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# Influence of Plasma on Relativistic Images of Gravitational Lensing

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In presence of plasma the gravitational lensing is chromatic. Even in the homogeneous plasma the gravitational deflection differs from the vacuum deflection angle, and the gravitational deflection angle in plasma depends on the frequency of a photon. We discuss influence of plasma on relativistic images formed by photons which perform one or several turns around the lens.

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## 1. Introduction

Gravitational lensing is well developed theory which combines a wide range of phenomena connected with the deflection of light rays by gravity. General Relativity predicts that a light ray passing near a spherical body of mass $M$ with large impact parameter $b$ is deflected by the Einstein angle:

$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4M}{b}, \quad G = c = 1\,. \qquad (1)$$

This expression is valid if $b \gg R_S$, where $R_S = 2M$ is the Schwarzschild radius of the gravitating body. Einstein angle does not depend on the photon frequency, so gravitational lensing in vacuum is achromatic.

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## 2. Relativistic images in vacuum

Theory of gravitational lensing usually considers small deflection angles ($\hat{\alpha} \ll 1$) and vacuum. One way to expand the usual consideration is to go beyond the weak deflection limit. If the photon impact parameter is close to its critical value, $0 < b/M - 3\sqrt{3} \ll 1$, then the photon makes one or several turns around the black hole near a radius $r = 3M$ and flies off to infinity. It is the case when the deflection angle is large, it is usually referred as strong deflection limit [1]. Photons from a distant source which undergo one or several loops around a central object (lens), and then go to observer form images (Fig. 1), which are called relativistic images [2].

Using an exact expression for the deflection angle, Virbhadra and Ellis [2] calculated numerically positions and magnifications of relativistic images for the Schwarzschild space-time. Relativistic images can be also studied analytically with using of the deflection angle in strong deflection limit [1, 3].

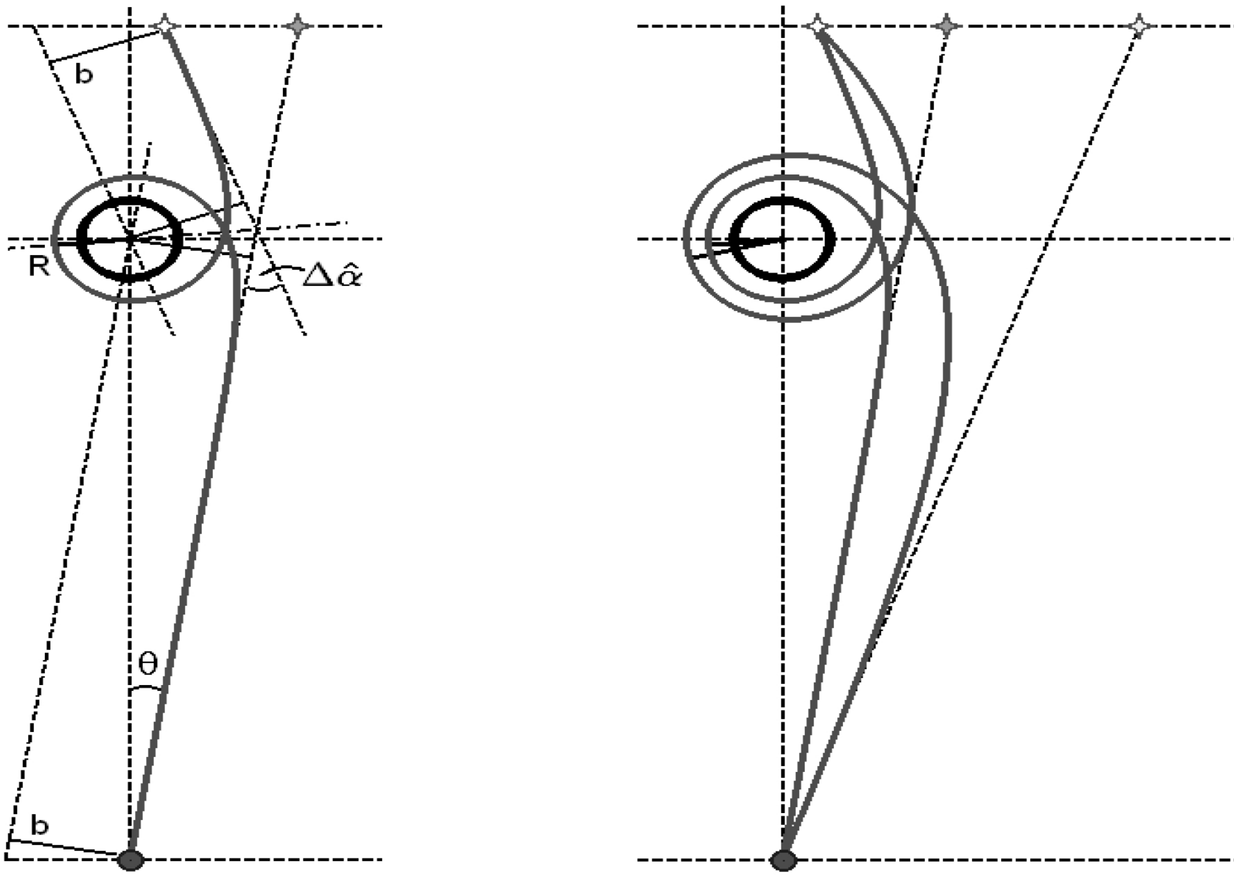


FIG. 1: A scheme of formation of relativistic images of a point source in vacuum (left) and plasma (right).

## 3. Relativistic images in plasma

Another way to expand the usual gravitational lens theory is to consider a cosmic plasma instead of vacuum. The most interesting effect in this case is chromatic gravitational deflection of light [4, 5]. It leads to angular difference in position of images at different frequencies. Gravitational lensing in plasma is considered in details in papers [4–6], see also [7, 8]. In particular, we have shown for the first time [4, 5] that due to dispersive properties of plasma even in a homogeneous plasma the gravitational deflection differs from the vacuum deflection angle, and the gravitational deflection angle in plasma depends on the frequency of the photon as

$$\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1-(\omega_e^2/\omega^2)}\right) . \qquad (2)$$

Here $\omega_e$ is the electron plasma frequency, $\omega$ is the photon frequency at infinity. This formula is written for homogeneous plasma and valid under condition of smallness of $\hat{\alpha}$.

Influence of plasma on relativistic images have been considered for the first time in paper [6].

The deflection angle of a photon moving in Schwarzschild metric in homogeneous plasma from infinity to central object ($R$ is the distance of the closest approach) and then to infinity is [6, 9]:

$$\hat{\alpha} = 2 \int_R^\infty \frac{dr}{\sqrt{r(r-2M)}\sqrt{\frac{h^2(r)}{h^2(R)} - 1}} - \pi \,,$$
$$h(r) = r\sqrt{\frac{r}{r-2M} - \frac{\omega_e^2(r)}{\omega^2}} \,. \tag{3}$$

In [6] we have derived for the first time the asymptotic analytic formulae for the gravitational deflection angle of photons in the Schwarzschild metric in homogeneous plasma, in a strong deflection limit:

$$\hat{\alpha}(b,x) = -\sqrt{\frac{1+x}{2x}} \ln\left[\frac{2\,z_1^2(x)}{3x}\frac{b-b_{cr}}{b_{cr}}\right] - \pi \,,$$
$$b_{cr} = \sqrt{3}\, r_M \sqrt{\frac{1+x}{3x-1}} \,,$$
$$z_1(x) = \frac{9x - 1 + 2\sqrt{6x(3x-1)}}{48x} \,,$$
$$r_M = 6M\,\frac{1+x}{1+3x} \,,$$
$$x = \sqrt{1 - \frac{8\omega_e^2}{9\omega^2}} \,. \tag{4}$$

This formula is valid for $b$ close to $b_{cr}$, $b_{cr}$ is the critical value of impact parameter under given $\omega_e^2/\omega^2$.

We apply these formulae for calculation of positions and magnifications of relativistic images in homogeneous plasma (see Fig. 1). We conclude that the presence of uniform plasma increases the angular size of relativistic rings or the angular separation of point images from the gravitating center. Presence of uniform plasma increases also magnifications of relativistic images. The angular separations and the magnifications can become significantly larger in comparison with vacuum case, when the photon frequency goes to the plasma frequency [6].

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