$$\dot{x}_2 = -x_1 + y_2, \quad \dot{y}_2 = -\frac{3x_2}{\|\mathbf{x}\|^3} - x_2 - y_1 + u_2,$$

$$\dot{x}_3 = y_3, \quad \dot{y}_3 = -\frac{3x_3}{\|\mathbf{x}\|^3} - x_3 + u_3,$$

where $\mathbf{x} = \langle \mathbf{x_1}; \mathbf{x_2}; \mathbf{x_3} \rangle$ is coordinate vector of the celestial body, $\mathbf{y} = \langle \mathbf{y_1}; \mathbf{y_2}; \mathbf{y_3} \rangle$ is an impulse vector, and $\mathbf{u} = \langle \mathbf{u_1}; \mathbf{u_2}; \mathbf{u_3} \rangle$ is the acceleration vector [1]. The center of mass of the Earth coincides with the origin of the coordinate system, while the Ox_1 axis is directed along the axis connecting the centers of mass of the Earth and the Sun. $\|\cdot\|$ is the Euclidean norm of a vector. The libration points L_1 and L_2 in the rotating coordinate system are stationary and have coordinates $\mathbf{x}^* = \langle \mathbf{1}; \mathbf{0}; \mathbf{0} \rangle$, $\mathbf{y}^* = \langle \mathbf{0}; \mathbf{1}; \mathbf{0} \rangle$ and $\mathbf{x}^{**} = \langle -\mathbf{1}; \mathbf{0}; \mathbf{0} \rangle$, $\mathbf{y}^{**} = \langle \mathbf{0}; -\mathbf{1}; \mathbf{0} \rangle$, respectively.

It is known that the libration points L_1 and L_2 are unstable. Hence, the question of retention of the celestial bodies motion in the neighborhood of these points is relevant. In some cases, instability may be a positive factor that helps reduce energy costs while maneuvering [2]. To intercept maneuvers in near-Earth space in [3] the developed scheme maneuvering is described. This scheme involves the selection and calculation of the orbit of expectations, and the construction of the active part of the trajectory.

For example, to solve the comet and asteroid hazard problem explores the idea of impact on a potentially dangerous object to alter its trajectory and thus prevent the threat of collision. To achieve this goal in [4] proposed to use gravity assist in near-Earth space. It uses properties of unstable trajectory collision. In this study, the use of another type of instability is proposed, namely, the instability property of the collinear libration points [3].

The study presents the results of research and numerical construction of the set of trajectories to intercept potentially dangerous objects in the near-Earth space.

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ON THE STABILITY OF PERIODIC POINTS OF THE THREE-DIMENSIONAL DIFFEOMORPHISMS

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Smooth C^r-diffeomorphisms of the three-dimensional space with a hyperbolic fixed point at the origin are considered, where $r \geqslant 1$ (case $r = \infty$ is included). The existence of non-transversal homoclinic point is assumed; i.e. intersection of the stable and unstable manifolds contains a point, referred to as a homoclinic point, and this point is a point of tangency of these manifolds.

It follows from results of Sh. Newhouse, B. F. Ivanov, L. P. Shil'nikov, S. V. Gonchenko and other authors (papers [1, 2, 3]) that, when the stable and unstable manifolds are tangent in a certain way, a neighborhood of the homoclinic point may contain stable periodic points, but at least one of the characteristic exponents for such points tends to zero with increasing the period.

Let f be a self-diffeomorphism of three-dimensional space of class C^r $(1 \leqslant r \leqslant \infty)$ with fixed hyperbolic fixed point at the origin. Obviously, there are two cases:

1) real matrix $Df(0)$ have real eigenvalues only;

2) real matrix $Df(0)$ have complex eigenvalues.

In papers [4, 5] the first case is considered. It is shown that under certain conditions imposed mainly on the character of tangency of the stable and unstable manifolds, a neighborhood of the homoclinic point contains an infinite set of stable periodic points whose characteristic exponents are negative and bounded away from zero. An example of such a diffeomorphism was considered in the monograph of V. A. Pliss [6].

The main purpose of the talk is to show that this result can be extended to the second case.

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