Product Stock Management: "Calculating stock provision with Triangular Fuzzy Numbers"

Adriana Santos Caballero [1], Jaime Gil-Lafuente [2] Maria Lluïsa Solé Moro [3]
Universidad de Barcelona, Av Diagonal, 690, 08034 Barcelona, http://www.ub.edu/ubbusiness
1) hadrisant@gmail.com; 2) j.gillafuente@gmail.com; 3) mlsolesole@ub.edu

***Abstract**: The aim of this paper is to analyse the distinctive characteristics of the Logistics System, namely: "Calculating stock provision" - in order to do this we have used data belonging to a company in the fashion sector. In order to carry out this investigation, the following sequence has been followed: firstly, the current content of the logistics function concept is defined through the study of its historical evolution in the academic and business sectors; after this evolutionary framework has been laid out, we proceed to identify the key elements characterized by uncertainty, in our approach we will be specifically using the Triangular Fuzzy Numbers technique, and then present a justification for the selection of the case. Finally, and fully in keeping with the above, we will propose a future line of investigation.*

Keywords: *Logistic function, Stock Management, Triangular Fuzzy Numbers*

1. INTRODUCTION

There is a fairly widespread opinion in the business world that the logistic function is one of the key factors in business management [1].

In the current environment, characterised by uncertainty, with increasingly globalised markets, and customer-oriented companies that are focused on their core businesses, where competition is no longer measured between companies, but rather between supply chains themselves, the design and organisation of the logistic function should be considered, from a strategic perspective, as a source of competitive advantages.

One of the most highlighted aspects of the activities performed in marketing refers to the process of getting the product to the consumer. This involves a set of heterogeneous and complex tasks, normally interconnected, that entail both time and costs. Three concepts, among others, stand out in this area: logistics, distribution and distribution channel. We will enter into this field to try and devise a technique relating to the uncertainty that is appropriate for dealing with some of the most significant problems presented by the current situation. To this end, we believe that the notion of "logistics" will be useful. [2]

This paper attempts to analyse the development of the logistics function, specifically Product Stock Management, using a fuzzy methodological approach, based on the knowledge of the key factors that characterise it.

2. THE LOGISTIC FUNCTION: DEFINITION AND DEVELOPMENT

Table 1. Summarises, in a simplified manner, the different meanings relating to the notion of the logistic function throughout history, beginning with the initial vision from the start of the 20th Century, which identified it exclusively as physical distribution within the marketing area, and then on to contemporary approaches, in which it acquired its maximum scope.

The variety of terminological meanings described in the literature review justifies the need to clarify the meaning adopted in this proposal. By taking into account the various contributions, we defined the logistic function as the planning, implementation and control of the physical flow of materials and associated information, which flows directly and inversely from the point of origin to the consumer, in order to meet customer demands.

In recent decades, as a result of globalisation and the intensive use of information technologies, the fashion industry has undergone a radical transformation. Specifically, the distance between the different actors involved in the manufacturing and distribution process has been reduced. This change has facilitated the emergence of a business model capable of meeting demand in a matter of weeks: fast fashion [3].

Table 1. Definitions of the logistic function.

Year	Author	Definition
1991	Schary and Coakley[4]	The management of goods and services, and related information, from point of origin to point of consumption.
1992	Christopher [5]	A strategic management process for the procurement, movement and storage of materials, intermediate products and finished products, and related information flows.
1998	Anaya [6]	The control of the flow of materials from the source of supply until the product is positioned at the point of sale.
2003	Council of Logistics Management [7]	Aspect of channel management that plans, implements and controls the efficient and effective management of forward and reverse flow of goods, services and related information between the point of origin and the consumer, in order to satisfy customer demands.
2006	Bowersox,	The responsibility for

	Closs, and Bixby [8]	designing and managing motion control systems and the geographical position of the flow of materials, intermediate products and finished products at the lowest possible cost.
2012	Council of Supply Chain Management Professionals [9]	An aspect of supply chain management that plans, implements and controls the efficient and effective management of forward and reverse flow and storage of goods, services and related information between the point of origin and the consumer, to satisfy customer requirements

Source: Adapted from [10] and [11]

3. PRODUCT STOCK MANAGEMENT

The movement of a product from its manufacture to its acquisition by the consumer, but in which, on some occasions, flow concepts and fund concepts alternate. One of the latter is known by the term “Stock” and relates to the accumulation of material in certain common points called ‘warehouses’. The economic study of the accumulation and decumulation of these amounts of product has been designated with the name "stock management"

We are proposing a simple way to deal with this problem. Some operational and management research exists that has explored the issue and shed light upon interesting models [12]. We will limit ourselves to reproducing one of the simplest schemes in the field of certainty to later transform it so that it is suitable for use in the field of uncertainty.

The fundamental elements of stock management are summarised in Table 2. In the following manner:

Table 2. Elements of stock management.

1. There is a finite number of wholesale and/or retail outlets that make orders, and these must be attended to within a given period of time.
2. The demand of the point can be known, with certainty, for each unit of time, in terms of probability through a statistically stable or unstable demand (stationary). Or in an uncertain manner.
3. Product outputs exhaust stock levels in warehouses if resupply does not take place in infinitely small time frames (continuum hypothesis) or more or less large finite time frames (discrete hypothesis)
4. The supply and replenishment of materials to create stock in warehouses incurs a cost every time this event takes place. The less replenishment performed in a given economic horizon, the lower the overall cost of this concept.
5. The stock of a quantity of product in a warehouse implies a cost, and the greater the amount deposited the larger the cost will be.
6. Optimization occurs when the sum of the two types of overall cost is minimised for a length of time spanning the economic horizon.
7. The variable to be found may be the amount of product to be replenished in each period and / or the time or times when the replenishment takes place.

Source: adapted from [12]

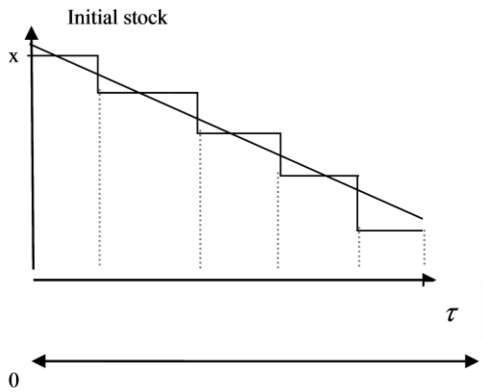
4. METHODOLOGY

The movement of stock within a finite period T that is not replenished, is produced by periodic outputs in different amounts as a consequence of wholesale or retail outlet orders, as represented in Figure 1.

Note the development of stocks that go from an initial stock at time 0 to a final stock at time T (the interval [0,T] is considered to be a supply period) in the form of an irregular staircase due to the height of the step and its scope. Along the economic horizon several periods typically occur that may or may not have the same duration. Evidently, it suggests that a theoretical model on this basis has little chance of success. Hence the adoption of a first hypothesis: "Stocks are uniformly reduced through a continuous function".

Considering the first hypothesis as accepted, we move into the replenishment problem, in other words, that of rebuilding the initial stock. Typically, from the time the shipment of the product is requested until its arrival at the warehouse at a time *t*.

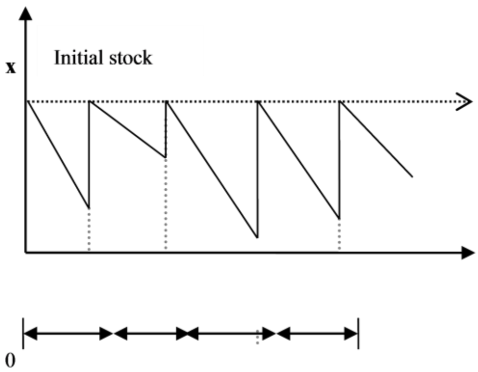
Fig.1 Hypothesis I



Source: adapted from [13].

Hypothesis I, of a constant replenishment time: For each certain number of units of time (days, for example), an amount of product is requested that always arrives during the same interval [0,*T*]. This theory can be seen in Fig. 2.

Fig. 2 Second hypothesis



Source: adapted from [14]. The times T - t, 2T - t, 3T – t, denote the request of the input batch. We can see that although at times *T*, *2T*, the arrival of the product gives rise to a stock that is constantly equal to the maximum, before it enters the warehouse minimum risk "points" are created, because a delay in the arrival of the product could lead to an inability to fulfil orders from wholesalers, with the consequent economic loss, in other words, the possibility exists of being "out of stock". As a counterpoint, the simplicity of the replenishment mechanism makes the adoption of this hypothesis very attractive.

Hypothesis II: Constantly replenished amount. When the stock level reaches a certain level, the input batch always comprises the same amount of product. As shown in fig.2. During the times *T1 – t*, *T1 + T2 – t*,….., the request for delivery of the product batch is made, and delivery will take place at *T1, T1 + T2*,….

In this case, being out of stock is not strictly possible, however, the systematisation of batch delivery times is very difficult when developing a general model.

Hypothesis III: replenishment request at a constant stock level.

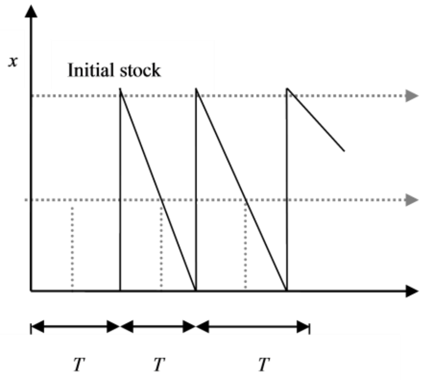
The batch send request occurs at times *T1 – t, T1 + T2 – t*, …., at a non-uniform rate, but, for this specific hypothesis, always at the same level and with the same amount of product [15].

Perhaps it is this last hypothesis that allows for an easier administrative organisation due to the easy automation of the issuance of input requests.

We have dismissed some general approaches in order to present an overall picture of what are deemed to be the 'classic' problems in stock management. We have one final hypothesis to present that, thanks to its restrictive nature, signifies a simplification, which constitutes the almost obligatory first step when embarking on a study of stock management in virtually all the specialist texts on this matter. We are of course referring to Wilson's model. Given the purpose of this paper, the transit of numerical and non-numerical marketing towards uncertainty.

Hypothesis IV: Constant demand. This assumption implies knowledge of orders received by intermediaries and that are of a regular nature, in the sense that continuous demand means that the same amount of product leaves the warehouse in each unit of time. This hypothesis allows for a joint solution to assumptions I and II. As indicated graphically in Fig. 3[1].

Fig.3. Hypothesis IV



Source: adapted from [16]

The warehouse supply period always takes place at the same time *T – t, 2T – t*,….

We can see that the slope of the straight line representing a decline in stock is always the same in each period *T*, which is deduced from the hypothesis raised of constant demand.

J. Gil-Lafuente cites the configurator elements of the model in his work 'Marketing for the New Millennium', as shown in Table 3.

Table 3. Model configurators

a) The batch input to the warehouse is constant in each period and is equal to *n*.
b) We will designate with a *T*, as we have done so far, the period of time that covers each input batch clearly along the economic horizon.
c) The costs of supply management are independent of the requested amount (input batch) and are increasingly equal along the economic horizon; we call them C^g .
d) Product requirements for the entire economic horizon are *N* units.
e) The storage cost of a product unit for economic horizon *N* is an average value that is also considered a constant and we call it C^a .
f) The duration of the economic horizon is a period of time equal to Θ .
g) The number of times that replenishment takes place along the economic horizon will be designated by *r*.

Source: adapted from [17]

Our aim is to determine the batch size *n* (the number of product units that it comprises) so that the overall cost of supply and storage is minimal. Given the conditions of the model, obtaining the "economic batch" $n^{(opt)}$ automatically provides the "optimal replenishment number" $r^{(opt)}$.

We can quickly see the certainty scheme in Table 4 as the model develops.

[1] Assuming the existence of a safety stock does not substantially change the model.

Table 4. Model equations scheme

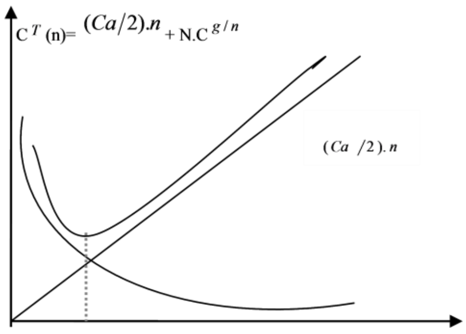
1.The measured stock of the batch inputs into the warehouse, given the linearity of the output, will be:	$\frac{(n+0)}{2} = \frac{n}{2}$
2.The total storage cost for the entire economic horizon *N* is:	$\frac{n}{2} \cdot C_a$
3.The replenishment number will be obtained by dividing the necessary amount of product for the entire economic horizon, *N*, by the size of each batch *n*, i.e.*:*	$V = \frac{N}{n}$
4. The total cost of supply management for the entire economic horizon *N* is:	$C_g \frac{N}{n}$
Therefore, the total cost to optimise for the economic horizon is	$C_T(n) = \frac{n}{2} C_a + \frac{N}{n} C_g$

Source: adapted from [18]

The value to be optimised (in this case, minimized) is the sum of two variable quantities $\frac{n}{2} C_a$ and $\frac{N}{n} C_g$. It can be observed that the product of these two quantities is a constant (here the only variable is *n).*

As is well known, "the minimum of the sum of two variable quantities whose product is a constant only occurs when these quantities are equal.

In other words: $n^{(opt)} = \sqrt{2 C_g \frac{N}{C_a}}$



Source: adapted from [19]

This same result can be obtained by differentiating $C_T(n)$ with respect to *n* and equating it to 0. In Effect:

$$\frac{dC_T(n)}{dn} = \frac{C_a}{2} + \frac{-N.C_g}{n^2} = 0$$

6. RESULTS

Considering this simple treatment of the problem of stock management in terms of certainty, we can move on to the area of uncertainty. To this end, we assume that certain elements can only be estimated using uncertain numbers. For example, in this paper we will contribute to this technique by working with triangular fuzzy numbers [20].

As already expressed:

$$\hat{C}_g = (C_g^{(1)}, C_g^{(2)}, C_g^{(3)})$$
$$\hat{C}_a = (C_a^{(1)}, C_a^{(2)}, C_a^{(3)}) \quad (1)$$

Where $\hat{C}_g$ supply management costs, $\hat{C}_a$ storage costs

And in the form of α - cuts:

$$C_g^{(\alpha)} = [C_g^{(1)} + (C_g^{(2)} - C_g^{(1)})\alpha, C_g^{(3)} - (C_g^{(3)} - C_g^{(2)})\alpha]$$
$$C_a^{(\alpha)} = [C_a^{(1)} + (C_a^{(2)} - C_a^{(1)})\alpha, C_a^{(3)} - (C_a^{(3)} - C_a^{(2)})\alpha]$$

If the optimisation formula is presented by separating the certain elements from the uncertain elements, it will be:

$$n^{(opt)} = \sqrt{2N} (\bullet) \sqrt{C_g / C_a}$$

Therefore it remains as a general formula for uncertainty expressed by *NTB:*

$$n^{(opt)} = \sqrt{2N} (\bullet) \left[\sqrt{\frac{[C_g^{(1)} + (C_g^{(2)} - C_g^{(1)})\alpha}{[C_a^{(3)} - (C_a^{(3)} - C_a^{(2)})\alpha}}, \sqrt{\frac{C_g^{(3)} - (C_g^{(3)} - C_g^{(2)})\alpha]}{C_a^{(1)} + (C_a^{(2)} - C_a^{(1)})\alpha]}} \right]$$

In order to demonstrate the utility of this expression, values are given to the quantities involved in $\hat{n}^{(opt)}$**.** That is why the component is called certain $\sqrt{A}$ and the behaviour uncertain $\sqrt{\hat{B}}$; it will be

$n^{(opt)} = \sqrt{A} (\bullet) \sqrt{\hat{B}}$ N = 100,000

$\hat{C}_g$ = **(**120,000, 140,000, 150,000**)**

$\hat{C}_a$ = (15,000, 17,500, 18,000)

For the certain part: $\sqrt{A}$ =150

For the uncertain part:

$$\sqrt{\hat{B}} = \left[\sqrt{\frac{[120.000 + 20.000\alpha}{[18.000 - 500\alpha}}, \sqrt{\frac{150.000 - 10.000\alpha}{15.000 + 2.500\alpha}} \right]$$

The endecadaria scale [21] will be used for $\alpha \in [0,1]$ in order to present a result with a clearer view of the size of the supply batch.

$$\left[150 \bullet \sqrt{\frac{[120.000 + 20.000\alpha}{[18.000 - 500\alpha}}, 150 \bullet \sqrt{\frac{150.000 - 10.000\alpha}{15.000 + 2.500\alpha}} \right]$$

1	424.26	
.9	420.62	428.85
.8	416.97	433.52
.7	413.31	438.28
.6	409.63	443.13
.5	405.94	448.07
.4	402.24	453.11
.3	398.52	458.26
.2	394.80	463.51
.1	391.06	468.86
0	387.30	474.34

As described by Gil-Lafuente, [20], the results obtained using this fuzzy number show that the optimal supply batch will be between 387.30 and 474.34 units of the product with a maximum output of 424.26 units. if the corresponding triangular approximation is assumed to be correct, after rounding up the numbers determined by treating them as product units, the amount will be:

$$\hat{n}^{(opt)} = \mathbf{(387, 424, 474)}$$

7. CONCLUSION

If the fuzzy number found as a result would give rise, as in this case, to a poor dimensioning of uncertainty, it would be advisable to use counter-expertise by any of the known methods, for example, through the R^+-expertons [22]

8. FUTURE LINE OF INVESTIGATION

As a future line of investigation in order to improve the model through fuzzy methodology, we propose working with triangulation.

Triangulation is a basic principle that is commonly followed in order to enhance the reliability and validity of the investigation [23]. Triangulation is generally conceived of as a means of protection against the investigator's tendencies and for confronting and submitting the accounts of different informants to a reciprocal control [24].

In order to obtain the triangulation of the information, we anticipate using a combination of multiple sources of evidence during the data collection phase [25] & [26]. This would merely be the beginning of a path that could lead to extraordinary results. The progress made in recent years in the field of stock management makes us think of the many possibilities offered by the arithmetic and non-numerical mathematics of uncertainty. And not only as a result of the transformation of the traditional models supplied by Operational Research, but rather by incorporating the uncertain elements of the new philosophies on the supply of materials, among which "just-in-time" [27] appears to be successful.

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