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Scalar-Tensor Theory of Gravitation in Minkowski Space–Time

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In Minkowski space–time we consider the scalar and tensor fields, which form together the effective Riemann metric for a matter lagrangian. The demand of minimal interaction, including tensor selfinteraction, leads to Einstein equations for the tensor field and the nonlinear equation for the scalar field. The cosmological scenario to a given theory leads to the existence of the slow-roll regime of cosmological expansion in agreement with modern observations.

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As known now, we live in the universe where there is an abundance of dark matter and dark energy and the acceleration of cosmological expansion is observed [1, 2]. One of possible approach to theoretical description of this observations is the modification of General Relativity. In this connection the interest in Brans–Dicke scalar theories is resumed [3]. In such an approach the gravitational field is depended on the metric and scalar potential, moreover the latter may be interpreted as dark energy or dark matter [4].

Scalar fields are commonly used as candidates for the dark energy. Nevertheless, there is no unambiguous criterion for the choice of the field lagrangian in scalar field theories. Moreover, any scalar field in the slow-roll regime can model the cosmological constant and hence leads to an appropriate cosmological scenario.

In present paper we consider the minimal possible scalar-tensor generalization of Relativistic Theory of Gravitation (RTG) [5]. In this theory the tensor gravitational field is regarded in Minkowski space–time and effective metric arises due to the demand of gauge invariance under Lie transformations of dynamical variables. The gravitational fields equations of RTG massless variant are the Einstein ones for this effective metric. The gauge invariance demand allows the generalization of space–time metric in the form

$$\tilde{f}^{ik} = f(\phi)\tilde{g}^{ik}, \tag{1}$$

$f(\phi)$ is certain function of the field ϕ, $\tilde{g}^{ik} = \sqrt{-\gamma}(\gamma^{ik} + q\psi^{ik})$ is the effective RTG metric, forming from Minkowski metric γ^{ik} and gravitational tensor potential ψ^{ik}; $g = \det g_{ik}$,$\gamma = \det \gamma_{ik}$, q is gravitational interaction constant. In the framework of Brans–Dicke theory such approach was proposed in [6].

Let us consider in Minkowski space the most general form of scalar field Lagrangian, leading to linear field equations

$$L_0^{\phi} = \left(\frac{1}{2}\phi_{,i}\,\phi_{,k}\,\gamma^{ik} - \frac{1}{2}m^2\phi^2 + a\phi - b\right)\sqrt{-\gamma} \tag{2}$$

where m, a, b are constants. Gauge invariant Lagrangian for interacting scalar and tensor fields may be obtained with the help of replacement in the expression (2) Minkowski metric by metric f^{ik} and adding field ψ^{ik} Lagrangian of RTG

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$$L = L^G + L^\phi = \sqrt{-g}\Big[-\frac{1}{16\pi q^2}R + \frac{1}{2}f\phi_{,i}\,\phi_{,k}\,g^{ik} - \frac{1}{2}f^2m^2\phi^2 + af^2\phi - bf^2\Big] \quad (3)$$

where R is the scalar curvature for metric g_{ik}. At presence of matter we must add to (3) the matter Lagrangian $L^M(Q_A, f^{ik})$, Q_A are dynamical variables of matter.

For minimal interaction tensor and scalar fields with a matter, the scalar field equations have the form

$$\frac{\delta L^\phi}{\delta\phi} = -\frac{1}{2}\sqrt{-g}\frac{f_{,\phi}}{f}T^M \quad (4)$$

where $T^M = T^M_{ik}g^{ik}$ is the trace of the energy-momentum tensor in space with metric g^{ik}.

We choose the function f in the form

$$f(\phi) = (2k\phi)^{-1}, \quad (5)$$

k is the scalar interaction constant.

The field equations are

$$G_{ik} = 8\pi q^2[T^M_{ik}(Q_A, \phi, g^{mn}) + T^\phi_{ik}(\phi, g^{mn})], \quad (6)$$

$$g^{ik}D_iD_k\phi - \frac{D_i\phi D^i\phi}{2\phi} = kT^M(Q_A, \phi, g^{mn}). \quad (7)$$

The equations, restricting spin states of the field ψ^{ik} $D_i\tilde{g}^{ik} = 0$ [5] and matter equations must be added (D_i is covariant derivative in Minkowski space).

Thus, the usage in this approach the effective metric f^{ik} allows to introduce in a universal manner the interaction between a matter with nonzero trace of the energy-momentum tensor and scalar field. As a result the scalar field does not interact with electromagnetic one, that allows to regard it as a candidate for dark matter.

If the tensor field ψ equals to zero, the Lagrangian of the nonlinear scalar field in Minkowski space has the potential V given as

$$V(\phi) = \frac{c}{\phi} - \frac{d}{\phi^2} \quad (8)$$

where $c = a/4k^2, d = b/4k^2$. This Lagrangian possesses the property of spontaneously broken symmetry, if constant d is positive, and has the energy minimum equals to $-3c^2/4d$ at $\phi_0 = 2d/c$.

We assume that the metric f^{ik} coincides with the metric g^{ik} in the vacuum state, then $\phi_0 = 1/2k$. In linear approximation for the vacuum state, the field equations corresponding to used Lagrangian, describe a massive gravitational field with graviton mass equal to $qc\sqrt{12\pi/d}$

Consider the scenario of the Universe evolution in the given scalar-tensor theory. The metric of the homogenous and isotropic Universe is the Robertson-Walker metric with a flat 3-dimensional space.

The reason for choosing the flat model is mostly the observational evidence. In addition, if the problem is solved in the framework of RTG, the flat model is the only acceptable one [5].

We are interested in evolution of the Universe at the epoch following the annihilation of electron-positron pairs. At the given epoch we can treat the matter as consisting of the cold matter that includes barionic and dark matter and the radiation that includes photons and three types of neutrino and antineutrino. The equation of state is $p^{CM} = 0$ for the cold matter and $p^r = \epsilon^r/3$ for radiation. The equations of matter evolution follow from the gravitational field equations and have the form

$$(T^{(CM)}{}^k_i + T^{(r)}{}^k_i + T^{\varphi}{}^k_i)_{;k} = 0 \quad (9)$$

where the indices CM, r, φ denote the quantities related to the cold matter, the radiation and the scalar field respectively.

The performed numeric simulations of the full system of field equations, describing the cosmological solution for the homogenous and isotropic Universe, shows that for certain restrictions on the parameters the slow-roll regime is possible. In the slow-roll regime the scalar field models the dark energy in agreement with modern observational data.

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