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To the Problem of Compton Rotation of Photons in a Strong Magnetic Field: Limit of Total Spin Polarization of Electrons

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A formula is obtained for Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas with high degree of spin polarization of electrons in quantizing magnetic field.

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1. Introduction

The effect of Compton rotation of the plane of polarization of X- and gamma-photons in the absence of a magnetic field was predicted by V.G. Baryshevsky and V.L. Luboshitz in 1965 and experimentally tested on iron at early 1970s; the degree of electron spin polarization did not exceed 8% [1]. At the presence of a strong magnetic field (at a neutron star surface) the spin polarization is almost total, electron wave functions are changed. That is why we need to recalculate the difference of Compton scattering forward amplitudes for right and left circular polarizations of a photon. The method applied here was developed in [2].

Let us consider a constant homogeneous magnetic field with a strength B directed along z axis. Let us choose the following gauge of the vector potential:

$$A_0 = A_x = A_z = 0, A_y = Bx. \quad (1)$$

At elastic forward scattering, the 4-momentum of initial and final photon (moving at angle θ to z axis) with a wave vector k and frequency ω do not change:

$$\begin{aligned} ck_0 &= \omega, \quad ck_x = 0, \\ ck_y &= \omega \sin\theta, \quad ck_z = \omega\cos\theta. \end{aligned} \quad (2)$$

A photon with linear polarization is a superposition of 2 circularly polarized photons with opposite helicities. The 4-vector of right (+) or left (–) circular photon polarization is (transposing is denoted by T):

$$\sqrt{2} e^T_{(\pm)} = \left[\, 0 \;\; \mp i \;\; \cos\theta \;\; -\sin\theta \,\right]. \quad (3)$$

The 4-momentum of a virtual electron in R-process (g) and S-process (f) is

$$\begin{aligned} &c\lambda_0^{\pm} = \varepsilon_0 \pm \hbar\omega, c\lambda_2^{\pm} = p_y c \pm \hbar\omega \sin\theta, \\ &c\lambda_3^{\pm} = p_z c \pm \hbar\omega\cos\theta, \lambda^+ = g, \lambda^- = f. \end{aligned} \quad (4)$$

If e is the electric charge, m, p_y, p_z are electron's mass and momentum along y, z axes, respectively, then one can write out the electron wave functions using the bispinors u_n and Hermitian functions H_n, $n \geq 0$:

$$\begin{aligned} &\Psi_0(x) = iA_0\sqrt{2eBc\hbar}u_0, \\ &U_n(x) = \frac{1}{\sqrt{2}}\exp(-\frac{x^2}{2})H_n(x), \\ &\varepsilon_0 = \sqrt{m^2c^4 + p_z^2c^2}, \\ &A_0^{-1} = \sqrt{2c\hbar\varepsilon_0(\varepsilon_0 + mc^2)\sqrt{Bec\hbar}}, \\ &u_0^T = \left[\, 0 \;\; -mc^2 - \varepsilon_0 \;\; 0 \;\; p_z c \,\right]. \end{aligned}$$

Dirac matrices γ_k (k = 0, 1, 2, 3) are written in standard presentation. Electron propagator

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contains structures of the following type – ($\varepsilon_{n\lambda}$ is the energy of virtual electron on intermediate n-th Landau level, Γ_n is the level width):

$$G_B(\lambda,x)=\sqrt{\frac{Be}{c\hbar}}\sum_{n=0}^{\infty}\frac{\hbar c^2}{c^2\lambda_0^2-\varepsilon_{n\lambda}^2-i\cdot 0}D_n(\lambda), Y(\lambda)=\gamma_0\lambda_0-\gamma_3\lambda_3+mc\ ,$$
$$D_n(\lambda)=U_n(x_1)U_n(x_2)Y(\lambda)\beta_1+(1-\delta_{0n})U_{n-1}(x_1)U_{n-1}(x_2)Y(\lambda)\beta_2$$
$$+(1-\delta_{0n})i\sqrt{\frac{2neB\hbar}{c}}(U_{n-1}(x_1)U_n(x_2)\gamma_1\beta_1-U_n(x_1)U_{n-1}(x_2)\beta_1\gamma_1)\ ,$$
$$2\beta_1=1+i\gamma_2\gamma_1, 2\beta_2=1-i\gamma_2\gamma_1,\quad \varepsilon_{n\lambda}=\sqrt{m^2c^4+2ne\hbar Bc+\lambda_3^2c^2}-\frac{i\Gamma_n}{2}\ .$$

2. Calculation of the rotation angle

If $k_x=0$ then the matrix element of the process is [2]:

$$\Omega=\frac{-i\alpha(2\pi\hbar)^4ce_\mu e_\nu'^{\,*}}{\hbar VS_0\omega}\delta^3(\mathbf{p}+\hbar\mathbf{k}-\mathbf{p'}-\hbar\mathbf{k'})\chi_{\mu\nu},\quad \alpha=\frac{e^2}{\hbar c},$$
$$\chi_{\mu\nu}=\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}dx_1dx_2\bar{\Psi}_0(\xi_1)(\gamma_\nu G_B(g,\rho)\gamma_\mu+\gamma_\mu G_B(f,\eta)\gamma_\nu)\Psi_0(\xi_2), \tag{5}$$

$$j_r(p)=\sqrt{\frac{eB}{c\hbar}}(x_r+\frac{cp}{eB}),\quad \xi_r=j_r(p_y),\quad \rho_r=j_r(g_2),\eta_r=j_r(f_2),\quad r=1,2. \tag{6}$$

Here V is the normalization volume for the photon, S_0 is the normalization area for the electron in (xy) plane. δ-function includes y-, z- and t-components. The formula for the Compton rotation angle of the plane of linear polarization of hard X-photon per unit path (n_e is the electron density) is [1]:

$$\frac{d\varphi}{dl}=\frac{\pi n_e c}{\omega}\,\Re e(F_{(-)}-F_{(+)}). \tag{7}$$

Here $F_\pm$ are Compton forward scattering amplitudes for right (+) and left (–) circular photon polarizations. The relationship between F in (7) and Ω in (5), according to [1] and [3] (making a substitution for the x-component of δ-function through V, S_0), is:

$$\Omega=(\varepsilon_0+\hbar\omega)$$
$$\times\frac{i(2\pi\hbar)^4cF}{VS_0\varepsilon_0\hbar\omega}\delta^3(\vec{p}+\hbar\vec{k}-\vec{p'}-\hbar\vec{k'}). \tag{8}$$

Subject to the expressions (5), (6), and (8) we obtain for Eq. (7) the following formula (μ_B is the Bohr magneton):

$$\frac{d\varphi}{dl} = \frac{(\pi\hbar c)^2 n_e \alpha \cos\theta}{\hbar\omega(\varepsilon_0 + \hbar\omega)} \exp\left(-\frac{\phi}{2}\right) \sum_{n=1}^{\infty} \phi^{n-1}\ \Re e(\Xi_n^{(+)}(g,\theta) - \Xi_n^{(-)}(f,\theta)),$$

$$\phi = \frac{\hbar\omega^2 \sin^2\theta}{cBe}, \quad \Lambda_n(\lambda) = c^2\lambda_0^2 - \varepsilon_{n\lambda}^2 + \frac{\Gamma_n^2}{4}, \quad \Gamma_n \approx \frac{16(2n-1)\alpha(\mu_B B)^2}{3mc^2},$$

$$\Xi_n^{(\pm)}(\lambda,\theta) = \frac{((c\lambda_0\varepsilon_0 - m^2c^4)\cos\theta - p_z c(c\lambda_3\cos\theta \pm \sqrt{2n}\hbar\omega\sin^2\theta))\Lambda_n(\lambda)}{\Lambda_n^2(\lambda) + \Gamma_n^2\varepsilon_{n\lambda}^2}. \quad (9)$$

For example, if $n_e \sim 10^{23}$ cm^{-3}, $\hbar\omega \sim 0.1$ MeV, $B \sim 10^{13}$ Gs, then $d\varphi/dl \sim 10^2 - 10^3$ rad/cm at the resonance on n-th Landau level of virtual electron.

3. Summary

The difference of the Compton forward scattering amplitudes in quantizing magnetic field is calculated for an electron in the ground state and circularly polarized X-photons moving at an arbitrary angle to magnetic field strength lines with opposite helicities. A corresponding formula is obtained for calculation of the Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas at the limit of total spin polarization of electrons.

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