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Occurrence of Squeezed and Entangled Gluon States in QCD and Their Influence on Intermittency of Hadrons

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Theoretical justification for the occurrence of squeezed and entangled color states in QCD is given. Reducing the value of the scaling exponent in the transition from coherent to squeezed states is showed by investigation of the influence of squeezing effect on intermittency and scaling of the final hadron states taking into account the phase transition from color particles to hadrons.

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1. Introduction

Many experiments at $e^+e^-, p\bar{p}, ep$ colliders were devoted to hadronic physics, since detailed studies of such processes are important for better understanding and testing both perturbative and non-perturbative QCD and also for finding manifestations of new physics. The discrepancies between theoretical calculations and experimental data, for example for the width of multiplicity distribution (MD), suggest that the non-perturbative evolution of the quark-gluon cascade plays an important role. New gluon states, evaluated at the non-perturbative stage, contribute to various features of hadronic physics. In particular, such a contribution to MD can be in the form of the sub-Poissonian distribution [1, 2].

It is known that such a property is inherent for squeezed states (SS), which are well studied in quantum optics (QO) [3–6]. Photon squeezed states can have both sub-Poissonian and super-Poissonian statistics corresponding to antibunching and bunching of photons.

Therefore we believe that the non-perturbative stage of gluon evolution can be one of sources of the gluon SS in QCD by analogy with nonlinear medium for photon SS in QO.

2. Squeezed and entangled gluon states

By analogy with QO [6] the squeezing condition for gluons with different colors h, g is written as

$$\left\langle \hat{N}\left((\widehat{\Delta X}^{h,g}_{\lambda})_{\frac{1}{2}}\right)^2\right\rangle < 0 \qquad (1)$$

where $\hat{N}$ is a normal ordering operator, $\widehat{\Delta X}^{h,g}_{\lambda} = \hat{X}^{h,g}_{\lambda} - \left\langle \hat{X}^{h,g}_{\lambda}\right\rangle$, the phase-sensitive Hermitian operators $(\hat{X}^{h,g}_{\lambda})_1 = \left[\hat{b}^h_\lambda + \hat{b}^g_\lambda + \hat{b}^{h+}_\lambda + \hat{b}^{g+}_\lambda\right]/(2\sqrt{2})$ and $(\hat{X}^{h,g}_{\lambda})_2 = \left[\hat{b}^h_\lambda + \hat{b}^g_\lambda - \hat{b}^{h+}_\lambda - \hat{b}^{g+}_\lambda\right]/(2i\sqrt{2})$ are linear combination of the annihilating (creating) operators $\hat{b}^h_\lambda$ $(\hat{b}^{h+}_\lambda)$, $h, g = \overline{1, 8}$ are gluon color charges, λ is a polarization index. Averaging in (1) is performed over the final state vector

$$|\mathrm{f}\rangle \simeq |\mathrm{in}\rangle - i\,t\left(\hat{H}^{(3)}_{\mathrm{I}}(0) + \hat{H}^{(4)}_{\mathrm{I}}(0)\right)|\mathrm{in}\rangle \qquad (2)$$

which describes the gluon system later at small time t. Operators $\hat{H}^{(3)}_{\mathrm{I}}$ and $\hat{H}^{(3)}_{\mathrm{I}}$ describing the three- and four-gluon selfinteractions include combinations of three and four annihilating and

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creating operators [7]. Initial state vector $|\mathrm{in}\rangle$ describes the gluon system at the end of the perturbative stage [8] and may be regarded as a product of the coherent states of the gluons with different colors and polarization indexes $|\mathrm{in}\rangle \equiv |\,\alpha\rangle = \prod_{\lambda=1}^{3}\prod_{b=1}^{8}|\alpha_{\lambda}^{b}\rangle$.

Thus, the two-mode squeezing condition is

$$\hat{N}\left((\widehat{\Delta X}_{\lambda}^{h,g})_{\frac{1}{2}}\right)^2\Big\rangle = \pm\frac{i\,t}{8}\Big\{\langle\alpha|\big[\big[\hat{H}_{\mathrm{I}}(0),b_{\lambda}^{h+}\big],b_{\lambda}^{h+}\big]|\alpha\rangle-\langle\alpha|\big[b_{\lambda}^{h},\big[b_{\lambda}^{h},\hat{H}_{\mathrm{I}}(0)\big]\big]|\alpha\rangle$$
$$+\langle\alpha|\big[\big[\hat{H}_{\mathrm{I}}(0),b_{\lambda}^{g+}\big],b_{\lambda}^{g+}\big]|\alpha\rangle-\langle\alpha|\big[b_{\lambda}^{h},\big[b_{\lambda}^{h},\hat{H}_{\mathrm{I}}(0)\big]\big]|\alpha\rangle+2\langle\alpha|\big[\big[\hat{H}_{\mathrm{I}}(0),b_{\lambda}^{h+}\big],b_{\lambda}^{g+}\big]|\alpha\rangle$$
$$-2\langle\alpha|\big[b_{\lambda}^{g},\big[b_{\lambda}^{h},\hat{H}_{\mathrm{I}}(0)\big]\big]|\alpha\rangle\Big\}<0 \qquad (3)$$

where $\hat{H}_{\mathrm{I}}(0)=\hat{H}_{\mathrm{I}}^{(3)}(0)+\hat{H}_{\mathrm{I}}^{(4)}(0)$.

It can be shown that only the four-gluon self-interaction can yield a two-mode squeezing effect since

$$\big[\big[\hat{H}_{\mathrm{I}}^{(3)}(0),\hat{b}_{\lambda}^{h+}\big],\hat{b}_{\lambda}^{h+}\big]=0,\quad \big[\big[\hat{H}_{\mathrm{I}}^{(3)}(0),\hat{b}_{\lambda}^{g+}\big],\hat{b}_{\lambda}^{g+}\big]=0,\ \big[\big[\hat{H}_{\mathrm{I}}^{(3)}(0),\hat{b}_{\lambda}^{h+}\big],\hat{b}_{\lambda}^{g+}\big]=0,$$
$$\big[\hat{b}_{\lambda}^{g},\big[\hat{b}_{\lambda}^{h},\hat{H}_{\mathrm{I}}^{(3)}(0)\big]\big]=0,\ \big[\hat{b}_{\lambda}^{h},\big[\hat{b}_{\lambda}^{h},\hat{H}_{\mathrm{I}}^{(3)}(0)\big]\big]=0,\quad \big[\hat{b}_{\lambda}^{g},\big[\hat{b}_{\lambda}^{g},\hat{H}_{\mathrm{I}}^{(3)}(0)\big]\big]=0. \qquad (4)$$

In particular for the collinear gluons we have the condition

$$\left\langle\hat{N}\left((\widehat{\Delta X}_{\lambda}^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\,t\,\frac{\alpha_s\pi}{4k_0}(f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahb}f_{agc}+f_{agb}f_{ahc})$$
$$\times\sum_{\lambda_1\neq\lambda}|\alpha_{\lambda_1}^{b}||\alpha_{\lambda_1}^{c}|\sin(\gamma_{\lambda_1}^{b}+\gamma_{\lambda_1}^{c})<0. \qquad (5)$$

Here $|\alpha_{\lambda_1}^{b}|$ and $\gamma_{\lambda_1}^{b}$ are an amplitude and a phase of the initial gluon coherent field, $\alpha_s=g^2/(4\pi)$ is a coupling constant, f_{ahb} is a structure constant of the color group $SU_{\mathrm{c}}(3)$. The two-mode squeezing condition (5) is fulfilled for any cases apart from if all initial gluon coherent fields are real or imaginary. Obviously, the larger are both the amplitudes of the initial gluon coherent fields with different colour and polarization indexes and coupling constant, the larger is the two-mode squeezing effect.

Thus, non-perturbative four-gluon self-interaction is the source of the squeezing effect.

One of the entangled condition is

$$0<y<1 \qquad (6)$$

where entangled measure is defined by the analogy with QO as

$$y\simeq 2\sqrt{2}\left|\left\langle\hat{N}\left((\widehat{\Delta X}_{\lambda}^{h,g})_{\frac{1}{2}}\right)^2\right\rangle\right|. \qquad (7)$$

Entangled condition for collinear gluons can be written as

$$0 < \left| t\,\frac{\alpha_s \pi}{\sqrt{2}k_0}(f_{ahb}f_{ahc} + f_{agb}f_{agc} + f_{ahb}f_{agc} + f_{agb}f_{ahc}) \sum_{\lambda_1 \neq \lambda} |\alpha^b_{\lambda_1}||\alpha^c_{\lambda_1}| \sin(\gamma^b_{\lambda_1} + \gamma^c_{\lambda_1}) \right| < 1. \quad (8)$$

Obviously the squeezed gluon states are simultaneously entangled if the amplitudes of the initial gluon coherent fields are small enough.

3. Intermittency and scaling

If the transition from quark-gluon system to hadrons are considered as the phase transition, the factorial moments are defined as

$$F_q = J_q J_1^{-q} J_0^{q-1} \quad (9)$$

where

$$J_q = (q!)^2 \sum_{n=0}^{q} \frac{1}{n!((q-n)!)^2} \left(\frac{2|\nu|}{\mu}\right)^n \int_0^{\infty} \mathrm{d}t \exp\left\{|a|t - \frac{|a|^2}{2x}t^2\right\} \left| H_{q-n}\left(\sqrt{|t-|\nu|^2|}\,(\mu A_1 - |\nu| A_1^*)\right)\right|^2, \quad (10)$$

H_{q-n} is Hermite polynomial, $\mu = \mathrm{ch}(r), \nu = \mathrm{sh}(r)e^{i\vartheta}$, r is a squeezing parameter, ϑ defines squeezing direction, a is a phase transition parameter, x is function of the phase transition parameter and bin width, A_1 in the case of the coherent squeezed states (CSS) [5] reads

$$A_1 = \frac{1}{\sqrt{2\mu\nu}}[\mu e^{i\phi} + \nu e^{-i\phi}] \quad (11)$$

and in the case of the squeezed coherent states (SCS) [5]

$$A_1 = \frac{e^{i\phi}}{\sqrt{2\mu\nu}}\left[\mu^2 + |\nu|^2 - \mu(\nu^* e^{2i\phi} + \nu e^{-2i\phi})\right]^{-1/2}. \quad (12)$$

We have the intermittency [9, 10] if

$$\ln F_q \propto \varphi_q(-\ln x) \quad (13)$$

and scaling of Ochs-Wosiek [11] if

$$\ln F_q \propto \beta_q \ln F_2, \quad (14)$$

where φ_q is an intermittency index and β_q is fitted by the function $(q-1)^\nu$, ν is a scaling exponent.

If for coherent hadron states we have $\nu = 1.369 \pm 0.002$, then for squeezed hadron states we obtain

1. CSS, SCS: $\nu = 1.3621 \pm 0.0013$ at $r = 0.6$ and $(\phi - \vartheta/2) = \pi/2$.
2. SCS: $\nu = 1.3642 \pm 0.0013$ at $r = 0.6$ and $(\phi - \vartheta/2) = \pi/3$.
3. CSS: $\nu = 1.3439 \pm 0.0003$ at $r = 2.0$ and $(\phi - \vartheta/2) = \pi/3$.
4. SCS: $\nu = 1.3155 \pm 0.0007$ at $r = 2.0$ and $(\phi - \vartheta/2) = \pi/3$.

Thus we observe reducing the value of the scaling exponent in the transition from a coherent to squeezed hadron states.

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