ANALYSIS AND APPLICATION OF G-NETWORK WITH INCOMES AND RANDOM WAITING TIME OF NEGATIVE CUSTOMERS

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Abstract

In the article an open Markov queueing G-network with incomes and random waiting time of negative customers has been considered. Negative customers destroy positive customers on the expiration of a random time. Queueing system (QS) receives a certain random income when positive customer arrives to it and loss when negative customer arrives to it. A technique for finding the expected incomes of the network QS has been proposed. In information systems and networks negative customers may describe behavior of requests for service, at which request is a command to stop the operation being performed or the behavior of computer viruses, the effects of which on the information (positive customer) occurs through a random time and has a damaging effect.

1 Introduction

Consider an open G-network [1] with n single-queues QS: $S_1,\ S_2,\ ...,\ S_n$. Lets introduce system S_0, from which Poisson flow of customers arrive to the network. The network state at time t described by the vector $k\,(t) = ((k_1(t), l_1(t))\,, ..., (k_n(t), l_n(t)))$, which forms a homogeneous Markov process with a countable number of states, where the state $(k_i(t), l_i(t))$ means, that at time t in QS S_i there are k_i positive customers and l_i negative customers, $i = \overline{1, n}$. We introduce the vectors $k\,(t) = (k_1\,(t), k_2(t), \ldots\,, k_n(t))$ and $l\,(t) = (l_1\,(t), l_2(t), \ldots\,, l_n(t))$.

External arrivals to the network, service times of rates and probabilities of customer transitions between QS depend on time, [2]. In QS S_i from the outside (from the system S_0) is coming a Poisson stream of the positive customers with the intensity $\lambda_{0i}^{+}(t)$ and Poisson stream of negative customers with the intensity $\lambda_{0i}^{-}(t)$, $i = \overline{1,\ n}$. All flows of customers incoming to the network are independent. Lets $\mu_i^{+}\,(k_i(t))$ – service rate of positive customers in QS S_i at time t, depend on count of customers at it system, $i = \overline{1,\ n}$. If in QS S_i at time t there are $k_i\,(t)$ customers, then the probability, that the positive customer serviced in QS S_i during time $[t, t + \Delta t)$, are equals $\mu_i^{+}\,(k_i(t))\,\Delta t + o\,(\Delta t)$. Positive customer, get serviced in S_i at time t with probability $p_{ij}^{+}(t)$ move to QS S_j as a positive customer, and with probability $p_{ij}^{-}(t)$ – as a negative customer, and with probability $p_{i0}(t) = 1 - \sum_{j=1}^{n}\left[p_{ij}^{+}(t) + p_{ij}^{-}(t)\right]$ come out from the network to external environment, $i,\ j = \overline{1,\ n}$.

Negative customer is arrived to QS increases the length of the queue of negative customers for one, and requires no service. Each negative customer, located in i-th

QS, stay in the queue random time according to a Poisson process of rate $\mu_i^- (l_i)$, $i = \overline{1,\ n}$. By the end this time, negative customer destroy one positive customer in the QS S_i and leave the network. If after this random time in the system there are no positive customers, then given negative customer leave network, without exerting any influence on the operation of the network as a whole. Wherein probability that, in QS S_i negative customer leave queue during $[t, t + \Delta t)$, on condition that, in this QS at time t there are l_i negative customers, equals $\mu_i^- (l_i) \Delta t + o(\Delta t)$.

2 Finding expected incomes of network systems

Consider the dynamics of income changes of a network system S_i. Denote by the $V_i(t)$ its income at moment time t. Let the initial moment time income of the system equal $V_i(0) = v_{i0}$. The income of its QS at moment time $t + \Delta t$ can be represented in the form

$$V_i(t + \Delta t) = V_i(t) + \Delta V_i(t, \Delta t), \tag{1}$$

where $\Delta V_i(t, \Delta t)$ – income changes of the systemS_i at the time interval $[t, t + \Delta t)$, $i = \overline{1, n}$. To find its value we write down the value of the conditional probabilities of events that may occur during Δt and the income changes of its QS, associated with these events:

1. With probability $p_i^{(1)} (t, \Delta t) = \lambda_{0i}^+(t)\Delta t + o(\Delta t)$ at moment time t to the system S_i from the external environment will come positive customer, which will bring an income to the amount of r_{0i}, where r_{0i} – random variable (RV), expectation (E) which is equals $E\ \{r_{0i}\} = a_{0i}$, $i = \overline{1, n}$.

2. With probability $p_i^{(2)} (t, \Delta t) = \lambda_{0i}^-(t)\Delta t + o(\Delta t)$ in the QS S_i at moment time t from the external environment will come a negative customer, $i = \overline{1, n}$; income change of this system this case will not occur.

3. If at the moment time t at the system S_i is located $k_i(t)$of positive customers, then with probability $p_i^{(3)} (t, \Delta t) = \mu_i^+ (k_i(t))\, u(k_i(t))\, p_{i0}(t)\Delta t + o(\Delta t)$, where $u(x)-$ Heaviside function, positive customer comes out from the network to the external environment, while the total amount of income of QS S_i is reduced by an amount which is equal to $-R_{i0}$, where $E\{R_{i0}\} = b_{i0}$, $i = \overline{1,\ n}$, $\mu_i^+ (0) = 0$.

4. With probability $p_i^{(4)} (t, \Delta t) = \mu_i^- (l_i(t))\, u(l_i(t))\, \Delta t + o(\Delta t)$ in QS S_i at the moment time t negative customer, destroying positive customer in QS S_i, will leave the network, $i = \overline{1, n}$; In this case income for the system S_i decreases by an amount $-R_{i0}^{neg}$, $E\{R_{i0}^{neg}\} = b_{i0}^{neg}$, $\mu_i^- (0) = 0$, $i = \overline{1, n}$.

5. If at the moment time t in QS S_i there were $l_i(t)$ negative customers and there were not positive customers, then negative customer leaves this QS with probability $p_i^{(5)} (t, \Delta t) = \mu_i^- (l_i(t))\,(1 - u(k_i(t)))\,\Delta t + o\ (\Delta t)$, $i = \overline{1, n}$. In this case income change of QS S_i will not occur, $i = \overline{1, n}$.

6. If at the moment time in the system S_i there is positive customer, then after finishing it servicing in QS S_i it move to QS S_j as a positive customer With probability $p_i^{(6)} (t, \Delta t) = \mu_i^+ (k_i(t))\, u(k_i(t))\, p_{ij}^+(t)\Delta t + o(\Delta t)$, $i, j = \overline{1,\ n}$, $i \neq j$; in such a transition

income of system S_i decreases by an amount R_{ij}, and income of system S_j will increase by this amount, where $E\{R_{ij}\} = a_{ij}$, $i, j = \overline{1, n}$, $i \neq j$.

7. If at the moment time t in system S_j there is positive customer, then serviced in S_j, it will move to system S_i with probability $p_i^{(7)}(t, \Delta t) = \mu_j^+(k_j(t))\, u(k_j(t))\, p_{ji}^+(t)\Delta t + o(\Delta t)$, at the same time income of QS S_i decreases by an amount r_{ji}, and income of QS S_j it will increase by the same amount, where $E\{r_{ji}\} = b_{ji}$, $i, j = \overline{1, n}$, $i \neq j$.

8. With probability $p_i^{(8)}(t, \Delta t) = \mu_i^+(k_i(t))\, u(l_i(t))\, p_{ij}^-(t)\Delta t + o(\Delta t)$ positive customer, serviced in QS S_i, at the moment time t forwarded to QS S_j as negative customer $i, j = \overline{1, n}$, $i \neq j$; in such a transition system income of S_i decreases by an amount R_{ij}^{neg}, and income of system S_j will not change, where $E\{R_{ij}^{neg}\} = a_{ij}^{neg}$, $i, j = \overline{1, n}$, $i \neq j$.

9. With probability

$$p^{(9)}(t, \Delta t) = 1 - \left\{ \lambda_{0i}^+(t) + \lambda_{0i}^-(t) + \mu_i^+(t)p_{i0}^+(t) + \mu_i^-(t) + \sum_{j=1}^{n} \mu_i^+(t)p_{ij}^+(t) + \right.$$

$$\left. + \sum_{j=1}^{n} \mu_j^+(t)p_{ji}^+(t) + \sum_{j=1}^{n} \mu_i^+(t)p_{ij}^-(t) \right\} \Delta t + o(\Delta t)$$

on time interval $[t, t + \Delta t)$ there will be no change of system S_i nothing is going to happen (not a positive customer or a negative customer is received and no customer is serviced), in this case, the total income of S_i may increase (decrease) to the amount of $r_i \Delta t$, where $E\{r_i\} = c_i$, $i = \overline{1, n}$.

It's obvious that $r_{ji}(\xi_j) = R_{ji}(\xi_j)$ with probability 1, i.e. $b_{ji} = a_{ji}, i, j = \overline{1, n}$. Suppose that at any instant of time RV r_{0i}, R_{i0}, R_{i0}^{neg}, R_{ij}, r_{ji}, R_{ij}^{neg} does not depend on RV r_i.

We will assume, that all network systems are single-queues and customers service rates in QS S_i has an exponential distribution with rate $\mu_i^+(t)$; let also negative customer, arrives to QS S_i, will leave it queue after a random time, that has an exponential distribution with rate $\mu_i^-(t)$. Consequently, in these cases, we obtain, that $\mu_i^+(k_i(t)) = u(k_i(t))\, \mu_i^+(t)$, $\mu_i^-(l_i(t)) = u(l_i(t))\, \mu_i^-(t)$, $i = \overline{1, n}$.

In addition suppose, that all systems operating under heavy-traffic regime, i.e. $k_i(t) > 0$ $\forall t > 0$, $i = \overline{1, n}$. In the simulation, the effect of virus penetration into a computer network or during a computer attack on it occurs just such a situation. Also, we assume, that $l_i(t) > 0$ $\forall t > 0$, $i = \overline{1, n}$. Then it follows from the foregoing

$$\Delta V_i(t, \Delta t) = \begin{cases} r_{0i} + r_i \Delta t \text{ with probability } \lambda_{0i}^+(t)\Delta t + o(\Delta t), \\ -R_{i0} + r_i \Delta t \text{ with probability } \mu_i^+(t)p_{i0}^+(t)\Delta t + o(\Delta t), \\ -R_{i0}^{neg} + r_i \Delta t \text{ with probability } \mu_i^-(t)q_{i0}^-(t)\Delta t + o(\Delta t), \\ -R_{ij} + r_i \Delta t \text{ with probability } \mu_i^+(t)p_{ij}^+(t)\Delta t + o(\Delta t), \\ r_{ji} + r_i \Delta t \text{ with probability } \mu_j^+(t)p_{ji}^+(t)\Delta t + o(\Delta t), \\ -R_{ij}^{neg} + r_i \Delta t \text{ with probability } \mu_i^+(t)p_{ij}^-(t)\Delta t + o(\Delta t), \\ r_i \Delta t \text{ with probability } 1 - \left\{\lambda_{0i}^+(t) + \lambda_{0i}^-(t) + \right. \\ + \mu_i^+(t)p_{i0}^+(t) + \mu_i^-(t) + \sum_{j=1}^{n} \mu_i^+(t)p_{ij}^+(t) + \\ \left. + \sum_{j=1}^{n} \mu_j^+(t)p_{ji}^+(t) + \sum_{j=1}^{n} \mu_i^+(t)p_{ij}^-(t) \right\} \Delta t, j = \overline{1, n},\ j \neq i\,. \end{cases} \quad (2)$$

Therefore, taking into account (2) $E\left\{\Delta V_i\left(t,\,\Delta t\right)\right\} = f_i(t)\Delta t + o\left(\Delta t\right)$, where

$$f_i(t) = \left(\lambda_{0i}^{+}(t) + \lambda_{0i}^{-}(t)\right) a_{0i} + \sum_{j=1}^{n}(\mu_j^{+}(t)p_{ji}^{+}(t)b_{ji}) +$$

$$+ \sum_{j=1}^{n} p_{ij}^{-}(t)\left[a_{ij}^{neg}\mu_i^{+}(t) + c_i\left(\mu_i^{+}(t) + \mu_i^{-}(t)\right)\right] +$$

$$+ \sum_{j=1}^{n}\left[p_{ij}^{+}(t)\mu_i^{+}(t)(2c_i - a_{ij})\right] - b_{i0}\mu_i^{+}(t)p_{ij}^{+}(t) + b_{i0}^{neg}\mu_i^{-}(t) + c_i.$$

For $v_i\left(t\right) = M\left\{V_i\left(t\right)\right\}$ from (1) we have $v_i\left(t + \Delta t\right) = v_i(t) + E\left\{\Delta V_i\left(t,\,\Delta t\right)\right\}$, where, passing to the limit $\Delta t \to 0$, we get linear inhomogeneous differential equations of first order $\frac{dv_i(t)}{dt} = f_i\left(t\right)$, $i = \overline{1,\,n}$, i.e.

$$\frac{dv_i\left(t\right)}{dt} = \left(\lambda_{0i}^{+}(t) + \lambda_{0i}^{-}(t)\right) a_{0i} + \sum_{j=1}^{n}(\mu_j^{+}(t)p_{ji}^{+}(t)b_{ji}) +$$

$$+ \sum_{j=1}^{n} p_{ij}^{-}(t)\left[a_{ij}^{neg}\mu_i^{+}(t) + c_i\left(\mu_i^{+}(t) + \mu_i^{-}(t)\right)\right] +$$

$$+ \sum_{j=1}^{n}\left[p_{ij}^{+}(t)\mu_i^{+}(t)(2c_i - a_{ij})\right] - b_{i0}\mu_i^{+}(t)p_{ij}^{+}(t) + b_{i0}^{neg}\mu_i^{-}(t) + c_i.$$

By setting the initial conditions $v_i(0) = v_{i0}$, $i = \overline{1,\,n}$, we can find the expected incomes of the network systems. In this way

$$v_i\left(t\right) = v_{i0}\left(0\right) + \int_0^t f_i\left(\tau\right)d\tau,\ \ i = 1, \ldots, n.$$

References

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