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Dirac Particle in the Presence of a Magnetic Charge in De Sitter Universe: Exact Solutions and Transparency of the Cosmological Horizon

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It has been shown that the monopole background does not affect the transparency properties of the de Sitter cosmological horizon for quantum particles with spin 1/2. One can expect that this conclusion is due only to the geometry of the de Sitter space-time, and it does not depend on spin of the particles.

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1. The analysis of the problem

The Dirac equation for a particle with spin 1/2 in presence of the Abelian monopole is solved exactly on the background of cosmological de Sitter model in static coordinates. Variables are separated with the use of technique of Wigner D-functions. The system of radial equations is solved in hypergeometric functions. The complete set of spinor wave solutions is constructed. For all values of quantum numbers, energy E and conserved extended angular momentum j, two pairs of linearly independent solutions are specified: running waves (to and from de Sitter horizon) and standing waves (regular and singular in the origin). It is shown that the known algorithm for calculation of the reflection coefficient $r_{E,j}$ on the background of the de Sitter space-time presumes an additional constrain on quantum numbers of solutions $ER/\hbar c \gg j$ where R is a curvature radius; beyond this part of solutions any constructive recipe to get expression for coefficient r_{Ej} does not exist. According to standard definition one straightforwardly gets $r_{Ej} = 0$. So, the monopole background does not affect the transparency properties of the de Sitter horizon. This conclusion does not depend on spin and mass of a particle.

2. Main result

In the de Sitter space model with the metric

$$dS^2 = \Phi dt^2 - \frac{dr^2}{\Phi} - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$
$$\Phi = 1 - r^2 \quad (1)$$

the Dirac monopole potential in Schwinger gauge can be represented as $A_\phi = g\cos\theta$. The Dirac equation on this background reads [1]

$$\left(i\frac{\gamma^0}{\sqrt{\Phi}}\partial_t + i\sqrt{\Phi}\gamma^3\partial_r + \frac{1}{r}\Sigma^k_{\theta,\phi} - M \right)\Psi = 0, \quad (2)$$

where

$$\Sigma^k_{\theta,\phi} = i\gamma^1\partial_\theta + \gamma^2\frac{i\partial_\phi + (i\sigma^{12} - k)\cos\theta}{\sin\theta}, \quad (3)$$

and $k \equiv eg/hc$ is quantized according to Dirac rule, $i\sigma^{12}$ stands for the third component of spin.

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There exist tree symmetry operators of the form

$$J_1 = l_1 + \frac{(i\sigma^{12} - k)\cos\phi}{\sin\theta} \,,$$
$$J_2 = l_2 + \frac{(i\sigma^{12} - k)\sin\phi}{\sin\theta} \,,$$
$$J_3 = l_3 \tag{4}$$

with l_i denoting components of the angular momentum.

Eigenvalue states of $\vec{J}^2$ and J_3, can be represented in the form

$$\Psi^k_{\epsilon jm}(t,r,\theta,\phi) = \frac{e^{-i\epsilon t}}{r} \left| \begin{array}{c} f_1(r) D_{k-1/2} \\ f_2(r) D_{k+1/2} \\ f_3(r) D_{k-1/2} \\ f_4(r) D_{k+1/2} \end{array} \right| . \tag{5}$$

Instead of spinor monopole harmonics, the technique of Wigner D-functions is used, $D_\sigma = D^j_{m,\sigma}(\phi,\theta,0)$. There exist an additional operator $\hat{K}^k$ which is diagonalized with simple eigenvalues K:

$$\hat{K}^k = -i\gamma^0\gamma^3\Sigma^k_{\theta,\phi},$$
$$K = -\delta\sqrt{(j+1/2)^2 - k^2} \tag{6}$$

leading to linear restrictions on the radial functions:

$$f_4 = \delta\ f_1 \ , \quad f_3 = \delta f_2, \quad \delta = \pm 1 \ . \tag{7}$$

The complete set of the solutions $\Psi_{\epsilon,j,m,\lambda}$ is constructed [2]. Special attention is given to the states of minimal values of the total angular moment $j_{\min}$. Detailed analysis of the system of radial equations is performed; for all values of quantum numbers, two pairs of linearly independent solutions are specified: running waves (to and from the horizon) and standing waves (regular and singular at the origin).

Let us introduce two notations

$$E = \mu\, mc^2, \quad \lambda = \frac{\hbar}{mc} \tag{8}$$

and make estimation $\frac{\lambda^2}{R^2} \sim 10^{-80}$.

Then, we can show that the known algorithm for calculation of the reflection coefficient R_{Ej} on the background of the de Sitter space-time results in following condition on quantum numbers of solutions

$$\mu^2 - 1 \gg \frac{\lambda^2}{R^2}\,[j(j+1)+2]. \tag{9}$$

In massless case, corresponding condition looks as follow

$$\frac{R^2\omega^2}{c^2} \gg [j(j+1)+2] \ , \tag{10}$$

or

$$\frac{R^2 4\pi^2}{\lambda^2} \gg [j(j+1)+2] \ . \tag{11}$$

When taking into account this constrain in the known result for $R_{Ej} = 0$.

Therefore, de Sitter horizon is completely transparent for spin 1/2 particles – see [1–12].

This statement is additionally substantiated through studying the expansion of the exact de Sitter's solutions in series on small parameter R^{-2}; The known recipe is based on zero order term in this series and cannot be improved anyhow.

The presence of the monopole background does not affect the transparency properties of the de Sitter horizon. This result does not depend on spin and mass of a particle.

The last but not the least mathematical remark should be given. All known quantum mechanical problems with potentials containing one barrier reduce to a second order differential equation with four singular points, the equation of Heun class. In particular, the most popular cosmological problem of that type is a particle in the Schwarzschild space-time background and it reduces to the Heun differential equation. Quantum mechanical problems of tunneling type are never linked to differential equation of hypergeometric type, equation with three singular points; but in the case of de Sitter model the wave equations for different fields, of spin 0, 1/2, and 1, after separation of variables are reduced to the second order differential equation with three singular points, and there exists no ground to search in these systems problems of tunneling class.

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