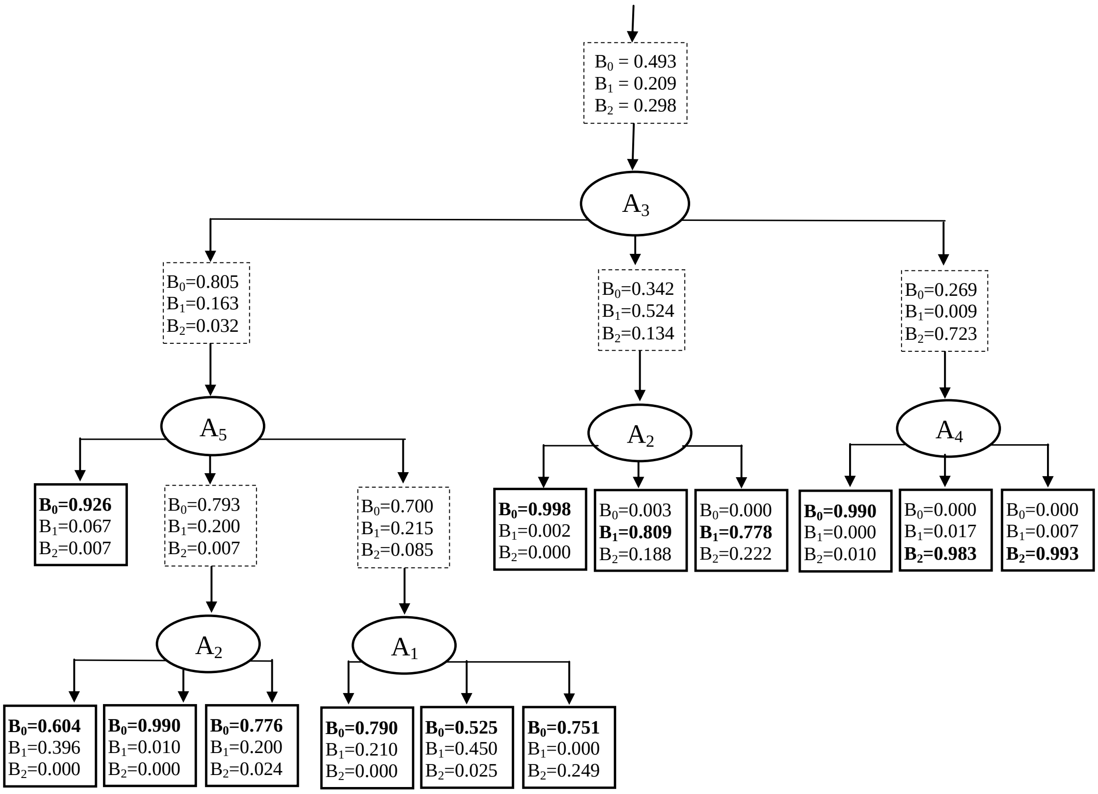

Fig.2 – Non-ordered FDT constructed based on the data obtained by the monitoring of the offshore electrical power generation system from Fig. 1

$A_i \in \mathbf{A}$, or in terms of reliability analysis: each non-leaf node is associated with a component. The non-leaf node agreeing with attribute A_i has m_i outgoing branches. The s-th outgoing branch (s = 0,…, m_i -1) from the non-leaf node corresponding to attribute A_i agrees with state s of the i-th component ($x_i = s$). A path from the root to a leaf defines one or more state vectors (according to the values of the input attributes (component states) occurred in the path) for which the structure function takes value determined by the value of the output attribute. If any input attribute is absent in the path, all possible states have to be considered for the associated component.

For example, consider construction of the structure function of the offshore electrical power generation system from Fig. 1 using the FDT depicted in Fig. 2. All possible component states (all state vectors) have to be used for calculation of the system performance level by the FDT to form the decision table (structure function). Let us explain this idea for the first level of the FDT in more detail.

Preliminary analysis of the data obtained based on the monitoring (see Table 2) shows that possible values of the output attribute B are distributed as follows: value 0 – with confidence 0.493, value 1 – with confidence 0.209 and value 2 – with confidence 0.298. These values are implied by frequency of every output value in the training test. Attribute A_3 is associated with the FDT root. So, analysis of the data starts from this attribute. Value $A_{3,0}$ of this attribute makes the output attribute B to be B_0 (the system is non-operational) with the confidence of 0.805. Other variants, B_1 and B_2, of output attribute B can be chosen with the confidence of 0.163 and 0.012 respectively. If the attribute A_3 has other values, i.e. $A_{3,1}$ or $A_{3,2}$, then the analysis is done similarly.

It is important to note that this method of construction of the structure function based on FDTs permits to compute (restore) data missing from the monitoring.

A representation of the system using the structure function allows calculating different indices and measures for estimation of system reliability. Probabilities of system performance levels (3) are one of them. Suppose that probabilities of the components states of the offshore electrical power generation system have values shown in Table 4. In this case, the probabilities of system performance levels are: A_2 = 0.73, A_1 = 0.20 and A_0 = 0.07. Other measures can be computed using the structure function too. For example, importance measures for this system can be calculated using the algorithms considered in [15, 24, 25].

Table 4. Components states probabilities

Component state, s	**Probabilities**				
	$p_{1,s}$	$p_{2,s}$	$p_{3,s}$	$p_{4,s}$	$p_{5,s}$
0	0.1	0.2	0.1	0.1	0.1
1	0.4	0.4	0.4	0.2	0.1
2	0.5	0.4	0.5	0.6	0.8

4. CONCLUSION

The new method for constructing the structure function is proposed in this paper. This method allows obtaining a structure function based on incompletely specified data (for example, data obtained from some monitoring). The term "incompletely specified" assumes uncertainties of two types.

The first type of uncertainty deals with some state vectors missing from the initial data. In practical application, it can be caused by the impossibility to obtain or indicate all possible combinations of system component states.

The second type of uncertainty results from ambiguity of initial data. In this case, the system performance level and components states can be defined with some possibilities. According to the typical definition of the structure function (1), performance level can have only one value for every state vector from set {0, …, M -1}. However, the boundary between two neighbouring values can be diffused in real applications. Both such values can be therefore indicated with some possibility. The proposed method takes such ambiguity into account and permits indicating performance level using some values ranging from 0 to M -1 with a possibility that is considered in the next steps of the method and is not disregarded.

5. ACKNOWLEDGEMENT

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