Справедлива следующая лемма.

Лемма. *Морфизм* ψ *является инъективным и доминантным. Следовательно,* $\overline{\mathcal{O}(A)}$ *– рациональное многообразие.*

Доказательство. Предположим, что для $X, Y \in T(A)$ мы имеем $XAX^{-1} = YAY^{-1}$. Тогда $X^{-1}Y = C \in Z(A)$, т. е. $Y = XC$. Учитывая, что $C = \mathrm{diag}(C_1, \ldots, C_s)$, где C_i имеет вид (1), и учитывая вид матриц X, Y из $T(A)$, получаем, что $C = E$ – единичная матрица, откуда $X = Y$. Значит, морфизм ψ инъективен.

Поскольку $\dim \overline{\mathcal{O}(A)} = n^2 - \dim Z(A) = \dim T(A)$, то из инъективности ψ следует его доминантность, т. е. ψ – бирациональный изоморфизм [7, предложение 3.17], и $\overline{\mathcal{O}(A)}$ бирационально изоморфно многообразию $T(A)$, которое, очевидно, рационально. Лемма доказана.

Завершим доказательство теоремы. Обозначим через h_A ограничение морфизма f_A на $Z(A) \times T(A)$. Несложное вычисление показывает, что h_A – инъективный морфизм. Действительно, если

$$h_A(C_1, X_1) = h_A(C_2, X_2),$$

то

$$X_1(A, B_0C_1)X_1^{-1} = X_2(A, B_0C_2)X_2^{-1}.$$

Значит, $X_1AX_1^{-1} = X_2AX_2^{-1}$, откуда

$$X_2 = X_1C_3, \qquad (3)$$

где $C_3 \in Z(A)$. Учитывая вид (1) матриц из централизатора $Z(A)$ и вид матриц из $T(A)$, получаем из (3), что $X_1 = X_2$. Тогда равенство $B_0C_1 = B_0C_2$ влечет $C_1 = C_2$.

Поскольку $\dim Z(A) \times T(A) = n^2 = \dim W(A)$, то морфизм h_A доминантен. Следовательно, h_A – бирациональный изоморфизм; $W(A)$ бирационально изоморфно рациональному многообразию $Z(A) \times T(A)$. Теорема доказана.

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L. A. PILIPCHUK, O. V. GERMAN, A. S. PILIPCHUK

THE GENERAL SOLUTIONS OF SPARSE SYSTEMS WITH RECTANGULAR MATRICES IN THE PROBLEM OF SENSORS OPTIMAL LOCATION IN THE NODES OF A GENERALIZED GRAPH

Рассмотрено построение общих решений разреженных систем с прямоугольными матрицами в задаче оптимального расположения сенсоров в узлах обобщенного графа. Исследуемые системы наряду с разреженной частью содержат уравнения общего вида. Матрица разреженной части недоопределенной системы является блочно-диагональной. Типы разреженности матричных блоков системы могут быть различными. Дополнительная часть системы может иметь общий вид. На основе теории декомпозиции опоры созданы эффективные методы решения линейных систем с прямоугольными разреженными матрицами и различными типами разреженности. Эти методы базируются на теоретико-графовой специфике структуры опоры и свойствах базиса пространства решений однородных разреженных систем специальных типов. Использованы фундаментальные результаты теории потоков и достижения в технологии построения аналитических и численных решений разреженных недоопределенных систем.

Ключевые слова: прямоугольная разреженная матрица; недоопределенная система; сенсор; обобщенный граф; невырожденный цикл; коэффициент преобразования дугового потока; опора; общее решение разреженной системы; декомпозиция.

The article is devoted to constructing the general solutions of sparse systems with rectangular matrices in the problem of sensors optimal location in the nodes of a generalized graph. The matrix of sparse part of the investigated underdetermined system is a block-diagonal matrix. The sparsity types of the matrix blocks of the system may be different. The additional part of the system may have a common form. On the basis of the theory of decomposition of support we construct the effective methods of solutions of linear systems with rectangular sparse matrices with different types of sparsity. These methods are based on the theoretic-graph specificities of the support structure and on the properties of the basis of the solution space of homogeneous sparse systems of special types. We apply the fundamental results of the theory of flows and advancements in the technology of construction the analytical and numerical solutions sparse underdetermined systems.

Key words: rectangular sparse matrix; underdetermined system; sensor; generalized graph; non-degenerate cycle; transformation coefficient of arc flow; support; general solution of a sparse system; decomposition.

The methods of decomposition and the theory of graphs partitioning are applied for constructing the general solutions of the systems with special sparse matrices. These systems arise in the Sensor Location Problem for one new application connected to the optimal sensors location in the nodes of a generalized graph.

Statement of problem. In [1] the problem of optimal location of the sensors in the nodes of a connected generalized graph (Sensor Location Problem for the generalized graph) was considered. To solve this problem has been investigated the sparse underdetermined linear system

$$\sum_{j\in I_i^+(U)} x_{i,j} - \sum_{j\in I_i^-(U)} \mu_{j,i} x_{j,i} = \begin{cases} x_i, \ i\in I^*, \\ 0, \ i\in I\setminus I^*. \end{cases} \tag{1}$$

Here $G=(I, U)$ is a finite oriented *connected symmetric* graph with set of nodes *I* and set of arcs *U*. I^* is the set of nodes with variable intensities x_i, $i\in I^*$, $I^*\subseteq I$. $\mu_{i,j}$ is transformation coefficient of the arc flow $x_{i,j}\ (i, j)\in U$.

On the basis of a priori given information in the Sensor Location Problem for the generalized graph [1] the system (1) can be converted to the following form:

$$\sum_{j\in I_i^+(\overline{U})} x_{i,j} - \sum_{j\in I_i^-(\overline{U})} \mu_{j,i} x_{j,i} = \begin{cases} x_i + b_i, \ i\in \overline{I}^*, \\ a_i, \ i\in \overline{I}\setminus \overline{I}^*, \end{cases} \tag{2}$$

$$\sum_{(i,j)\in\overline{U}} \lambda_{i,j}^p x_{i,j} = 0, \ p=\overline{1,q}. \tag{3}$$

Here graph $\overline{G}=(\overline{I},\overline{U})$ with a set of nodes $\overline{I}$ and a set of arcs $\overline{U}$ can be *not connected and not symmetric*; $\overline{I}^*$ is a set of nodes of the graph $\overline{G}$ with variable intensities x_i+b_i, $i\in\overline{I}^*$, $\overline{I}^*\subseteq\overline{I}$. Graph $\overline{G}$ consists from *m connected components* $G^n=(I^n, U^n)$, $n=\overline{1,m}$, where I_n^* is a set of nodes with variable intensities $x_i+b_i, i\in I_n^*$ for the connected component $G^n=(I^n, U^n)$, $I_n^*\subseteq\overline{I}^*$. Here $x_{i,j}$ is the arc flow, $(i, j)\in\overline{U}$, *q* is the number of additional equations (3), a_i, b_i, $\lambda_{i,j}^p$ are rational numbers. $I_i^+(\overline{U})$, $I_i^-(\overline{U})$ are defined as follows: $I_i^+(\overline{U})=\{j\in\overline{I} : (i, j)\in\overline{U}\}$, $I_i^-(\overline{U})=\{j\in\overline{I} : (j,i)\in\overline{U}\}$. $\mu_{i,j}$ is transformation coefficient of the arc flow $x_{i,j}, (i, j)\in\overline{U}$ [2]. The sets $I_i^+(U^n)$, $I_i^-(U^n)$ are defined as follows: $I_i^+(U^n)=\{j\in I^n : (i, j)\in U^n\}$, $I_i^-(U^n)=\{j\in I^n : (j,i)\in U^n\}$.

To solve the system (2)–(3) we decompose the sparse part of the system of equations and its additional part. The sparse part (2) of the equations of the system (2)–(3) represents a sparse structure of the system. The matrix of the system (2) is a block-diagonal one. We do not change the sparse part of the block-diagonal matrix of the system (2). The sparsity types of the matrix blocks of the system (2) may be different. The additional part (3) of the equations of the system (2)–(3) may have a general form. We start the process of solution by considering the network part (2) of the sparse underdetermined system (2)–(3) and first explore the types of sparsity and a structure of support [2, 3] for each block of the sparse matrix of the system (2).

Types of sparse systems. Each block with number n of the matrix of the system (2) corresponds to some connected component $G^n = (I^n, U^n),\ n = \overline{1, m}.$ We consider the types of sparse systems of the form (2) for each connected component $G^n = (I^n, U^n),\ n = \overline{1, m}.$ These types of sparse systems were obtained as a result of usage of a priori given information in the Sensor Location Problem [1]. There are 4 types of sparse systems listed below.

Type 1. If for any connected component G^n there is no transformation of arc flows (the transformation coefficient $\mu_{i,j}$ of the arc flow $x_{i,j}$ is equal to 1 for each arc $(i, j) \in U^n$) and $I_n^* = \varnothing,$ then the system (2) for block with number n has the form:

$$\sum_{j \in I_i^+(U^n)} x_{i,j} - \sum_{j \in I_i^-(U^n)} x_{j,i} = a_i,\ i \in I^n. \tag{4}$$

Type 2. If for any connected component $G^n = (I^n, U^n)$ there is no transformation of arc flows (the transformation coefficient $\mu_{i,j}$ of the arc flow $x_{i,j}$ is equal to 1 for each arc $(i, j) \in U^n$) and $I_n^* \neq \varnothing,$ then the system (2) for block with number n has the form [3, 4]:

$$\sum_{j \in I_i^+(U^n)} x_{i,j} - \sum_{j \in I_i^-(U^n)} x_{j,i} = \begin{cases} x_i + b_i,\ i \in I_n^*, \\ a_i,\ i \in I^n \setminus I_n^*. \end{cases} \tag{5}$$

Type 3. If the connected component $G^n = (I^n, U^n)$ contains at least one non-degenerate cycle [2] (transformation coefficient $\mu_{i,j}$ for some arcs $(i, j) \in U^n$ is not equal to 1, i. e. there is transformation of arc flows) and $I_n^* = \varnothing,$ then the system (2) for the block with number n has the form [2, 3]:

$$\sum_{j \in I_i^+(U^n)} x_{i,j} - \sum_{j \in I_i^-(U^n)} \mu_{j,i} x_{j,i} = a_i,\ i \in I^n. \tag{6}$$

Type 4. If the connected component $G^n = (I^n, U^n)$ contains at least one non-degenerate cycle (coefficients $\mu_{i,j}$ for some arc flows $x_{i,j},\ (i, j) \in U^n,$ are not equal to type 1 and $I_n^* \neq \varnothing,$ then the system (2) for block with number n has the form [5]:

$$\sum_{j \in I_i^+(U^n)} x_{i,j} - \sum_{j \in I_i^-(U^n)} \mu_{j,i} x_{j,i} = \begin{cases} x_i + b_i,\ i \in I_n^*, \\ a_i,\ i \in I^n \setminus I_n^*. \end{cases} \tag{7}$$

With help of the types of the sparse systems (4), (5), (6) and (7) for the connected components $G^n = (I^n, U^n),\ n = \overline{1, m}$ and on the basis of the theory of decomposition of support one can construct the effective methods of solutions of linear systems with rectangular sparse matrices with different types of sparsity. These algorithms are based on the theoretic-graph specificities of the structure of the support and on the properties of the basis of the solution space of homogeneous sparse systems of special types. The decomposition of the support, as a result, the decomposition of sparse underdetermined systems is one of the main steps in the solution of linear systems with rectangular sparse matrices [6]. Combinatorial aspects of the Sensor Location Problem for the graph are discussed in [7].

Graph-theoretic properties of the support. Now, let us represent the graph-theoretic properties of the support R of the graph $G^n = (I^n, U^n)$ for each type of the sparse systems (4), (5), (6) and (7).

Consider the structure of the support R_n of the graph $G^n = (I^n, U^n)$ for the system (4). Let $R_n = U_R^n$ be a support of a graph $G^n = (I^n, U^n)$ for the system (4), $I_n^* = \varnothing.$

Theorem 1. *The support $R_n = U_R^n$ of the graph $G^n = (I^n, U^n)$ for system* (4) *is a spanning tree.*

The proof can be found in [2, 3, 8].

Consider the structure of the support R_n of the graph $G^n = \left(I^n, U^n\right)$ for the system (5). Let $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ be a support of a graph $G^n = \left(I^n, U^n\right)$ for system (5), where $I_R^{*n} \neq \varnothing,\ I_R^{*n} \subseteq I_n^*$.

Theorem 2. *The support $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ of the graph $G^n = \left(I^n, U^n\right)$ for system* (5) *is the forest of trees and each tree of the forest contains exactly one node from the set $I_R^{*n} \neq \varnothing,\ I_R^{*n} \subseteq I_n^*$.*

The proof see in [3, 8].

Consider the structure of the support $R_n = U_R^n$ of the graph $G^n = \left(I^n, U^n\right)$ for the system (6).

Theorem 3. *The support $R_n = U_R^n$ of the graph $G^n = \left(I^n, U^n\right)$ for system* (6)*, $I_n^* = \varnothing$ consists of the connected components, each of which has a single non-degenerate cycle* [2, 3].

Let $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ be a support of graph $G^n = \left(I^n, U^n\right)$ for system (7), $I_n^* \neq \varnothing$.

Theorem 4. *The support $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ of the graph $G^n = \left(I^n, U^n\right)$ for system* (7)*, $I_R^{*n} \subseteq I_n^*$ consists of the connected components, each of which may have one of the following structures:*

- *the connected component of the support $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ is a tree which has exactly one node from the set $I_R^{*n} \neq \varnothing, I_R^{*n} \subseteq I_n^*$;*
- *the connected component of the support $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ does not contain nodes from the set I_R^{*n} and contains a single non-degenerate cycle.*

The proof see in [5, 9].

General solutions of sparse systems. The theoretic-graph properties of the support R_n of the graph $G^n = \left(I^n, U^n\right)$ for the sparse systems (4), (5), (6) and (7) with different type of sparsity are investigated. The decomposition theory for a graph will be applied to construct the solutions of linear algebraic systems with rectangular sparse matrices with different types of sparsity. We apply also the theoretic-graph properties of the support R_n of the graphs $G^n = \left(I^n, U^n\right),\ n = \overline{1, m}$ for constructing the basis of the solution space to sparse homogeneous algebraic systems, generated by sparse systems (4), (5), (6) and (7) respectively.

Theorem 5. *The general solution of a sparse system* (4) *and* (6) *has the following form:*

$$x_{i,j} = \sum_{(\tau,\rho)\in U^n \setminus U_R^n} x_{\tau,\rho}\delta_{i,j}^{\tau,\rho} + \left(\tilde{x}_{i,j} - \sum_{(\tau,\rho)\in U^n \setminus U_R^n} \tilde{x}_{\tau,\rho}\delta_{i,j}^{\tau,\rho}\right), (i, j) \in U_R^n, \tag{8}$$

where $\delta(\tau,\rho) = \left(\delta_{i,j}^{\tau,\rho}, (i, j) \in U^n\right), (\tau,\rho) \in U^n \setminus U_R^n$ form the fundamental systems of solutions of the homogeneous systems, generated by the system (4) *or* (6) *correspondingly. The vectors $\tilde{x} = \left(\tilde{x}_{i,j}, (i, j) \in U^n\right)$ are partial solutions for the non-homogeneous system* (4) *and* (6) *accordingly.*

Corollary 1. *If the partial solutions of the non-homogeneous system* (4) *or* (6) *constructed according to the rules: non-supporting elements the vectors $\tilde{x}$ are equal to zeros and supporting elements satisfy to the system* (4) *or* (6) *accordingly, then the general solution of a sparse system* (4) *and* (6) *has the following form:*

$$x_{i,j} = \sum_{(\tau,\rho)\in U^n \setminus U_R^n} x_{\tau,\rho}\delta_{i,j}^{\tau,\rho} + \tilde{x}_{i,j}, (i, j) \in U_R^n. \tag{9}$$

Effective algorithms for finding the fundamental system of solutions of the homogeneous system, generated by the system (4) are presented in [3, 8]. Also, the effective algorithms for finding partial solutions $\tilde{x} = \left(\tilde{x}_{i,j}, (i, j) \in U^n\right)$ of the sparse linear non-homogeneous systems (4) and (6) are obtained in [3]. The number of operations for computing non-zero components of every vector $\delta(\tau,\rho) = \left(\delta_{i,j}^{\tau,\rho}, (i, j) \in U^n\right)$ of the fundamental system of solutions of the homogeneous system, generated by the system (4) or (6), is proportional to the number of non-zero components of every vector $\delta(\tau,\rho)$, $(\tau,\rho) \in U^n \setminus U_R^n$ [3].

Theorem 6. *The general solution of a sparse system* (5) *and* (7) *has the following form:*

$$x_{i,j} = \sum_{(\tau,\rho)\in U^n\setminus U_R^n} x_{\tau,\rho}\delta_{i,j}^{\tau,\rho} + \sum_{\gamma\in I_n^*\setminus I_R^{*n}} x_\gamma \delta_{i,j}^\gamma + \left(\tilde{x}_{i,j} - \sum_{(\tau,\rho)\in U^n\setminus U_R^n} \tilde{x}_{\tau,\rho}\delta_{i,j}^{\tau,\rho} - \sum_{\gamma\in I_n^*\setminus I_R^{*n}} \tilde{x}_\gamma \delta_{i,j}^\gamma\right), (i,j)\in U_R^n, \quad (10)$$

$$x_i = \sum_{(\tau,\rho)\in U^n\setminus U_R^n} x_{\tau,\rho}\delta_i^{\tau,\rho} + \sum_{\gamma\in I_n^*\setminus I_R^{*n}} x_\gamma \delta_i^\gamma + \left(\tilde{x}_i - \sum_{(\tau,\rho)\in U^n\setminus U_R^n} \tilde{x}_{\tau,\rho}\delta_i^{\tau,\rho} - \sum_{\gamma\in I_n^*\setminus I_R^{*n}} \tilde{x}_\gamma \delta_i^\gamma\right), i\in I_R^{*n}. \quad (11)$$

Corollary 2. *If the partial solution* $\tilde{x} = (\tilde{x}_{i,j}, (i,j)\in U^n;\ \tilde{x}_i, i\in I_n^*)$ *constructed according to the rules: non-supporting elements of the vectors* $\tilde{x}$ *are equal to zeros, then the general solution of the systems* (5) *and* (7) *has the following form:*

$$x_{i,j} = \sum_{(\tau,\rho)\in U^n\setminus U_R^n} x_{\tau,\rho}\delta_{i,j}^{\tau,\rho} + \sum_{\gamma\in I_n^*\setminus I_R^{*n}} x_\gamma \delta_{i,j}^\gamma + \tilde{x}_{i,j},\ (i,j)\in U_R^n, \quad (12)$$

$$x_i = \sum_{(\tau,\rho)\in U^n\setminus U_R^n} x_{\tau,\rho}\delta_i^{\tau,\rho} + \sum_{\gamma\in I_n^*\setminus I_R^{*n}} x_\gamma \delta_i^\gamma + \tilde{x}_i,\ i\in I_R^{*n}, \quad (13)$$

where the vectors

$$\delta(\tau,\rho) = \left(\delta_{i,j}^{\tau,\rho}, (i,j)\in U^n;\ \delta_i^{\tau,\rho}, i\in I_n^*\right),\ (\tau,\rho)\in U^n\setminus U_R^n, \quad (14)$$

$$\delta(\gamma) = \left(\delta_{i,j}^{\gamma}, (i,j)\in U^n;\ \delta_i^{\gamma}, i\in I_n^*\right),\ \gamma\in I_n^*\setminus I_R^{*n} \quad (15)$$

form a fundamental system of solutions of the homogeneous system of linear algebraic equations, generated by the systems (5) *or* (7) *respectively.*

The effective algorithms for computing non-zero components of characteristic vectors (14) and (15) and partial solutions of the systems (5), (7) are suggested in [3].

Decomposition of the system. Let substitute the general solution (9) for each of the systems (4), (6) to (3) for the some connected component $G^n = (I^n, U^n)$. As a result, we obtain the system (16) for the connected component $G^n = (I^n, U^n)$:

$$\sum_{(\tau,\rho)\in U^n\setminus U_R^n} \Lambda_{\tau,\rho}^p x_{\tau,\rho} = -\sum_{(i,j)\in U_R^n} \lambda_{i,j}^p \tilde{x}_{i,j},\ p=\overline{1,q_n}, \quad (16)$$

where

$$\Lambda_{\tau,\rho}^p = \lambda_{\tau,\rho}^p + \sum_{(i,j)\in U_R^n} \lambda_{i,j}^p \delta_{i,j}^{\tau,\rho},\ p=\overline{1,q_n},\ (\tau,\rho)\in U^n\setminus U_R^n,$$

q_n is the number of additional equations (3) for the some connected component G^n and $R_n = U_R^n$ is a support of graph $G^n = (I^n, U^n)$ for considered system in the form (4) or (6) respectively, $I_n^* = \varnothing$. Thus, for some connected component $G^n = (I^n, U^n)$ we excluded the unknown $x_{i,j}, (i,j)\in U_R^n$, which correspond to the arcs of supports $R_n = U_R^n$ of the graphs for the systems (4) and (6) respectively.

Similarly, for each systems (5), (7) we substitute the general solution (12) to the system (3) for the some connected component $G^n = (I^n, U^n)$:

$$\sum_{(i,j)\in U_R^n} \lambda_{i,j}^p x_{i,j} + \sum_{(\tau,\rho)\in U^n\setminus U_R^n} \lambda_{\tau,\rho}^p x_{\tau,\rho} = \sum_{(i,j)\in U_R^n} \lambda_{i,j}^p \left[x_{i,j} = \sum_{(\tau,\rho)\in U^n\setminus U_R^n} x_{\tau,\rho}\delta_{i,j}^{\tau,\rho} + \sum_{\gamma\in I_n^*\setminus I_R^{*n}} x_\gamma \delta_{i,j}^\gamma + \tilde{x}_{i,j}\right] +$$
$$+ \sum_{(\tau,\rho)\in U^n\setminus U_R^n} \lambda_{\tau,\rho}^p x_{\tau,\rho} = 0,\ p=\overline{1,q_n}, \quad (17)$$

where q_n is the number of additional equations (3) for the some connected component G^n and $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ is a support of graph $G^n = \left(I^n, U^n\right)$ for considered system (5), (7) respectively, $I_n^* \neq \varnothing$. We change the summation order in (17)

$$\sum_{(\tau,\rho)\in U^n \setminus U_R^n} x_{\tau,\rho}\left[\lambda_{\tau,\rho}^p + \sum_{(i,j)\in U_R^n} \lambda_{i,j}^p\, \delta_{i,j}^{\tau,\rho}\right] + \sum_{\gamma\in I_n^* \setminus I_R^{*n}} x_\gamma \left[\sum_{(i,j)\in U_R^n} \lambda_{i,j}^p\, \delta_{i,j}^{\gamma}\right] =$$

$$= -\sum_{(i,j)\in U_R^n} \lambda_{i,j}^p\, \tilde{x}_{i,j},\ \ p = \overline{1, q_n}. \qquad (18)$$

We denote

$$\Lambda_{\tau,\rho}^p = \lambda_{\tau,\rho}^p + \sum_{(i,j)\in U_R^n} \lambda_{i,j}^p\, \delta_{i,j}^{\tau,\rho},\ \ (\tau,\rho)\in U^n \setminus U_R^n,$$

$$\Lambda_{\gamma}^p = \sum_{(i,j)\in U_R^n} \lambda_{i,j}^p\, \delta_{i,j}^{\gamma},\ \ \gamma \in I_n^* \setminus I_R^{*n},\ \ A_p = -\sum_{(i,j)\in U_R^n} \lambda_{i,j}^p\, \tilde{x}_{i,j},\ \ p = \overline{1, q_n}.$$

The system (18) gets to the form:

$$\sum_{(\tau,\rho)\in U^n \setminus U_R^n} \Lambda_{\tau,\rho}^p x_{\tau,\rho} + \sum_{\gamma\in I_n^* \setminus I_R^{*n}} \Lambda_\gamma^p x_\gamma = A_p,\ \ p = \overline{1, q_n}. \qquad (19)$$

Thus, we excluded the unknown $x_{i,j}$, $(i,j)\in U_R^n$, x_i, $i \in I_R^{*n}$ from systems (5) and (7) which correspond to the support $R_n = \left\{U_R^n,\ I_R^{*n}\right\}$ of the graph for the system (5) and (7) respectively for the connected component $G^n = \left(I^n, U^n\right)$.

For the system of a linear algebraic inequalities was suggested enabling one to get the substitutions for non basic variables in the redundant algebraic systems [10]. However, the suggested method does not point which variables to be selected as basic ones.

For the blocks of the sparse matrix of the underdetermined system (2) we apply the fundamental results of the theory of flows in networks, as well as advancements in the technology of construction their analytical and numerical solutions.

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