GRAPHS WITH EQUAL DISTANCE PARAMETERS

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1. Introduction. The concepts of distance packing and covering in graphs was introduced by Meir and Moon in [1]. We consider finite, simple, undirected graphs without loops and multiple edges. A set P of vertices in a graph is called a *k-packing* (or a *k-independent set*) if the distance between any two distinct vertices in this set is larger than k. The maximum size of the k-packings in a graph G is called the *k-packing number* of G and is denoted by $\rho_k(G)$. A set D of vertices in a graph G is called a *k-covering* (or a *k-domination set*) if for any vertex v in $V(G)$ there is a vertex in D at a distance no more than k from v. The minimum size of the k-domination sets in a graph G is called the *k-domination number* of G and is denoted by $\gamma_k(G)$. A set I of vertices in a graph is called a *k-independent domination set* if it is both a k-packing and a k-covering. The minimum size of the k-independent domination sets in a graph G is called the *k-independent domination number* of G and is denoted by $i_k(G)$. For every graph G, the inequality $\gamma_k(G) \leq i_k(G) \leq \rho_k(G)$ holds.

The relation between the distance packing, domination and independent domination numbers has been widely studied in the literature. In [1] it is shown that the equality $\gamma_k(T) = \rho_{2k}(T)$ holds for any tree T. In [2] this equality is proved for a larger class of sun-free chordal graphs, which includes line graphs of trees, interval graphs and powers of block graphs. In [3] the graphs with equal k-packing and $2k$-packing numbers are characterized. This characterization implies a simple polynomial recognition algorithm for such graphs.

2. Recognition of k-equipackable graphs. A graph G is called *k-equipackable* if $i_k(G) = \rho_k(G)$. For $k = 1$ such graphs have been widely studied under the name *well-covered*, see the survey by Plummer [4]. In [5] it is shown that deciding whether a graph is not k-equipackable is an NP-complete problem. Lesk and Plummer [6] obtained that the recognition of line 1-equipackable graphs is a polynomially solvable problem. In [7] it is proved that recognizing non-2-equipackable line graphs is an NP-complete problem. Our following result establishes the computational complexity for the problem of recognizing k-equipackable line graphs for $k \geq 2$.

Theorem 1. *Deciding whether a given line graph is not k-equipackable is an NP-complete problem for any fixed $k \geq 2$.*

Corollary 1. *Let G be a line graph. Deciding whether G^k is not well-covered is an NP-complete problem for any fixed $k \geq 2$.*

3. Subclasses of k-equipackable graphs. Let k be a positive integer and $\mathcal{R}_k$ be the class of graphs with $\rho_k(G) = \rho_{2k}(G)$ for every $G \in \mathcal{R}_k$. In [3] a simple characterization of the class $\mathcal{R}_k$ is obtained. In [2] it is proved that for every sun-free chordal graph G the equality $\gamma_k(G) = \rho_{2k}(G)$ holds. Using this results, we obtain the following characterization.

Theorem 2. *The following statements are equivalent for a sun-free chordal graph G:*
1) $\gamma_k(G) = \rho_k(G)$;
2) $G \in \mathcal{R}_k$.

Corollary 2. *The problem of recognizing sun-free chordal graphs G having $\gamma_k(G) = \rho_k(G)$ is polynomially solvable.*

Using the characterization of the graphs with equal k-packing and $2k$-packing numbers from [3], we obtain the following.

Theorem 3. *Every graph in $\mathcal{R}_k$ is k-equipackable.*

Thus, all the sun-free chordal graphs with $\gamma_k(G) = \rho_k(G)$ are k-equipackable. It is an open question whether there are any other k-equipackable sun-free chordal graphs.

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