ON DOUBLY CHORDAL GRAPHS

M. Talmaciu[1], **V.V. Lepin**[2]

[1] Vasile Alecsandri University of Bacau, România,
Department of Mathematics, Informatics and Education Sciences
mtalmaciu@ub.ro
[2] Institute of Mathematics, National Academy of Sciences of Belarus
11 Surganov str., 220072 Minsk, Belarus lepin@im.bas-net.by

The triangulated graphs (chordal) class has been noticed because of their properties. Among these properties we mention: perfection, recognition algorithms and ability to solve some combinatorial optimization problems (determining the stability number and minimum number of covering cliques) with linear complexity algorithms. Because of this, various ways of generalizing this notion were introduced.

Interest for strongly chordal (M. Farber [45], see [3], [5]. A graph G is strongly chordal if and only if every induced subgraph of G has a simple vertex. A vertex v of the graph G is simple in G if the set $\{N[u] : u \in N[v]\}$ is linearly ordered by inclusion.) graphs arises in several ways. The problems of locating minimum weight dominating sets and minimum weight independent dominating sets in strongly chordal graphs with real vertex weights can be solved in polynomial time, whereas each of these problems is NP-hard for chordal graphs.

Graphs with maximum neighborhood orderings were characterized and turned out to be algorithmically useful. These graphs are dual (in the sense of hypergraphs) to chordal graphs [1]. The graph G is *dually chordal* [1] iff G has a maximum neighborhood ordering. In [9] specifies that the doubly chordal graphs holds: clique problem can be solved in polynomial time, independent set in linear time, the recognition problem in linear time.

Many problems efficiently solvable for strongly chordal and doubly chordal graphs remain efficiently solvable for dually chordal graphs too [2]. A. Brandstadt, V. Chepoi, F. Dragan, in [2] gives an algorithm for solving the connected r-domination and Steiner tree problem in linear time on doubly chordal graphs and in quadratic time on dually chordal graphs.

We say that v is simplicial in G if $N[v]$ is complete. A vertex $u \in N[v]$ is a *maximum neighbor* of v if for all $w \in N[v]$ the inclusion $N[w] \subseteq N[u]$ holds (note that $u = v$ is not excluded). A vertex v is *doubly simplicial* if it is simplicial and has a maximum neighbor. A graph is *doubly chordal* if it admits a *doubly perfect elimination ordering* $v_1, v_2, ..., v_n$ of vertices such that for each $1 \le i \le n$, v_i is doubly simplicial in the subgraph induced by $\{v_i, ..., v_n\}$.

A set $A \subset V(G)$ is called a *weak set* of the graph G if $N_G(A) \neq V(G) - A$ and $G[A]$ is connected. If A is a weak set, maximal with respect to set inclusion, then $G[A]$ is called a weak component.

Let $G = (V, E)$ be a connected and non-complete graph. If A is a weak set, then the partition $\{A, N(A), V \setminus A \setminus N(A)\}$ is called a *weak decomposition* of G with respect to A.

A graph G is *hereditary doubly chordal* if any induced subgraph of G is doubly chordal. A new characterization of hereditary doubly chordal graphs, using weakly decomposition, is given below.

Theorem. *Let $G = (V, E)$ be a connected and non-complete graph, $G[A]$ is a weak component. The graph G is hereditary doubly chordal if and only if the following hold: (1) $N(A)$ is clique; (2) $G - A - N(A)$, $G(A \cup N(A))$ are hereditary doubly chordal graphs.*

The above results lead to a recognition algorithm for hereditary doubly chordal graphs in $O(n(n + m))$ time.

Corollary. *Let $G = (V, E)$ be a connected and non-complete graph with $G(A)$ a weak component in G. If G is hereditary doubly chordal then*

$$\alpha(G) = \max\{\alpha(G(A)) + \alpha(R), \alpha(G(A \cup N(A)))\};$$

$$\omega(G) = \max\{|N| + \omega(R), \omega(G(A \cup N(A)))\},$$

where $R = G - A - N(A)$.

The Corollary implies an algorithm for the construction of a stable set of maximum cardinal and a clique of maximum cardinal in a hereditary doubly chordal graph in $O(n(n+m))$ time.

Литература

1. Brandstadt A., Dragan F., Chepoi V., Voloshin V. *Dually chordal graphs* // SIAM J.Discrete Math. 1998. Vol. 11, P. 437–455.

2. Brandstadt A., Chepoi V., Dragan F. *The algorithmic use of hypertree structure and maximum neighbourhood orderings* // Discrete Applied Mathematics 1998. Vol. 82, P. 43–77.

3. Dahlhaus E., Manuel P.D., Miller M. *A characterization of strongly chordal graphs*, // Discrete Mathematics 1998. Vol. 187, P. 269–271.

4. Farber M., *Characterizations of Strongly Chordal Graphs* // Discrete Mathematics 1983. Vol. 43, P. 173–189.

5. McKee T. A., *A new characterization of strongly chordal graphs* // Discrete Mathematics 1999. Vol. 205, P. 245–247.

6. Moscarini M., *Doubly chordal graphs. Steiner trees and connected domination* // Network 1993. Vol. 2.3, P. 59–69.

7. Talmaciu M., Croitoru C. *Structural Graph Search* // Stud.Cercet.Stiint., Ser.Mat., 16 (2006), Supplement, Proceedings of ICMI 45, Bacau, Sept.18-20, 2006, P. 573–588

8. Talmaciu M., *Fast algorithms of dually chordal graphs* // Scientific Studies and Research, Series Mathematics and Informatics, 2015. Vol. 25, No. 1, P. 77–86.

9. http://www.graphclasses.org/classes/gc_181.html