ANALYSIS OF SELF-SIMILARITY PROPERTY OF α-STABLE PROCESSES

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Abstract

Self-similarity of network traffic has a strong impact on performance of the network and is a common property of modern telecommunication networks, which makes the study relevant to the industry needs [1].

One of the first steps in modelling network traffic with α-stable processes is research of self-similarity property to find out if the considered process has the long range dependency property. One of the most important parameters related to the self-similarity property is Hurst exponent. There are multiple methods of estimation of the Hurst exponent from the existing data. This work is aimed to compare statistical properties of the most popular estimation techniques applied to α-stable processes.

The aim of the work is to estimate Hurst exponent H from samples of α-stable processes and to perform comparative analysis of statistical properties of the estimates, retrieved using different methods. The following methods are covered in this paper: R/S analysis, variance-time analysis, wavelet analysis and detrended fluctuation analysis. Stable process is considered as a model random process with fractal properties.

The results of the modelling α-stable processes with Hurst exponent H are presented in the work, varying $0.5 < H < 1$. For the retrieved sample the H is estimated using considered methods. The theoretical and empirical research of the statistical properties of the estimates is performed, in particular bias, standard deviation and consistency of the estimates [2].

1 Introduction

Self-similar processes play an important role in probability because of their connection to limit theorems and they are widely used to model natural phenomena. For instance, persistent phenomena in internet traffic, hydrology, geophysics or financial markets are known to be self-similar. Stable processes have attracted growing interest in recent years: data with "heavy tails" have been collected in fields as diverse as economics, telecommunications, hydrology and physics of condensed matter, which suggests using non-Gaussian stable processes as possible models. Self-similar α-stable processes have been proposed to model some natural phenomena with heavy tails.

The stochastic process $X(t)$ is statistically self-similar if $x(at) \overset{d}{=} a^H X(at)$, where $a > 0$. Long-range dependence means slow (hyperbolic) decay in the time of the

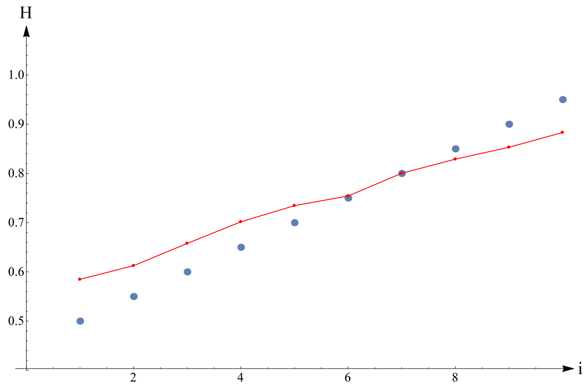

Figure 1: Hurst exponent estimates using R/S analysis

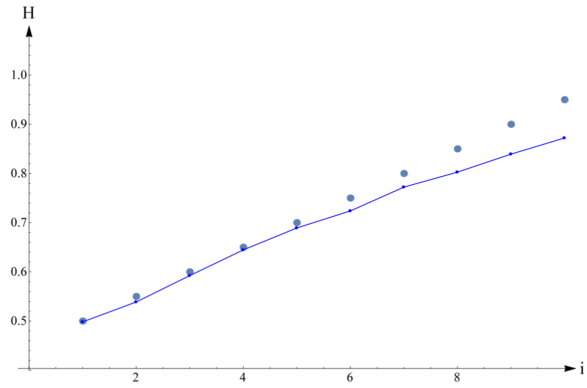

Figure 2: Hurst exponent estimates using variance-time analysis

autocorrelation function of a process. The parameter H (in general $0 < H < 1$) is called the Hurst exponent and is a measure of self-similarity or a measure of duration of long-range dependence of a stochastic process.

Stable distributions are a class of probability laws, and they have intriguing theoretical and practical features. The α-stable distributions are quite effective in the analysis of the financial time series because they can generalize the normal distribution and allow heavy tails and skewness. Despite the fact that the student-t, hyperbolic and normal inverse Gaussian distributions have heavy tail features, the most important reason for preferring the α-stable distributions is that they are supported by the generalized Central Limit Theorem. There is no a close form of α-stable distribution except for Normal, Cauchy and Levy distributions. However, one dimensional stable distribution can be described by the following characteristic function of $X\ S_\alpha(\beta, \gamma, \sigma)$:

$$\phi(t) = \begin{cases} \exp\left\{-\sigma^\alpha |t|^\alpha \left[1 - i\beta \operatorname{sgn}(t) \tan(\frac{\pi\alpha}{2})\right] + i\mu t\right\} & \text{if } \alpha \neq 1 \\ \exp\left\{-\sigma |t| \left[1 + i\beta \operatorname{sgn}(t) \left(\frac{2}{\pi}\right) \log|t|\right] + i\mu t\right\} & \text{if } \alpha = 1 \end{cases}$$

where $0 < \alpha \leq 2$, $-1 \geq \beta \leq 1$, $\mu, \sigma \in R$, $\sigma > 0$.

In accordance with the fractional Brownian motion, the non-integer alphas in the range $1 < \alpha \leq 2$ are described via long memory and statistical self-similarity properties; these are fractals. Additionally, α is the fractal dimension of the probability space of the time series and can be shown as $\alpha = \frac{1}{H}$, where H is the Hurst exponent and measures the statistical self-similarity.

2 Estimation of Hurst exponent

The estimation is performed using the following approach: at first 100 samples from stable process of size 1024 is generated for each of the considered H values. After that the estimation of Hurst exponent $\widehat{H}$ is performed for the samples using each method, the mean and standard deviation of estimates are calculated.

R/S analysis. This empirical method suggested by G. Hurst is still one of the most popular methods of research of fractal series of different nature.

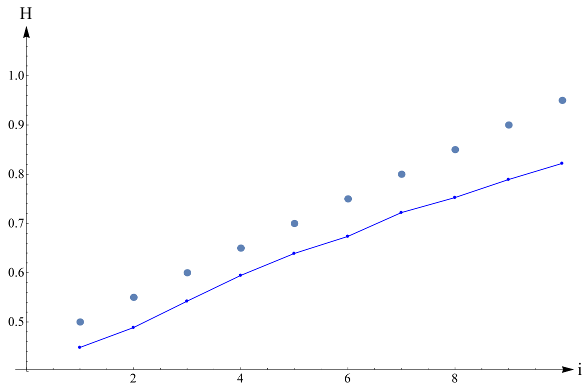

Figure 3: Hurst exponent estimates using detrended fluctuation analysis

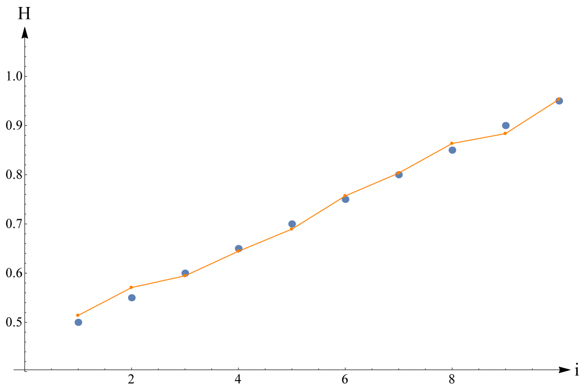

Figure 4: Hurst exponent estimates using wavelet analysis

Table 1: Standard deviation of estimates of Hurst exponent

Estimation method	Standard deviation $S_{\widehat{H}}$	Depends on real H
R/S analysis	$0.029 \leq S_{\widehat{H}} \leq 0.044$	Increases with H
Variance-time analysis	$S_{\widehat{H}} \approx 0.03$	No
Detrended fluctuation analysis	$0.028 \leq S_{\widehat{H}} \leq 0.041$	Increases with H
Wavelet analysis	$S_{\widehat{H}} \approx 0.04$	No

The estimates for Hurst exponent using R/S analysis are biased (see Figure 1) and their standard deviation increases with real H values (see Table 1).

Variance-time analysis is most often used to processes researches in telecommunication networks.

The estimates for Hurst exponent using variance-time analysis are biased (see Figure 2) and their standard deviation does not depend on real H values (see Table 1).

Detrended fluctuation analysis (DFA). DFA is the main method of determining self-similarity for nonstationary time series nowadays. This method is based on the ideology of onedimensional random walks.

The estimates for Hurst exponent using detrended fluctuation analysis are biased (see Figure 3) and their standard deviation increases with real H values (see Table 1).

Wavelet-based estimation. Of recent, the effective tool for a time series analysis is the multiresolution wavelet analysis, whichs main idea consists in the expansion of a time series on an orthogonal base, formed by shifts and the multiresolution copies of the wavelet function.

The estimates for Hurst exponent using wavelet analysis are unbiased (see Figure 4) and their standard deviation does not depend on real H values (see Table 1).

3 Conclusion

The best estimates of Hurst exponent for samples from alpha-stable processes can be retrieved using wavelet analysis, which is confirmed by their theoretical statistical properties [3]. The estimates retrived using this method are unbiased and their standard deviation does not depend on real H values. Estimates retrived using the other methods are biased, which is confirmed empirically. Additionally, the standard deviation of estimates decreases with the increase of sample size for all methods. In case of small sample size to achieve better estimation accuracy the one may want to use the average of fixed unbiased estimates retrieved using multiple methods.

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