CALCULATION OF EUROPEAN OPTIONS WITH ABSOLUTE CRITERIA

N. M. Zuev
Belarusian State University
Minsk, BELARUS
e-mail: ZuevNM@bsu.by

Abstract

The problem of European options calculation is considered. The recurrent equations for the major characteristics are obtained.
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1 Introduction

The problem of European options calculation is an important problem not only from the practical point, but also for the theory. In this paper we consider a (B, S)-market [1] and construct a sequence of recurrent equations for calculation of major characteristics of European type options for a self-financing portfolio.

2 Main Result

Let S_n be the cost of a risky active unit at the time moment n. Suppose it follows the model:

$$S_n = S_0(1 + \rho_1) \dots (1 + \rho_n),$$

where ρ_k, $k = 1, \dots, n$, are the interest rates with stochastic changes. Let B_n be the cost of a non-risky active unit at the time moment n that depends on the random variables $\rho_1, \dots, \rho_{n-1}$ only. Denote by $\pi_n = (\beta_n, \gamma_n)$ the portfolio at the time moment n, where β_n is the number of non-risky active units at the time moment n, γ_n is the number of risky active units at the time moment n, $n = 1, \dots, N$; N is the terminal moment, at this moment the option is executed. Denote by f_N the payment function that depends on random variables $\rho_1, \dots, \rho_N$ only. In case of a standard purchase of the European option, $f_N = (S_N - K)^+$ are the losses of the option seller for a risky active unit. Let K be the contract price for the purchase of a risky active unit at the time moment N. The variables β_n, γ_n are under prediction, and are supposed to depend on $\rho_1, \dots, \rho_{n-1}$ only. Let $X_n = \beta_n B_n + \gamma_n S_n$ be the portfolio cost at the time moment n.

The problem of calculation of a European option is concentrated on the choice of the starting capital $X_0 > 0$ (the option cost) and the portfolio $\pi_n = (\beta_n, \gamma_n)$, $n = 1, \dots, N$, so that $X_N - f_N = 0$ would be minimal.

In [2] – [4] the formulae to calculate the options with quadratic criteria are obtained.

In this paper we give the formula for calculation of options while minimizing $E\{|X_N - f_N|\}$, the minimum is 0.

Let us denote: $\tilde{X}_n = X_n/B_n$, $\tilde{S}_n = S_n/B_n$, $\tilde{f}_n = f_n/B_n$.

Theorem 1. *For the self-financing portfolio $\pi_n = (\beta_n, \gamma_n)$, $n = 1, \dots, N$, the values X_0, β_n, γ_n are calculated recursively from the following equations:*

$$E\left\{|\tilde{f}_{n-1} + \gamma_n \Delta(\tilde{S}_n) - \tilde{f}_n| / \bar{\rho}_{n-1}\right\} = \min, \tag{1}$$

for $n \leq N$;

$$\beta_n = \beta_{n-1} - \tilde{S}_{n-1}(\gamma_n - \gamma_{n-1}), \ \tilde{X}_0 = \tilde{f}_0, \tag{2}$$

$$\tilde{f}_{n-1} = E\{\tilde{f}_n \mid \bar{\rho}_{n-1}\} - \gamma_n E\{\Delta(\tilde{S}_n) \mid \bar{\rho}_{n-1}\}, \ n = 1, \dots, N.$$

In (1) the expectation is taken conditionally w.r.t. $\bar{\rho}_{n-1} = (\rho_1, \dots, \rho_{n-1})$.

Proof. In case of a self-financing portfolio the value of X_n follows the law: $X_n = \beta_n B_n + \gamma_n S_n = \beta_{n+1} B_n + \gamma_{n+1} S_n$. From it we get $\frac{X_{n+1}}{B_{n+1}} - \frac{X_n}{B_n} = \gamma_{n+1}\left(\frac{S_{n+1}}{B_{n+1}} - \frac{S_n}{B_n}\right)$, or

$$\Delta\left(\frac{X_n}{B_n}\right) = \gamma_n \Delta\left(\frac{S_n}{B_n}\right).$$

From here we obtain:

$$\frac{X_N}{B_N} = \frac{X_0}{B_0} + \sum_{n=1}^{N} \gamma_n \Delta\left(\frac{S_n}{B_n}.\right) \tag{3}$$

From (3) we get $\tilde{X}_N - \tilde{f}_N = \tilde{X}_0 - \tilde{f}_0 + \sum_{n=1}^{N}\left(\tilde{f}_{n-1} + \gamma_n \Delta(\tilde{S}_n) - \tilde{f}_n\right)$, where the functions $\tilde{f}_n$, $n = 1, \dots, N-1$ that depend on $\bar{\rho}_n$, are chosen below. □

Choose the values $\tilde{f}_{n-1}$ and γ_n from relations (1). From (1), (2) we find $\tilde{f}_n$, γ_n, β_n for $n \leq N$. The value of $\tilde{X}_0$ is set to $\tilde{f}_0$. The values β_n are found from (2) and the portfolio cost at the time moment n.

In case where the random variables ρ_n take only two values for all n, equation (1) for arbitrary probabilities of these values turn into two linear equations $\tilde{f}_{n-1} + \gamma_n \Delta(\tilde{S}_n) - \tilde{f}_n = 0$ with $\tilde{f}_{n-1}$ and γ_n unknown.

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