ON PARAMETER ESTIMATION OF STATIONARY GAUSSIAN TIME SERIES OBSERVED UNDER RIGHT CENSORING

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Abstract

Stationary Gaussian time series observed under right censoring are considered. Statistical estimators of the model parameters are constructed by using the method of moments for special auxiliary time series. Consistency of constructed estimators is proved under some additional general conditions.

Consider Gaussian time series x_t observed under right censoring. It means that instead of the exact values $x_1, \dots, x_T$ at the time moments $T_c = \{t : x_t \geq c\}$ only random events are observed [2, 3]:

$$A_t^* = \{x_t \in [c, +\infty)\},\ t \in T_c,$$

where $c \in \mathbb{R}$ is the censoring level, $T \in \mathbb{N}$ is the length of the observation process.

Let $X = (x_1, \dots, x_T)' \in \mathbb{R}^T$ be the vector of the exact observations. Then for Gaussian time series the vector X has a normal distribution $\mathcal{L}(X) = \mathcal{N}(\mu, \Sigma)$, where the mathematical mean μ and the covariance matrix Σ depend on some unknown parameter $\theta \in \Theta \subseteq \mathbb{R}^m$ of the time series model (e.g. for AR(p) model $\theta = (\varphi_1, \dots, \varphi_p, \sigma^2)$, where $\varphi_1, \dots, \varphi_p$ are the autoregression coefficients and σ^2 is the variance of the Gaussian innovation process [1]).

Define the auxiliary time series y_t for the right censored time series x_t [3]:

$$y_t = f_c(x_t) = \begin{cases} x_t, & t \in \{1, \dots, T\} \backslash T_c \\ c, & t \in T_c \end{cases} = \min\{x_t, c\}.$$

Using the method of moments for auxiliary time series y_t, the m values of the second moments $\sigma_\tau = \mathbb{E}\{x_t x_{t+\tau}\}$ for the initial time series x_t can be estimated, i.e. estimators $\hat{\sigma}_\tau$, $0 \leq \tau < m$, can be found. These $\hat{\sigma}_\tau$ with help of the method of moments for initial time series x_t allow to obtain estimators of the model parameters $\hat{\theta}$. The example of this estimation procedure is proposed for the AR(p) model.

The consistency of the constructed estimators $\hat{\theta}$ is proved.

References

[1] Anderson T.W. (1971). *The Statistical Analysis of Time Series*. Wiley, NY.

[2] Kharin Yu.S. (2013). *Robustness in Statistical Forecasting*. Springer, NY.

[3] Park J.W., Genton M.G., Ghosh S.K. (2007). Censored time series analysis with autoregressive moving average models. *Canadian J. Stat.* Vol. **35**(1), pp. 151-168.