- 3. Ознакомление с готовым решением аналогичной военно-прикладной задачи на персональном компьютере с помощью пакета прикладных программ в системе Mathcad в виде графиков функций  $m_1(t)$  и  $m_2(t)$ , по которым курсанты учатся определять время окончания боя и количество оставшихся боевых единиц у победившей стороны.
- 4. Ввод боевых данных  $\{N_i, \lambda_i, P_i\}$  группировок  $A_i, i=1,2$ , своего варианта исходной задачи и изучение двух случаев, когда боевые возможности группировками  $A_1$  постоянны, а группировки  $A_2$  изменяются произвольным разумным образом и наоборот. В зависимости от изменения параметров  $\{N_i, \widetilde{\lambda}_i, P_i\}, i=1,2$ , очевидно, будут изменяться время окончания боя и его исход.
- 5. Заполнение таблиц результатов расчетов и дополнительных расчетов, проведенных на персональных компьютерах, расчетов сравнительного анализа боевых действий двух группировок и других бланков отчета о выполнении данной лабораторной работы согласно указанным общим правилам оформления отчетов по лабораторным работам. Формулировка оперативно-тактических выводов: какие оперативно-тактические решения необходимо принять командованию группировки  $A_1$  ( $A_2$ ), чтобы закончить бой за минимальное время и с наименьшими потерями.

Целью данной лабораторной работы является: закрепление математических знаний и умений, полученных курсантами на лекциях и практических занятиях по высшей математике, приобретение оперативно-тактического мышления для принятия командных решений по руководству боевыми действиями, освоение методов экспериментальных исследований, повышение практических навыков работы с вычислительной компьютерной техникой и стандартными прикладными пакетами программ.

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## INTERDISCIPLINARY CONNECTIONS IN A DIFFERENTIAL RESEARCH MODEL OF OSCILLATIONS IN BIOLOGICAL SYSTEMS

### H.M. Hubal

When teaching students we must take into account that in many research studies one needs interdisciplinary connections. Interdisciplinary connections are most effective when mathematicians understand the sciences, which knowledge is necessary to solve the problems, and the specialists of the discipline understand mathematical methods to solve the problems.

This research study uses interdisciplinary connections.

Many environmental phenomena of life show that there exist biological clocks in animals and plants, including even single-celled animals. This is evidenced by the rhythmic heart contraction, closing the corollas of many flowers after dark, periodic change of the rate of photosynthesis in plants, fluctuations in the size of the cell nuclei and so on.

Thus, nature needs the oscillatory processes synchronized to the time of day.

Construct a differential equation for the chemical reaction in a homogeneous medium.

Let the substance  $P_1$  be in excess in some volume, i.e. the costs of the substance  $P_1$  are almost invisible in the reaction process. The molecules of the substance  $P_1$  with the constant velocity  $\lambda_0$  are converted into the molecules of the substance X (here we deal with the zeroth-order reaction). The substance X can convert into the substance Y. This is the second-order reaction since the greater the concentration of the substance Y, the greater its rate. In the following kinetic scheme the reverse arrow above the symbol Y indicates this dependence. The molecules of the substance Y, in turn, are irreversibly decomposed, this results the formation of the substance  $P_2$  (the first-order reaction).

Let us write the kinetic scheme of this reaction in the form:  $P_1 \xrightarrow{\lambda_0} X \xrightarrow{\lambda_1} Y \xrightarrow{\lambda_2} P_2$ .

Construct a mathematical model of this reaction, denoting by X, Y and  $P_2$  the concentrations of the corresponding substances:

$$\frac{dX}{dt} = \lambda_0 - \lambda_1 XY, \quad \frac{dY}{dt} = \lambda_1 XY - \lambda_2 Y, \quad \frac{dP_2}{dt} = \lambda_2 Y. \tag{1}$$

Since the first two equations do not depend on  $P_2$ , they can be considered separately. Find out whether this reaction goes so that the rate of formation of the substance  $P_2$  remains constant. This will be in the case when the concentrations X and Y do not change with time, i. e. dX/dt=0, dY/dt=0.

Using these conditions, according to (1) we obtain two algebraic equations relating the equilibrium concentrations  $\overline{X}$  and  $\overline{Y}$ :

$$\lambda_0 - \lambda_1 \overline{X} \overline{Y} = 0, \quad \lambda_1 \overline{X} \overline{Y} - \lambda_2 \overline{Y} = 0.$$
 (2)

The solution of the system (2) is  $\overline{X} = \lambda_2/\lambda_1$ ,  $\overline{Y} = \lambda_0/\lambda_2$ .

For the concentrations X and Y we define small deviations x(t) and y(t) from equilibrium concentrations  $\overline{X}$  and  $\overline{Y}$ , i.e.  $X(t) = \overline{X} + x(t)$ ,  $Y(t) = \overline{Y} + y(t)$ .

Substituting these expressions into the equation (1), taking into account the solution of the system (2) and the fact that the values  $\overline{X}$  and  $\overline{Y}$  are constant, we obtain the system of differential equations for the deviations x(t) and y(t):

$$\frac{dx}{dt} - \lambda_2 y - \lambda_1 x y - \frac{\lambda_1 \lambda_0}{\lambda_2} x, \quad \frac{dy}{dt} = \frac{\lambda_1 \lambda_0}{\lambda_2} x + \lambda_1 x y.$$

Neglecting terms that contain the values of the second order of smallness xy, we obtain the linearizable system for deviations:

$$\frac{dx}{dt} = -\lambda_2 y - \frac{\lambda_1 \lambda_0}{\lambda_2} x, \quad \frac{dy}{dt} = \frac{\lambda_1 \lambda_0}{\lambda_2} x. \tag{3}$$

Denote  $\lambda_1 \lambda_0 / \lambda_2 = 2\delta$ ,  $\lambda_1 \lambda_0 = \delta_1^2$  and get a single differential equation of the second order for x(t). Then we can write the system (3) in the form:

$$\frac{dx}{dt} = -\frac{\delta_1^2}{2\delta}y - 2\delta x, \quad \frac{dy}{dt} = 2\delta x.$$

By differentiating the first obtained equation with respect to t, we obtain:

$$\frac{d^2x}{dt^2} = -\frac{\delta_1^2}{2\delta} \frac{dy}{dt} - 2\delta \frac{dx}{dt}.$$

By substituting the value dy/dt from the second equation of the system into obtained differential equation, we obtain the following differential equation:

$$\frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + \delta_1^2 x = 0. (4)$$

Denote  $\delta^2 - \delta_1^2 = -\omega^2$ ,  $\sqrt{C_1^2 + C_2^2} = A$ ,  $C_1/\sqrt{C_1^2 + C_2^2} = \sin \alpha$ ,  $C_2/\sqrt{C_1^2 + C_2^2} = \cos \alpha$ . Then we obtain the general solution (4) in the form:

$$x = Ae^{-\delta t}\sin(\omega t + \alpha). \tag{5}$$

The expression (5) indicates that there are damped oscillations, where  $\omega$  is the oscillation frequency,  $Ae^{-\delta t}$  is the oscillation amplitude, which is equal to A at initial moment of time (t=0) and it decreases with time.

Therefore, the considered chemical reaction in biological systems goes in an oscillatory mode. As you can see, one needs to use the knowledge on ecology, biology, chemistry, physics in addition to mathematical knowledge in this research study.

# AN ACTIVE LEARNING E-ENVIRONMENT FOR THE COURSE «DISCRETE MATHEMATICS» INTENDED FOR SPECIALTY «SOFTWARE OF INFORMATIONAL TECHNOLOGIES»

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The goal of the course Discrete mathematics for specialty "Software of informational technologies" is to give students all of the mathematical foundations they need for their future studies and an understanding of important mathematical concepts together with a sense of why these concepts are important for applications, to show the practicality of discrete mathematics.

We wanted to design a comprehensive course using web-technologies and Problem Based Learning. We hope that we have achieved these goals. We have created practically oriented, engaging, online learning course with different kinds of student-to-student and student-to-professor interaction via the Internet. We have provided e-support of the course through university data portal including e-versions of lectures, laboratory classes, seminars, home tasks, individual home tasks, tests, projects for quick access to information. We propose two languages of teaching: English and Russian. We hope English variant of the course will give students an opportunity to integrate in European educational and professional areas.

The list of topics under consideration in the course is as follows.

**Propositional logic.** Propositions. Compound propositions. Conditional statements. Truth tables of compound propositions. Tautologies and contradictions. Logical equivalences. Propositional satisfiability. Satisfiability problem.

**Predicates and quantifiers.** Predicates. Quantifiers. Quantifiers with restricted domains. Logical equivalences involving quantifiers. Negating quantified expressions. Nested quantifiers.

Valid arguments Rules of inference. Valid arguments in propositional logic Rules of inference. Rules of inference for quantified statements. Using rules of inference to build arguments.

**Set Theory.** Terminology. Venn diagrams. Set operations. Boolean algebra of sets. Computer representation of sets. The multiplication principle. The addition principle. The pigeonhole principle. The principle of inclusion-exclusion.

**Relations.** Relations, properties of relations. Equivalence relations. Partial orderings. Hasse diagrams. The topological sorting algorithm.

**Combinatorics.** Permutations. Combinations. The binomial theorem. Pascal's identity and triangle. Permutations with repetition. Combinations with repetition. Rearrangement theorem.

Solving linear recurrence relations. Linear homogeneous recurrence relations with constant coefficients. Solving linear homogeneous recurrence relations with constant coefficients. Solving linear homogeneous recurrence relations with constant coefficients of degree two and of degree three. Linear nonhomogeneous recurrence relations with constant coefficients. Generating functions. Using generating functions to solve recurrence relations.