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<sup>1</sup>L. Meyer and F. Reif, Phys. Rev. **110**, 279 (1958).

<sup>2</sup>F. Reif and L. Meyer, Phys. Rev. (to be published).

<sup>3</sup>J. L. Yarnell, G. P. Arnold, P. J. Bendt, and E. C. Kerr, Phys. Rev. **113**, 1379 (1959).

<sup>4</sup>K. R. Atkins, H. Seki, and E. U. Condon, Phys.

Rev. **102**, 582 (1956). The onset temperature for superfluidity was about 1.35°K in the Vycor used.

<sup>5</sup>K. R. Atkins, Phys. Rev. **116**, 1339 (1959).

<sup>6</sup>More exactly,  $D_{i3} \propto (M + M^2/M_3)^{-1/4}$ , where  $M_3$  is the effective mass of a  $\text{He}^3$  atom. A reasonable estimate is  $M_3 = 2M_{\text{He}}$ .

<sup>7</sup>This is deduced from the temperature range from about 0.6 to 0.5°K where the mobility differences are sufficiently large for the subtraction analysis to be feasible.

<sup>8</sup>I. M. Khalatnikov and V. N. Zharkov, J. Exptl. Theoret. Phys. U.S.S.R. **32**, 1108 (1957) [translation: Soviet Phys. -JETP **5**, 905 (1957)].

## SHIFT OF AN ELECTRON INTERFERENCE PATTERN BY ENCLOSED MAGNETIC FLUX

R. G. Chambers

H. H. Wills Physics Laboratory, University of Bristol, Bristol, England

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Aharonov and Bohm<sup>1</sup> have recently drawn attention to a remarkable prediction from quantum theory. According to this, the fringe pattern in an electron interference experiment should be shifted by altering the amount of magnetic flux passing between the two beams (e.g., in region *a* of Fig. 1), even though the beams themselves pass only through field-free regions. Theory predicts a shift of *n* fringes for an enclosed flux  $\Phi$  of  $n\hbar c/e$ ; it is convenient to refer to a natural "flux unit,"  $\hbar c/e = 4.135 \times 10^{-7}$  gauss  $\text{cm}^2$ . It has since been pointed out<sup>2</sup> that the same conclusion had previously been reached by Ehrenberg and Siday,<sup>3</sup> using semiclassical arguments, but these authors perhaps did not sufficiently stress the remarkable nature of the result, and their work appears to have attracted little attention.

Clearly the first problem to consider, experimentally, is the effect on the fringe system of stray fields not localized to region *a* but extending, e.g., over region *a'* in Fig. 1. In addition

to the "quantum" fringe shift due to the enclosed flux, there will then be a shift due simply to curvature of the electron trajectories by the field. A straightforward calculation shows that in a "biprism" experiment,<sup>4</sup> such a field should produce a fringe displacement which exactly keeps pace with the deflection of the beams by the field, so that the fringe system appears to remain undisplaced relative to the envelope of the pattern. A field of type *a*, on the other hand, should leave the envelope undisplaced, and produce a fringe shift within it. In the Marton<sup>5</sup> interferometer, conditions are different, and a field of type *a'* should leave the fringes undisplaced in space. This explains how Marton *et al.*<sup>5</sup> were able to observe fringes in the presence of stray 60-cps fields probably large enough to have destroyed them otherwise; this experiment thus constitutes an inadvertent check of the existence of the "quantum" shift.<sup>2</sup>

To obtain a more direct check, a Philips EM100 electron microscope<sup>6</sup> has been modified so that it can be switched at will from normal operation to operation as an interferometer. Fringes are produced by an electrostatic "biprism" consisting of an aluminized quartz fiber *f* (Fig. 1) flanked by two earthed metal plates *e*; altering the positive potential applied to *f* alters the effective angle of the biprism.<sup>4</sup> The distances *s*-*f* and *f*-*o* (Fig. 1) are about 6.7 cm and 13.4 cm, respectively. With this microscope it was not possible to reduce the virtual source diameter below about 0.2  $\mu$ , so that it was necessary to use a fiber *f* only about 1.5  $\mu$  in diameter and a

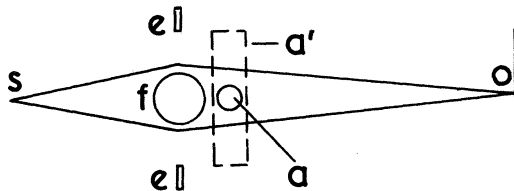


FIG. 1. Schematic diagram of interferometer, with source *s*, observing plane *o*, biprism *e*, *f*, and confined and extended field regions *a* and *a'*.

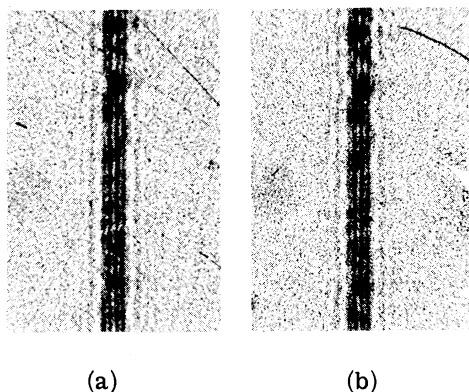


FIG. 2. (a) Fringe pattern due to biprism alone. (b) Pattern displaced by 2.5 fringe widths by field of type  $a'$ .

very small biprism angle, to produce a wide pattern of fringes which would not be blurred out by the finite source size. The fringe pattern obtained is shown in Fig. 2(a); the fringe width in the observing plane  $o$  is about  $0.6 \mu$ .

We first examined the effect of a field of type  $a'$ , produced by a Helmholtz pair of single turns 3 mm in diameter just behind the biprism. Fields up to 0.3 gauss were applied, sufficient to displace the pattern by up to 30 fringe widths, and as predicted the appearance of the pattern was completely unchanged. Figure 2(b), for instance, shows the pattern in a field producing a displacement of about 2.5 fringe widths. In the absence of the "quantum" shift due to the enclosed flux, this pattern would have had the light and dark fringes interchanged. We also verified that with this interferometer, unlike Marton's, a small ac field suffices to blur out the fringe system completely. These results confirm the presence of the quantum shift in fields of type  $a'$ .

Of more interest is the effect predicted for a field of type  $a$ , where intuition might expect no effect. Such a field was produced by an iron whisker,<sup>7</sup> about  $1 \mu$  in diameter and 0.5 mm long, placed in the shadow of the fiber  $f$ . Whiskers as thin as this are expected theoretically<sup>8</sup> and found experimentally<sup>9</sup> to be single magnetic domains; moreover they are found to taper<sup>9</sup> with a slope of the order of  $10^{-3}$ , which is extremely convenient for the present purpose. An iron whisker  $1 \mu$  in diameter will contain about 400 flux units; if it tapers uniformly with a slope of  $10^{-3}$ , the flux content will change along the length at a rate  $d\Phi/dz$  of about 1 flux unit per

micron. Thus if such a whisker is placed in position  $a$  (Fig. 1), we expect to see a pattern in which the envelope is undisplaced, but the fringe system within the envelope is inclined at an angle of the order of one fringe width per micron. Since the fringe width in the observing plane is  $0.6 \mu$ , and there is a "pin-hole" magnification of  $\times 3$  between the biprism-fiber assembly and the observing plane, we thus expect the fringes to show a tilt of order 1 in 5 relative to the envelope of the pattern. Precisely this is observed experimentally, as shown in Fig. 3(a). It will be seen that the whisker taper is not uniform, but in this example becomes very small in the upper part of the picture.

In fact the biprism is an unnecessary refinement for this experiment: Fresnel diffraction into the shadow of the whisker is strong enough to produce a clear fringe pattern from the whisker alone. Thus Fig. 3(b) shows the same section of whisker as Fig. 3(a), moved just out of the shadow of the biprism fiber. The biprism fringes are now unperturbed; the Fresnel fringes in the shadow of the whisker show exactly the same pattern of fringe shifts along their length as in Fig. 3(a). Figure 3(c) shows a further example of these fringes, from a different part of the same whisker, with the biprism moved out of the way. The whisker here is tapering more rapidly.

These fringe shifts cannot be attributed to direct interaction between the electrons and the surface of the whisker, since in Fig. 3(a) the whisker

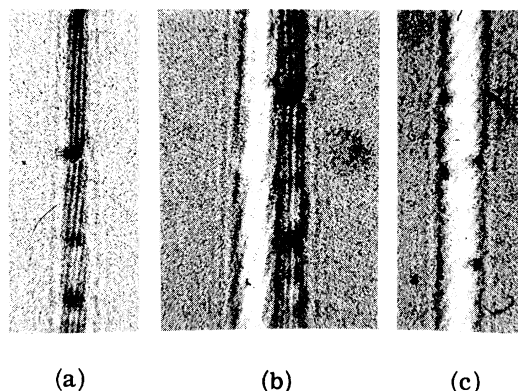


FIG. 3. (a) Tilted fringes produced by tapering whisker in shadow of biprism fiber. (b) Fresnel fringes in the shadow of the whisker itself, just outside shadow of fiber. (c) Same as (b), but from a different part of the whisker, and with fiber out of the field of view.

is completely in the shadow of the fiber. Nor can they be attributed to a return field  $H_z$  parallel to the whisker in the region  $a'$  outside it, due to the flux emerging from the sides and ends, for two reasons. An estimate of the magnitude of this field shows that it might be strong enough to displace the pattern by perhaps one fringe width, but not more, and observation confirms that the displacement of the envelope is very small; secondly, we have seen experimentally that an extended field  $H_z$  in fact produces a completely different effect (Fig. 2). Thus the patterns of Fig. 3 might be taken to demonstrate the existence of the predicted quantum shift. Indeed they do; nevertheless the tilt of the fringes can be attributed to a leakage field, as Pryce has pointed out to me, and it is illuminating to consider this.<sup>10</sup> Immediately outside a tapering whisker, the leakage field is in fact primarily radial and is given by  $H_r = (d\Phi/dz)/2\pi r$ . This field exerts a force on the electron and gives it a momentum  $p_z = \pm \frac{1}{2}(e/c)d\Phi/dz$ , the different signs applying to paths on either side of the whisker. The two beams which converge to interfere at  $o$  are thus tilted one above and one below the plane of Fig. 1, thus skewing the interference fringes. There is a progressive change in the phase difference between the two beams as one moves in the  $z$  direction. This is easily calculated from  $p_z$  by de Broglie's relation, and amounts to a phase-difference gradient of  $(e/\hbar c)d\Phi/dz$ . This is precisely the rate of change of the "quantum" phase difference  $e\Phi/\hbar c$  calcu-

lated by Aharonov and Bohm. One thus sees fairly intuitively how the "quantum" phase difference is progressively built up from the free end of the whisker, where it is zero, to any section where the interference is being observed. It remains true, however, that the total displacement of a given fringe is a direct measure not of the leakage field from that section but of the flux enclosed within it, and that a displacement will occur even in a parallel-sided region of the whisker where the radial leakage field is zero.

I am indebted to Mr. Aharonov and Dr. Bohm for telling me of their work before publication, and to them and to Professor Pryce for many discussions.

<sup>1</sup>Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).

<sup>2</sup>F. G. Werner and D. R. Brill, Phys. Rev. Letters **4**, 344 (1960).

<sup>3</sup>W. Ehrenberg and R. E. Siday, Proc. Phys. Soc. (London) **B62**, 8 (1949).

<sup>4</sup>G. Möllenstedt and H. Düker, Z. Physik **145**, 377 (1956).

<sup>5</sup>L. Marton, J. A. Simpson, and J. A. Suddeth, Rev. Sci. Instr. **25**, 1099 (1954).

<sup>6</sup>A. C. van Dorsten, H. Nieuwdorp, and A. Verhoeff, Philips Tech. Rev. **12**, 33 (1950).

<sup>7</sup>The use of a whisker was first suggested by Dr. J. W. Mitchell. The whiskers were grown by the method of S. S. Brenner, Acta Met. **4**, 62 (1956).

<sup>8</sup>C. Kittel, Phys. Rev. **70**, 965 (1946).

<sup>9</sup>R. W. DeBlois, J. Appl. Phys. **29**, 459 (1958).

<sup>10</sup>The following analysis is due to Professor Pryce.

## ACOUSTICALLY MODULATED $\gamma$ RAYS FROM Fe<sup>57</sup>

S. L. Ruby and D. I. Bolef

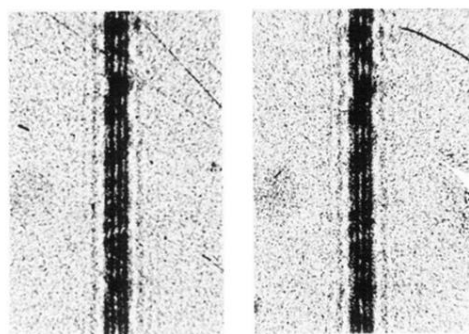
Westinghouse Electric Corporation, Pittsburgh, Pennsylvania

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The relationship between the emission of  $\gamma$  rays by nuclei bound in a crystal and the creation (or destruction) of phonons has been discussed by Visscher,<sup>1</sup> and suggests that a careful study of the "off-resonance" line shape in a Mössbauer-type experiment may be used to observe the frequency distribution of lattice vibrations in the crystal. Unfortunately, a direct attempt at such a study seems difficult since it requires the measurement of nuclear  $\gamma$ -ray absorption cross sections much smaller than the photoelectric cross sections for the same atom. In an attempt

to investigate the interactions between phonons and emitting nuclei, therefore, it was decided to generate low-energy phonons acoustically, and to study their effect on the  $\gamma$ -ray spectrum.

Source and absorber were one-mil thick 321 stainless steel (18% chromium, 8% nickel) foils. The source, into which had been diffused Co<sup>57</sup>, could be driven by either or both of two methods: (1) a low-frequency (15 cps) drive utilizing a loud speaker, and (2) a piezoelectric quartz crystal drive mounted on the rear of the source foil. The quartz crystal is driven by a radio-



(a)

(b)

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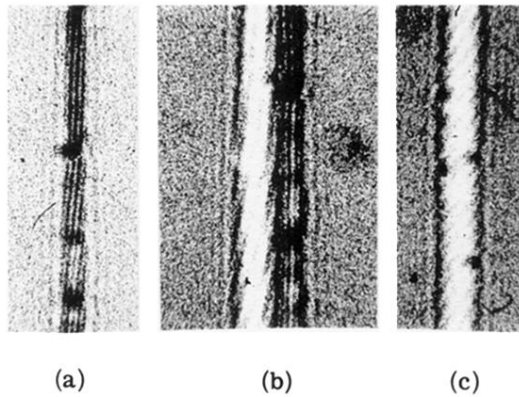


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