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DYNAMIC LOAD EFFECT IN THE VICINITY OF GOAFS WITHIN ROCK MASSES

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The problems of the concentrated-dynamic-load effect in an elastic isotropic space containing a spherical enclose in an elastic isotropic plane with a circular hole were solved.

Elastic space, elastic plane, concentrated force

INTRODUCTION

Investigation into the state of rocks under mining-induced or natural dynamic loading in the vicinity of a goaf involves a class of actual problems of the current geomechanics. The present authors report solution of model problems on evaluation of the stress-strain state (SSS) of a disturbed rock mass containing a goaf under the acting impulse load, which may result from either dynamic events, explosions, or other reason.

Mostly, a goaf's shape can be approximated as a circular one in the case of a plane problem statement, or as a spherical surface, whereas the daylight surface effect can be neglected. In terms of Saint-Venant principle, the disturbance source effect on a rock mass with a goaf can be modeled as a concentrated force if this disturbance source is at a rather large distance from the goaf.

MODEL PROBLEM SOLUTION

Consider problems of the dynamic concentrated force effect in an elastic isotropic medium with a spherical enclosure when the force direction coincides with radius-vector of its application point, and in an elastic isotropic plane with a circular hole under the arbitrarily directed force. Let the force application point be at distance aR from the center of a mine working of radius R, where a be a dimensionless parameter. Coordinates for 3D case are: the origin is in the sphere center, axis x_3 coincides with radius-vector of the force application point, two other axes are normal to the third one. Respectively, in 2D case the origin of coordinates is in the center of hole, axis x_1 superposes with radius-vector of the force application point, axis x_2 is normal to it, M is dimensionality of the problem.

The resulting system for the formulated model problems involves the Láme equations:

$$\rho\frac{\partial^2 u_i}{\partial t^2}=(\lambda+\mu)\frac{\partial\theta}{\partial x_i}+\mu\Delta u_i+f(t)\delta_{i3}\delta(x_1)\delta(x_2)\delta(x_3-aR),\ \ i=1,2,3 \tag{1}$$

for 3D case and:

$$\rho\frac{\partial^2 u_i}{\partial t^2}=(\lambda+\mu)\frac{\partial\theta}{\partial x_i}+\mu\Delta u_i+f(t)\delta_{ij}\delta(x_1-aR)\delta(x_2),\ \ i=1,2 \tag{1'}$$

and for 2D case (j indicates force direction);

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boundary conditions are written in spherical r, φ, β and polar coordinates r, α:

$$\begin{cases} \alpha_1 u_r|_{r=R} + \beta_1 \sigma_{rr}|_{r=R} = q_1(\beta) \\ \alpha_2 u_\beta|_{r=R} + \beta_2 \sigma_{r\beta}|_{r=R} = q_2(\beta); \end{cases} \tag{2}$$

$$\begin{cases} \alpha_1 u_r|_{r=R} + \beta_1 \sigma_{rr}|_{r=R} = q_1(\alpha) \\ \alpha_2 u_\alpha|_{r=R} + \beta_2 \sigma_{r\alpha}|_{r=R} = q_2(\alpha). \end{cases} \tag{2'}$$

Attenuation conditions should be fulfilled at infinity:

$$u_\beta, u_r \underset{r\to\infty}{\to} 0, \quad M=3; \qquad u_\alpha, u_r \underset{r\to\infty}{\to} 0, \quad M=2. \tag{3}$$

Assume that initial disturbances are absent:

$$u_\beta|_{t=0}=0, \quad u_r|_{t=0}=0, \quad \dot{u}_\beta|_{t=0}=0, \quad \dot{u}_r|_{t=0}=0, \quad M=3,$$

$$u_\alpha|_{t=0}=0, \quad u_r|_{t=0}=0, \quad \dot{u}_\alpha|_{t=0}=0, \quad \dot{u}_r|_{t=0}=0, \quad M=2. \tag{4}$$

Let us describe the solution of the formulated model problems. First, we solve the problem on the concentrated force in the infinite space (plane). The similar model problems are described by the equations (1), boundary conditions (3) and initial conditions (4). Denote displacement components for the obtained solutions of such problems as u_i^1.

Next, consider the problem of the boundary disturbance propagation from hole in the infinite space and in the plane with no acting mass forces, characterized by a specific kind of boundary conditions. Denote displacement components for these solutions as u_i^2.

This problem is described by the equations (in terms of the plane problem):

$$\rho \frac{\partial^2 u_i^2}{\partial t^2} = (\lambda+\mu)\frac{\partial \theta^2}{\partial x_i} + \mu \Delta u_i^2, \quad i=1,2$$

and the boundary conditions

$$\begin{cases} \alpha_1 u_r^2|_{r=R} + \beta_1 \sigma_{rr}^2|_{r=R} = q_1(\alpha) - \alpha_1 u_r^1|_{r=R} - \beta_1 \sigma_{rr}^1|_{r=R} \\ \alpha_2 u_\alpha^2|_{r=R} + \beta_2 \sigma_{r\alpha}^2|_{r=R} = q_2(\alpha) - \alpha_2 u_\alpha^1|_{r=R} - \beta_2 \sigma_{r\alpha}^1|_{r=R} \end{cases}$$

complemented with the conditions (3) and (4).

So, solution of the model problem is $u_i = u_i^1 + u_i^2$.

Solution of the first of additional model problems is a dynamic analog of Kelvin's fundamental solution [1] and in general can be written as [2]:

$$u_i^1(x_1,x_2,x_3,t) = U_i^1(x_1,x_2,x_3-aR,t), \quad M=3, \quad u_i^1(x_1,x_2,t) = U_{ij}^1(x_1-aR,x_2,t), \quad M=2. \tag{5}$$

For $M = 3$:

$$U_i^1(x_1,x_2,x_3,t) = f_+(\tau) * G_i(x_1,x_2,x_3,t),$$

$$G_i = \frac{1}{4\pi r\rho}\left(\left(\frac{x_i x_3}{c_1^2 r^2}\delta(t-t_1) + \frac{1}{c_2^2}\left(\delta_{i3} - \frac{x_i x_3}{r^2}\right)\delta(t-t_2)\right) - \frac{t}{r^2}\left(\delta_{i3} - \frac{3x_i x_3}{c_1^2 r^2}\right)\left(H(t-t_1) - H(t-t_2)\right)\right),$$

$$f_+(t)=f(t)H(t)\,,\quad c_1=\sqrt{\frac{\lambda+2\mu}{\rho}}\,,\quad c_2=\sqrt{\frac{\mu}{\rho}}\,,$$

where H is Heaviside function, σ is delta function, σ_{ij} is bivalent mixed tensor.

Calculating integral convolutions, find:

$$U_i^1=-\frac{x_i x_3}{4\pi\rho\, r^3}\sum_{j=1}^{2}(-1)^j\left(\frac{1}{c_j^2}f_+\left(t-t_j\right)+\frac{3}{r^2}\left(f_2\left(t-t_j\right)+\frac{r}{c_j}f_3\left(t-t_j\right)\right)\right),\quad i=1,2\,;$$

$$U_3^1=\frac{1}{4\pi\rho r}\left(\frac{x_3^2}{c_1^2r^2}f_+\left(t-t_1\right)+\frac{1}{c_2^2}\left(1-\frac{x_3^2}{r^2}\right)f_+\left(t-t_2\right)+\frac{1}{r^2}\left(1-\frac{3x_3^2}{r^2}\right)\sum_{j=1}^{2}(-1)^j\left(f_2\left(t-t_j\right)+\frac{r}{c_j}f_3\left(t-t_j\right)\right)\right);\tag{6}$$

$$f_2(t)=f_+(t)*t_+,\quad t_+=tH(t),\quad f_3(t)=f_+(t)*H(t).$$

At $M=2$ for (5) we have:

$$U_{ij}^1=-\frac{\delta_{ij}}{2\pi}\left(\frac{\delta_{ij}}{c_1r^2}N_{11}(r,t)-\frac{\delta_{ij}}{c_2}\left(\frac{1}{r^2}N_{12}(r,t)+N_{-12}(r,t)\right)+\frac{x_ix_j}{r^2}\sum_{j=1}^{2}\frac{(-1)^j}{c_j}\left(\frac{2}{r^2}N_{1j}(r,t)+N_{-1j}(r,t)\right)\right),$$

$$N_{sj}(r,t)=H(t-t_j)\int_{t_j}^{t}f(t-\tau)(c_j^2\tau^2-r^2)^{s/2}d\tau,\tag{6′}$$

$$N_{1j}(r,t)=H(c_j\tau-r)\int_{r/c_j}^{t}f(t-\tau)\sqrt{c_j^2\tau^2-r^2}\,d\tau.$$

When considering the second model problem, it is more convenient to take the following dimensionless parameters:

$$\widetilde{x}_i=\frac{x_i}{R}\,,\quad \widetilde{u}_i=\frac{u_i}{R}\,,\quad \widetilde{\sigma}_{ij}=\frac{\sigma_{ij}}{\lambda+2\mu}\,,\quad \tau=\frac{c_1t}{R}\,,\quad \kappa=\frac{\lambda}{\lambda+2\mu}\,,\quad \eta=\frac{c_1}{c_2}\,.$$

Mathematical statement of the second model problem in dimensionless coordinates is as follows: $M=3$:

$$\eta^2\frac{\partial^2\widetilde{u}_r^2}{\partial\tau^2}=(\eta^2-1)\frac{\partial\widetilde{\theta}}{\partial r}+\left(\Delta\widetilde{u}_r^2-\frac{2}{r^2}\left(\frac{1}{\sin\beta}\frac{\partial}{\partial\beta}(\widetilde{u}_\beta^2\sin\beta)\right)+\widetilde{u}_r\right),$$

$$\eta^2\frac{\partial^2\widetilde{u}_\beta^2}{\partial\tau^2}=(\eta^2-1)\frac{1}{r}\frac{\partial\widetilde{\theta}}{\partial\beta}+\left(\Delta\widetilde{u}_\beta^2+\frac{1}{r^2}\left(2\frac{\partial\widetilde{u}_r^2}{\partial\beta}-\frac{\widetilde{u}_\beta^2}{\sin^2\beta}\right)\right),$$

$$\begin{cases}\widetilde{\alpha}_1\,\widetilde{u}_r^2\,|_{r=1}+\widetilde{\beta}_1\widetilde{\sigma}_{rr}^2\,|_{r=1}=\widetilde{q}_1(\beta)-\widetilde{\alpha}_1\,\widetilde{u}_r^1\,|_{r=1}-\widetilde{\beta}_1\widetilde{\sigma}^1{}_{rr}\,|_{r=1},\\ \widetilde{\alpha}_2\,\widetilde{u}_\beta^2\,|_{r=1}+\widetilde{\beta}_2\widetilde{\sigma}_{r\beta}^2\,|_{r=1}=\widetilde{q}_2(\beta)-\widetilde{\alpha}_2\,\widetilde{u}_\beta^1\,|_{r=1}-\widetilde{\beta}_2\widetilde{\sigma}^1{}_{r\beta}\,|_{r=1},\end{cases}$$

$$\widetilde{\alpha}_1=\alpha_1R,\widetilde{\beta}_1=\beta_1(\lambda+2\mu)\,,$$

$$\widetilde{q}_1(\beta)=q_1(\beta)-\widetilde{\alpha}_1\,\widetilde{u}_r^1\,|_{r=1}-\widetilde{\beta}_1\widetilde{\sigma}^1{}_{rr}\,|_{r=1}\,,$$

$$\widetilde{q}_2(\beta)=q_2(\beta)-\widetilde{\alpha}_2\,\widetilde{u}_\beta^1\,|_{r=1}-\widetilde{\beta}_2\widetilde{\sigma}^1{}_{r\beta}\,|_{r=1}\,,\tag{7}$$

$$\widetilde{u}_\beta^2, \widetilde{u}_r^2 \underset{r\to\infty}{\to} 0,$$

$$\widetilde{u}_\beta^2 |_{\tau=0} = 0, \quad \widetilde{u}_r^2 |_{\tau=0} = 0, \quad \dot{\widetilde{u}}_\beta^2 |_{\tau=0} = 0, \quad \dot{\widetilde{u}}_r^2 |_{\tau=0} = 0;$$

$M = 2$:

$$\eta^2 \frac{\partial^2 \widetilde{u}_r^2}{\partial \tau^2} = (\eta^2 - 1)\frac{\partial \widetilde{\theta}}{\partial r} + \left(\Delta \widetilde{u}_r^2 - \frac{1}{r^2}\left(2\frac{\partial \widetilde{u}_\alpha^2}{\partial \alpha} + \widetilde{u}_r^2 \right)\right),$$

$$\eta^2 \frac{\partial^2 \widetilde{u}_\alpha^2}{\partial \tau^2} = (\eta^2 - 1)\frac{1}{r}\frac{\partial \widetilde{\theta}}{\partial \beta} + \left(\Delta \widetilde{u}_\alpha^2 + \frac{1}{r^2}\left(2\frac{\partial \widetilde{u}_r^2}{\partial \alpha} - \widetilde{u}_\alpha^2 \right)\right),$$

$$\begin{cases} \widetilde{\alpha}_1 \widetilde{u}_r^2 |_{r=1} + \widetilde{\beta}_1 \widetilde{\sigma}_{rr}^2 |_{r=1} = \widetilde{q}_1(\alpha), \\ \widetilde{\alpha}_2 \widetilde{u}_\alpha^2 |_{r=1} + \widetilde{\beta}_2 \widetilde{\sigma}_{r\alpha}^2 |_{r=1} = \widetilde{q}_2(\alpha), \end{cases}$$

$$\widetilde{\alpha}_1 = \alpha_1 R, \widetilde{\beta}_1 = \beta_1(\lambda + 2\mu), \qquad (7')$$

$$\widetilde{q}_1(\alpha) = q_1(\alpha) - \widetilde{\alpha}_1 \widetilde{u}_r^1 |_{r=1} - \widetilde{\beta}_1 \widetilde{\sigma}^1{}_{rr} |_{r=1},$$

$$\widetilde{q}_2(\alpha) = q_2(\alpha) - \widetilde{\alpha}_2 \widetilde{u}_\alpha^1 |_{r=1} - \widetilde{\beta}_2 \widetilde{\sigma}^1{}_{r\alpha} |_{r=1},$$

$$\widetilde{u}_\alpha^2, \widetilde{u}_r^2 \underset{r\to\infty}{\to} 0,$$

$$\widetilde{u}_\alpha^2 |_{\tau=0} = 0, \quad \widetilde{u}_r^2 |_{\tau=0} = 0, \quad \dot{\widetilde{u}}_\alpha^2 |_{\tau=0} = 0, \quad \dot{\widetilde{u}}_r^2 |_{\tau=0} = 0.$$

Hereinafter omit symbol “wave” in dimensionless parameters.

The solution procedure for the formulated mathematical problems (7) is described in [2]. In turn, the Laplace transformation based on the solution of sets (7) can be written as ($\gamma_1 = 1$, $\gamma_2 = \eta$):

for $M = 3$:

$$u_r^{2L} = \sum_{n=0}^{\infty} \left(\frac{1}{r^{n+2}} \sum_{j,k=1}^{2} U_{njk}^L(r,s) \widetilde{q}_{kn}^L(s) e^{-\gamma_j (r-1)s} \right) P_n(\cos\beta), \qquad (8)$$

$$u_\beta^{2L} = -\sin\beta \sum_{n=1}^{\infty} \left(\frac{1}{r^{n+2}} \sum_{j,k=1}^{2} V_{njk}^L(r,s) \widetilde{q}_{kn}^L(s) e^{-\gamma_j (r-1)s} \right) C_{n-1}^{3/2}(\cos\beta), \qquad (9')$$

where P_n, $C_{n-1}^{3/2}$ are Legendre and ultra-spherical polynomials,

$$\widetilde{q}_1^L(s) = \sum_{n=0}^{\infty} \widetilde{q}_{1n}^L(s) P_n(\cos\beta),$$

$$\widetilde{q}_2^L(s) = -\sin\beta \sum_{n=1}^{\infty} \widetilde{q}_{2n}^L(s) C_{n-1}^{3/2}(\cos\beta),$$

$$U_{n11}^L = -\frac{a_{n22} R_{n1}(rs)}{X_n(s)}, \quad U_{n12}^L = \frac{a_{n12} R_{n1}(rs)}{X_n(s)}, \quad U_{n21}^L = n(n+1)\frac{a_{n21} R_{n0}(\eta rs)}{X_n(s)}, \quad U_{n22}^L = -n(n+1)\frac{a_{n11} R_{n0}(\eta rs)}{X_n(s)},$$

$$X_n(s) = -\widetilde{\alpha}_1 \widetilde{\beta}_2 D_{n1}(s,\eta s) + \widetilde{\beta}_1 \widetilde{\beta}_2 D_{n2}(s,\eta s) - \widetilde{\alpha}_1 \widetilde{\alpha}_2 D_{n3}(s,\eta s) + \widetilde{\beta}_1 \widetilde{\alpha}_2 D_{n4}(s,\eta s),$$

$$D_{n1}(x,y) = R_{n1}(x) Q_{n3}(y) - n(n+1) Q_{n2}(x) R_{n0}(y),$$

$$D_{n2}(x,y)=Q_{n1}(x)Q_{n3}(y)-n(n+1)Q_{n2}(x)Q_{n2}(y),$$

$$D_{n3}(x,y)=R_{n1}(x)R_{n3}(y)-n(n+1)R_{n0}(x)R_{n0}(y),$$

$$D_{n4}(x,y)=Q_{n1}(x)R_{n3}(y)-n(n+1)R_{n0}(x)Q_{n2}(y),$$

$$a_{n11}(s)=\tilde{\beta}_1 Q_{n1}(s)-\tilde{\alpha}_1 R_{n1}(s),\quad a_{n12}(s)=n(n+1)(\tilde{\beta}_1 Q_{n2}(\eta s)-\tilde{\alpha}_1 R_{n0}(\eta s)),$$

$$a_{n21}(s)=\tilde{\alpha}_2 R_{n0}(s)+\tilde{\beta}_2 Q_{n2}(s),\quad a_{n22}(s)=\tilde{\alpha}_2 R_{n3}(\eta s)+\tilde{\beta}_2 Q_{n3}(\eta s),$$

$$R_{n0}(z)=\sum_{k=0}^{n}A_{nk}z^{n-k},\quad R_{n1}(z)=\sum_{k=0}^{n+1}B_{nk}z^{n+1-k},\quad R_{n2}(z)=\sum_{k=0}^{n+2}C_{nk}z^{n+2-k},\quad R_{n3}(z)=R_{n+1,0}(z)-(n+1)R_{n0}(z),$$

$$A_{nk}=\begin{cases}\dfrac{(n+k)!}{(n-k)!k!2^k}, & 0\le k\le n,\\ 0, & k<0 \text{ or } k>n,\end{cases}\qquad B_{nk}=A_{nk}+kA_{n,k-1}, C_{nk}=B_{nk}+kB_{n,k-1},$$

$$Q_{n1}(z)=R_{n2}(z)-\kappa(2R_{n1}(z)+n(n+1)R_{n0}(z)),\quad Q_{n2}(z)=(1-\kappa)(R_{n1}(z)+R_{n0}(z)),$$

$$Q_{n3}(z)=\frac{1-\kappa}{2}(R_{n2}(z)+(n+2)(n-1)R_{n0}(z)).$$

And for $M = 2$:

$$u_r^{2L}=\frac{1}{r}\sum_{n=-\infty}^{\infty}\left(\sum_{j,k=1}^{2}u_{nj}^L(\gamma_j rs)Y_{njk}(s)\tilde{q}_{kn}^L(s)\right)e^{in\alpha},\tag{8′}$$

$$u_\alpha^{2L}=\frac{1}{r}\sum_{n=-\infty}^{\infty}\left(\sum_{j,k=1}^{2}v_{nj}^L(\gamma_j rs)Y_{njk}(s)\tilde{q}_{kn}^L(s)\right)e^{in\alpha},\tag{9′}$$

where:

$$\tilde{q}_1^L(s)=\sum_{n=-\infty}^{\infty}\tilde{q}_{1n}^L(s)e^{in\alpha},\quad \tilde{q}_2^L(s)=\sum_{n=-\infty}^{\infty}\tilde{q}_{2n}^L(s)e^{in\alpha},$$

$$Y_{n11}(s)=\frac{a_{n22}(s)}{X_n(s)},\quad Y_{n12}=-\frac{a_{n12}(s)}{X_n(s)},\quad Y_{n21}=-\frac{a_{n21}(s)}{X_n(s)},\quad Y_{n22}=\frac{a_{n11}(s)}{X_n(s)},$$

$$X_n(s)=a_{n11}(s)a_{n22}(s)-a_{n12}(s)a_{n21}(s),$$

$$u_{n1}^L(s)=-v_{n2}^L(s)=nK_n(s)-sK_{n+1}(s),$$

$$u_{n2}^L(s)=v_{n1}^L(s)=inK_n(s),$$

$$\sigma_{rn1}^L(s)=((1-\kappa)n(n-1)+s^2)K_n(s)-(1-\kappa)sK_{n+1}(s),$$

$$\sigma_{rn2}^L(s)=\sigma_{an1}^L(s)=(1-\kappa)in((n-1)K_n(s)-sK_{n+1}(s)),$$

$$\sigma_{an2}^L(s)=-(1-\kappa)\left(\left(n(n-1)+\frac{s^2}{2}\right)K_n(s)+sK_{n+1}(s)\right),$$

K_n is modified Bessel's function of n-th order.

Fulfilling the Laplace transformation of (8) and (9), we obtain the solution of the second model problem concurrently with the solution of initial model problems.

SOLUTION OF THE SECOND PROBLEM

First, we find coefficients of the Fourier series $\tilde{q}_{kn}(t)$. Analytical derivation of these coefficients for a plane problem is rather difficult. Interpolation was used. For spatial problems we can find these coefficients through computer algebra. As the analytical conversion of the Laplace transformation was impossible in the plane case, the approximated inversion was fulfilled by formulae cited in [3]. Based on the properties of the Laplace transformation [2], the complete inversion (8) and (9) is performed by numerical convolution.

EXAMPLES OF THE PROBLEM SOLUTIONS

Dimensionless parameters are used in solving the problems cited below.

Example 1. Consider the action of concentrated force, which time dependence has a shape of triangular impulse:

$$f(\tau) = 10^{12}(\tau H(\tau) - 2(\tau - 1/2)H(\tau - 1/2) + (\tau - 1)H(\tau - 1)), \text{ N.}$$

Points of the force application are at a distance of 10 radii from the center of an underground void (mine working) simulated by a sphere. The sphere surface is stress-free. We took the data on the potassium salt mass: the mine working radius, $R = 3$ m; Young modulus $E = 10$ GPa, Poisson's ratio $\nu = 0.3$; density $\rho = 2200$ kg/m^3. The first six terms of the series (7) and (8) are taken to plot the solution, namely, $\alpha_1 = \alpha_2 = 0$, $\beta_1 = \beta_2 = 1$.

The radial and tangential displacements within rocks between the disturbance source and the mine working boundary are shown in Fig. 1; the same for the mine working contour is in Fig. 2.

Example 2. Consider action of a concentrated force on a rock mass with a spherical enclosure. The functional time dependence of the force is expressed as "smoothed" triangle impulse:

$$f(\tau) = 10^{12}((4 + (-16 + 16(\tau - 1/2))(\tau - 1/2))\tau^2)(H(\tau) - H(\tau - 1)), \text{ N.}$$

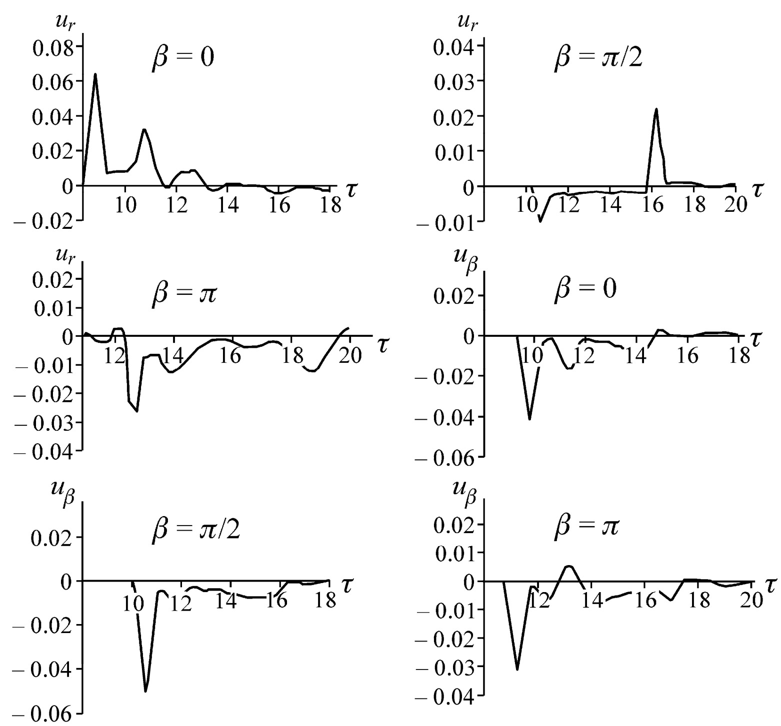

Fig. 1. Displacements in rock mass at $r = 2$

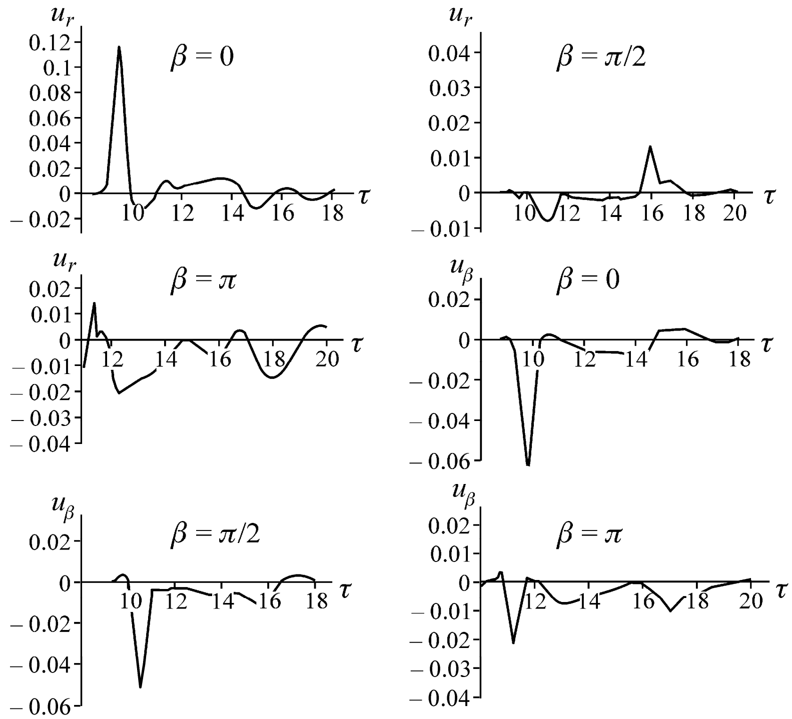

Fig. 2. Displacements at mine working contour

The point of the force application locates at distance of 10 radii of the sphere from its center. Consider a working with a stiff fill: the fill material has stiffness few times higher than that of the rock material. The values used in the first example are taken as initial ones.

Calculation data for this model problem are shown in Figs. 3 and 4.

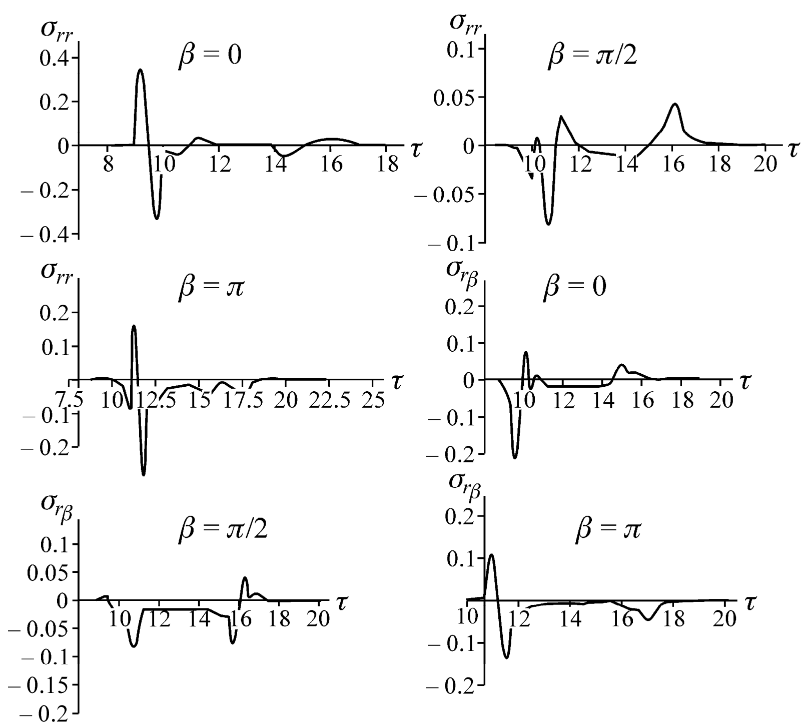

Fig. 3. Stresses at mine working contour

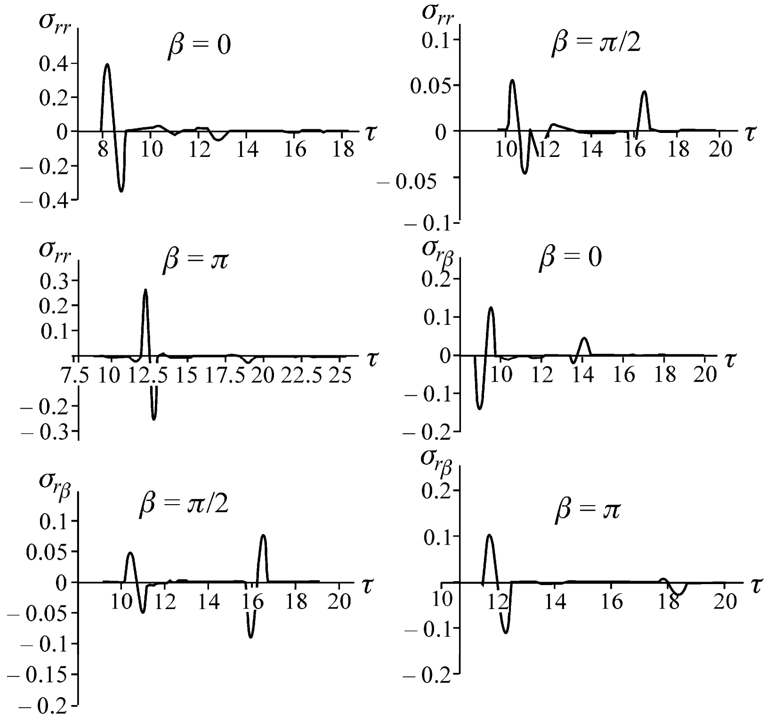

Fig. 4. Stresses in rock mass at $r = 2$

Example 3. Examine a probable arise of failure zones in a rock mass with a goaf under action of concentrated force. Different criteria of dynamic failure are elaborated and practiced [4]. Let us employ a generalized form of the simplest one $\max\{\sigma_1, \sigma_2, \sigma_3\} = \sigma_c$, where σ_1, σ_2 and σ_3 are principal stresses σ_c is compressive strength. In this case we take dimensionless value σ_c = 0.00032.

Apply the above criterion to the calculation data for the second model problem. At $\beta = 0$ stress σ_{rr} is principal. Curve for σ_{rr} at the mine working contour and straight line for $\sigma_{rr} = 1000\sigma_c$ are shown in Fig. 5.

Thus, the failure is to occur at the working contour under load 1000 times lower than that, cited in example 2, namely, 10^9 N. It can be concluded from the analysis of the stress state of the study rock mass at the section between a failure source and a mine working that the failure in this very area is also possible under load 500 times lower than the magnitude reported in the example.

Example 4. Examine the concentrated force action in an elastic space with a circular hole. Direction of the force action coincides with the radius-vector of the force application point, its functional time dependence is a triangle impulse:

$$f(\tau) = 10^9(\tau H(\tau) - 2(\tau - 1/2)H(\tau - 1/2) + (\tau - 1)H(\tau - 1)), \text{ N.}$$

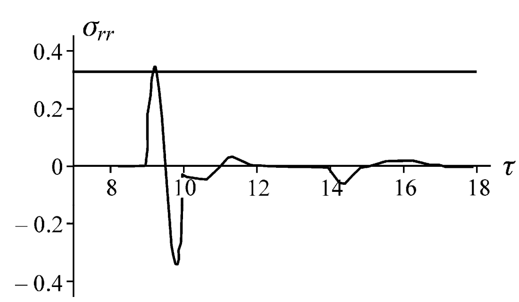

Fig. 5. Comparison of σ_{rr} with the limit value

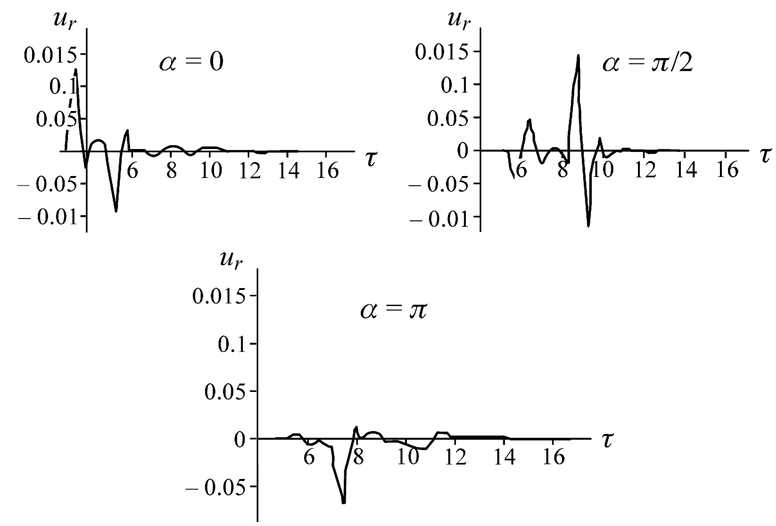

Fig. 6. Radial displacements at $r = 2$

The point of the force application is at a distance of 5 radii from the center of working modeled as a circle with a stiffly fixed boundary. The source data correspond to the potassium salt mass: $R = 3$ m, $E = 10$ GPa; $\nu = 0.3$; $\rho = 2200$ kg/m^3.

The first seven terms of the series (8′) and (9′) were taken and $\alpha_1 = \alpha_2 = 1$, $\beta_1 = \beta_2 = 0$.

The radial displacements at distance of two radii from the mine working center are demonstrated in Fig. 6.

CONCLUSION

The model problems discussed in the present paper have wide scope of application as mineral exploitation severely disturbs dynamic equilibrium of environment.

In the recent years we observe a strong and growing interest in the investigations into dynamic phenomena of a wide intensity range in mining areas all over the world. One of promoting factors is the greater frequency of “large-scale catastrophic events” in regions with the developed mining industry. The large-scale extraction of rocks induces considerable changes of their stress state, which works as the background for dangerous rock pressure manifestations, intensive fracturing, motions along faults, and so on.

To identify stresses and strains caused by mining, and conditions for release of accumulated energy is of prime importance to predict production-induced catastrophes. The safe exploitation of mineral resources in complex areas, in particular, requires development of models to describe properly the respective processes and their manifestations.

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