ON THE COMPLEXITY OF THE CLUSTERING MINIMUM BICLIQUE COMPLETION PROBLEM

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We consider the complexity results for the Clustering Minimum Biclique Completion problem restricted to subclasses of bipartite graphs.

A finite undirected graph $G = (V, E)$ is bipartite if its vertex set V can be partitioned into two sets $X, Y \subseteq V$ (partite sets) such that every edge of G has its ends in different sets X, Y. For a vertex $v \in V$, the set of vertices of the graph G adjacent to v is denoted by $N_G(v)$. Let $G = (X \cup Y, E)$ be an arbitrary bipartite graph with non-empty partite sets X, Y and let p be a positive integer such that $p \leq |X|$. If we add all edges of the set $\overline{E} = \{\{x, y\} : x \in X, y \in Y, \{x, y\} \notin E\}$ to the graph G, we obtain a complete bipartite graph $G' = (X \cup Y, E \cup \overline{E})$ whose the partite sets X can be partitioned into p non-empty sets $X_1, X_2, \ldots, X_p$ with the following condition: $N_{G'}(x) = N_{G'}(x')$ for every pair of vertices $x, x' \in X_i$, $i \in \{1, 2, \ldots, p\}$. The Clustering Minimum Biclique Completion problem asks for a minimum cardinality set $E' \subseteq \overline{E}$ to be added to the graph G so that the partite set X of the resulting bipartite graph $G' = (X \cup Y, E \cup E')$ can be partitioned into p non-empty sets $X_1, X_2, \ldots, X_p$ with the same condition. Let $\xi_p(G) = |E'|$. The decision version of the problem can be stated as follows:

Clustering Minimum Biclique Completion

Instance: A bipartite graph $G = (X \cup Y, E)$ with non-empty parts X and Y, two positive integers $p \leq |X|$ and k.

Question: Can G be transformed by adding at most k additional edges connecting vertices from different sets X, Y into a bipartite graph G' whose the partite set X can be partitioned into p non-empty sets $X_1, \ldots, X_p$ such that $N_{G'}(x) = N_{G'}(x')$ for any two vertices $x, x' \in X_i$, $i \in \{1, 2, \ldots, p\}$? Equivalently, is $\xi_p(G) \leq k$?

This problem, also known as the Multicast Partition problem, has been introduced by N. Faure [1, 2] and arises in telecommunication network technologies [2]. The computational complexity of the Clustering Minimum Biclique Completion problem for various subclasses of bipartite graphs is little-studied. To the best of our knowledge, there is only two results. N. Faure et. al. in [2] showed that: (a) the problem is NP-complete for fixed $p = 2$ (by a reduction from the Maximum Edge Biclique problem) and (b) the problem restricted to bipartite graphs $G = (X \cup Y, E)$ with degrees of vertices of Y at most 1 can be solved in strongly polynomial time by a dynamic programming algorithm. On the other hand, the problem is well-studied from a mathematical programming point of view [3–6].

We provide several well-studied subclasses of bipartite graphs, for which the considered problem remains NP-complete. Recall that a graph G is H-free if G does not contain an isomorphic copy of the graph H as an induced subgraph.

Theorem 1. Clustering Minimum Biclique Completion *for P_4-free bipartite graphs is NP-complete.*

Corollary 1. Clustering Minimum Biclique Completion *is NP-complete for the following subclasses of bipartite graphs* (*for definitions we refer to* [7, 8])*: bipartite permutation graphs, convex graphs and chordal bipartite graphs.*

A bipartite graph $G = (X \cup Y, E)$ is $(3, 2)$-regular if the degree of every vertex of X is 3 and the degree of every vertex of Y is 2.

Theorem 2. Clustering Minimum Biclique Completion *for C_4-free $(3, 2)$-regular bipartite graphs and $p = 2$ is NP-complete.*

On the positive side, $\xi_p(G)$ for $2K_2$-free bipartite graphs $G = (X \cup Y, E)$ can be computed in $O(|X|^4|Y|p)$ time.

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