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Geometric Scalar Gravity

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We review the main properties of the recently proposed the Geometric Scalar Gravity (GSG), emphasizing its agreement with classical tests of the gravitational field in the solar system and the amplitude of the gravitational radiation.

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1. Introduction

Since general relativity (GR) is becoming centenary, in the near past few papers [1] have revisited its construction and previous theories of gravity that failed in trying to generalized the Newtonian one which, of course, include the scalar gravity. The main drawbacks of these ancient proposals can be summarized as follows: (i) there is a preferred-frame, represented by the global Lorentz invariance; (ii) the gravitational field source is the trace of the energy-momentum tensor. (iii) the flat Minkowski background is observable.

Notwithstanding, some theories involving scalar fields in different scenarios have been proposed up to now, trying to explain the observational tests where GR is not enough and, here and there, bringing in new ideas.

2. GSG in a nutshell

First of all, let us remark the main properties of the GSG [2]: (i) it satisfies the general covariance principle; (ii) the description of the gravitational interaction is done through a scalar field Φ; (iii) the dynamics of Φ is nonlinear; (iv) all kinds of matter and energy interact with Φ only through the gravitational metric

$$q^{\mu\nu} = \alpha\, \eta^{\mu\nu} + \beta\, \frac{\partial^\mu\Phi\partial^\nu\Phi}{w}$$

where $w = \eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$; (v) test particles follow geodesics relative to $q^{\mu\nu}$. We have $\alpha = \alpha(\Phi)$ and $\beta = \beta(\Phi)$. Also, the auxiliary metric $\eta^{\mu\nu}$ is non observable. All fields of the standard model are coupled to $q^{\mu\nu}$ and its derivatives in a covariant, universal and minimal way. It means that gravity is a geometrical phenomenon.

There are many ways of seeing the appearance of a curved space-time in this context [3]. The simplest one can be derived from the statement below:

Theorem 1. *Consider the Lagrangian* $L = V(\Phi)w$*, whose equation of motion corresponds to a nonlinear dynamics. This equation is equivalent to the Klein-Gordon one* $\Box\Phi = 0$ *in the metric* $q^{\mu\nu}$*, with* $\alpha + \beta = \alpha^3 V$.

The Newtonian limit for a test particle geodesic motion in $q_{\mu\nu}$ is reproduced if $\alpha = e^{-2\,\Phi}$. To determine β, we use the classical tests of gravity for a static and spherically symmetric configuration. In this case, the symmetries suggest that $\Phi = \Phi(R)$. Then, we take the background metric in spherical coordinates (t, R, θ, ϕ) and do the change $R = \sqrt{\alpha(r)}\, r$. This gives a gravitational metric like $ds^2 = (1/\alpha)\, dt^2 - B\, dr^2 - r^2\, d\,\Omega^2$ where we denote $B \equiv \frac{\alpha}{\alpha+\beta}\left(\frac{1}{2\alpha}\frac{d\alpha}{dr}\, r + 1\right)^2$.

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By successive approximations, we obtained $V(\Phi) = (1/4)(e^{\Phi} - 3e^{3\Phi})^2$ which yields

$$\Phi = \frac{1}{2}\ln\left(1 - \frac{r_H}{r}\right) \tag{1}$$

where $r_H \equiv 2MG/c^2$ due to the weak field limit of the scalar field Φ. Finally, the line element can be written down as

$$ds^2 = \left(1 - \frac{r_H}{r}\right) dt^2 - \left(1 - \frac{r_H}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{2}$$

which is equal to the Schwarzschild metric. If new observations require modification of the metric near to a compact object, then the shape of the potential $V(\Phi)$ should be changed.

3. GSG dynamics in the presence of matter

Now we briefly describe how matter couples to Φ through $q^{\mu\nu}$. We start with the action for Φ written in the Minkowski metric and then, from the variation with respect to Φ, we obtain an expression that should be rewritten in terms of $q^{\mu\nu}$ by using the Theorem. After, we add the Lagrangian of the matter, makes the variation w.r.t $q^{\mu\nu}$ to define $T_{\mu\nu}$ as usual and the general covariance guarantees its conservation. Ultimately, once $\delta q^{\mu\nu}$ is functionally dependent of $\delta\Phi$, we rewrite the results in terms of $\delta\Phi$ and, after some calculation, we get

$$\sqrt{V}\,\Box\Phi = \kappa\chi. \tag{3}$$

This equation describes the dynamics of GSG in the presence of matter. The quantity χ involves a non-trivial coupling between $\nabla_\mu\Phi$ and $T_{\mu\nu}$. In the Newtonian limit, the Poisson equation is achieved by fixing $\kappa \equiv 8\pi G/c^4$.

4. Gravitational Radiation

Following the main lines presented in [4], we provide an estimation of the order of magnitude of the gravitational waves (GW) using GSG. We bound the discussion to the simplest case of the dust. Then, we use the Theorem to rewrite Eq. (3) and the energy density $\rho = \alpha\rho_b$ in the Minkowski background. The equation of motion for Φ becomes

$$\Box\Phi = \frac{k}{2}\left[\frac{2}{k}\frac{\alpha - 9}{\alpha - 3}\,w - 2\left(\frac{1}{\alpha\sqrt{V}}\right)^3 \frac{\alpha^2}{\alpha - 3}\,\varrho_b\right]. \tag{4}$$

Matter conservation implies that there is a conserved mass which is the non-radiative term of the scalar field. By using the Green functions to solve Eq. (4), the scalar field Φ is proportional to the integral of the right hand side of Eq. (4) evaluated at the retarded time $t - |\mathbf{x} - \mathbf{x}'|$. Then, in the wave zone regime, after doing a power law expansion and correct approximations, we obtain the leading-order radiative term in the spherically symmetric case

$$\Phi(t, r) = -\frac{4\pi G}{r}\left\{\int dr' r'^2 \left[\varrho_0\left(11\Phi + v^2\right)\right]_{ret} + \mathcal{O}(2)\right\}, \tag{5}$$

Now we use Eq. (5) and the virial relation to estimate Φ for a source of mass M, radius R and velocity v, in the weak-field regime:

$$\Phi \sim \frac{GM}{rc^2}\left(\frac{v}{c}\right)^2. \tag{6}$$

The definition of the total rate of energy emission $d\mathcal{E}/dt$ in the wave zone is proportional to $(r\dot{\Phi})^2$ where we used the virial relation and $T_{\mu\nu}$ in flat space to express the radiative energy flux as $T^{0r}_{\Phi} = (1/4\pi G)\dot{\Phi}^2$. Finally, we get

$$\frac{d\mathcal{E}}{dt} \sim \frac{c^5}{G}\left(\frac{v}{c}\right)^{10} \sim \frac{c^5}{G}\left(\frac{GM}{Rc^2}\right)^5. \tag{7}$$

Note that GSG provides a monopole radiation for a spherically symmetric mass distribution, which is of the same order of magnitude of the quadrupole radiation in GR.

5. Concluding Remarks

GSG satisfies many requirements of a good theory of gravity: it is covariant, background independent (similarly to the field theory formulation of GR) and matter interacts with Φ only through the gravitational metric $q^{\mu\nu}$. So, the traditional drawbacks of scalar gravity are overcome.

It is clear that a careful analysis on GW in this context should be done. The aim of Sec. 4 was to show that GSG provides the same order of magnitude for the radiation G/c^5 in a completely different matter distribution. The crucial result will be the precise evaluation of the binary pulsar timing. This is non-trivial issue, but we will do this in the near future.

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