STATISTICAL INFERENCE FOR RANDOM FIELDS IN THE SPECTRAL DOMAIN BASED ON TAPERED DATA

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Let $X(t)$, $t \in I$, be a real-valued measurable strictly stationary zero-mean random field, where I is R^d or Z^d endowed with the measure $\nu(\cdot)$ which is the Lebesgue measure or the counting measure ($\nu(\{t\}) = 1$) respectively. Suppose that all order moments exist and the field $X(t)$ has spectral densities of all orders $f_k(\lambda_1, ..., \lambda_{k-1}) \in L_1(S^{k-1})$, $k = 2, 3, ...,$ where $S = R^d$ or $(-\pi, \pi]^d$ for the continuous-parameter or discrete-parameter cases respectively.

Let the field $X(t)$ be observed over the domain $D_T = [-T, T]^d \subset I$. Consider the problem of estimation of integrals of cumulant spectra of orders $k = 2, 3, ...$

$$J_k(\varphi_k) = \int_{S^{k-1}} \varphi_k(\lambda) f_k(\lambda) d\lambda \tag{1}$$

for appropriate functions $\varphi_k(\lambda)$ with $\varphi_k(\lambda) f_k(\lambda) \in L_1(S^{k-1})$. The functionals (1) can be used to represent some characteristics of stochastic processes and fields in nonparametric setting and also appear in the parametric estimation in the spectral domain, e.g., when the minimum contrast (or quasi-likelihood) estimators are studied.

We will base our analysis on tapered data $\{h_T(t) X(t), t \in D_T\}$, where $h_T(t) = h(t/T)$, $t \in R^d$, and the taper $h(t)$ satisfies some conditions. The use of tapers leads to the bias reduction of estimates, which is important when dealing with spatial data: tapers can help to fight the so-called "edge effects".

Denote $H_{k,T}(\lambda) = \int h_T(t)^k e^{-i(\lambda,t)} \nu(dt)$ and define the finite Fourier transform of tapered data: $d_T^h(\lambda) = \int h_T(t) X(t) e^{-i(\lambda,t)} \nu(dt)$, $\lambda \in S$, the tapered periodograms of the second and the third orders:

$$I_{2,T}^h(\lambda) = \frac{|d_T^h(\lambda)|^2}{(2\pi)^d H_{2,T}(0)}, \quad I_{3,T}^h(\lambda_1, \lambda_2) = \frac{d_T^h(\lambda_1) d_T^h(\lambda_2) d_T^h(-\lambda_1 - \lambda_2)}{(2\pi)^{2d} H_{3,T}(0)}$$

(provided that $H_{2,T}(0) \neq 0$, $H_{3,T}(0) \neq 0$) and the tapered periodogram of k-th order:

$$I_{k,T}^h(\lambda_1, ..., \lambda_{k-1}) = \frac{1}{(2\pi)^{(k-1)d} H_{k,T}(0)} \prod_{i=1}^{k} d_T^h(\lambda_i), \lambda_i \in S,$$

(provided that $H_{k,T}(0) \neq 0$), where $\sum_{i=1}^{k} \lambda_i = 0$, but no proper subset of λ_i has sum 0.

We consider the empirical spectral functional of k-th order

$$J_{k,T}(\varphi_k) = \int_{S^{k-1}} \varphi_k(\lambda) I_{k,T}^h(\lambda) d\lambda. \tag{2}$$

as an estimate for the spectral functional (1). We discuss the questions: (i) evaluation of bias and (ii) conditions for asymptotic normality of (2). We pay special attention to the case $k = 2$ and present applications for parameter estimation of particular models of random fields.